Binding energy, mass defect of deuteron, some related phenomena and generalized mass energy equation

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Abstract

There are two inherent observations; firstly masses of nucleons are fundamental constants i.e. same inside and outside the nucleus in all cases and secondly nuclei possess BE ($\Delta mc^2$) due to mass defect. If the observations of deuteron (BE= 2.2244 MeV) are explained on the basis of $\Delta E = \Delta mc^2$, then difference in masses of nucleons must be $2.388 \times 10^{-3}$ u or about 0.11854% of masses of nucleons outside nucleus. On the basis of $\Delta E = \Delta mc^2$, theoretically masses of nucleons must be less in nucleus, which is not justified as masses of nucleons are fundamental physical constants. If the applications of the generalized equation $\Delta E = A \Delta m$ are speculated in this regard, then it is capable of explaining both the observations simultaneously i.e. equality of masses of nucleons (assuming infinitesimally small mass defect) and binding energy. As according to $\Delta E = A \Delta m$ (used even on ad hoc basis) even due to infinitesimally small mass defect ($2.388 \times 10^{-13}$ u, say) the binding energy of deuteron can be 2.2244 MeV due to presence of conversion factor A. The value of A other than unity can also be justified in total kinetic energy of fission fragments of U$^{235}$ or Pu$^{239}$. Till date $\Delta E = \Delta mc^2$ is not confirmed in chemical reactions. Till date $\Delta E = \Delta mc^2$ is not confirmed in chemical reactions but regarded as true which is not justified.
1.0 Decrease in masses of proton and neutron in deuteron to explain Binding Energy.

In the experimental and theoretical nuclear physics the masses of nucleons (protons and neutrons) are fundamental physical constants in category of atomic and nuclear constants, and binding energy (energy required to break the nucleus) is an inherent property of all nuclei. The mass energy inter conversions are universally explained on the basis of $E=\Delta mc^2$, where $\Delta m$ is mass defect.

**Mass defect** = Mass of nucleons out side nucleus – Mass of nucleons inside nucleus  \hspace{1cm} (1)

**Binding energy** = $\Delta mc^2$  \hspace{1cm} (2)

The mass defect and BE in terms of mass of atom

\[ \Delta m = \left\{ \left[ Z(m_p + m_e) + (A-Z)m_n \right] - M_{\text{atom}} \right\} \]  \hspace{1cm} (3)

with all terms usual meanings.

\[ \text{BE} = \left\{ \left[ Z(m_p + m_e) + (A-Z)m_n \right] - M_{\text{atom}} \right\} c^2 \]  \hspace{1cm} (4)

This aspect is critically discussed in view of **deuteron**, which contains just one neutron and proton.

(a) The mass of proton is experimentally measured equal to $1.672621 \times 10^{-27}$ kg, (1.007276 u or 938.272029 MeV) and is same in all cases. Also the mass of neutron is $1.674927 \times 10^{-27}$ kg (1.008664 u or 939.565360 MeV)

(b) Experimentally binding energy (BE) of deuteron is measured by various methods [1-3] has been found to be 2.2244 MeV (1amu = 931.494 MeV, 1amu = 1.660 5381 × 10⁻²⁷ kg), which is equivalent to 0.002388 u (3.984 × 10⁻³⁰ kg) on the basis of $\Delta E = \Delta mc^2$.

The expected mass of $^1\text{H}_2$ atom is 2.0165 u which is sum of mass of $^1\text{H}_1$ atom (1.0078 u) plus mass of neutron (1.0087 u). However the measured mass of $^1\text{H}_2$ is 2.0141 u. Thus it is the mass defect (2.0165 u – 2.0141 u = 0.0024 u) which is converted to binding energy, which is law of conservation of matter.

2.0 $\Delta E=\Delta mc^2$ implies mass of nucleons decreases in nucleus, which is not justified.

The binding energy of the deuteron is experimentally [1-3] observed as 2.2244 MeV, according to $\Delta E = \Delta mc^2$ it is equal to mass defect 0.002388 u. It means in the nucleus of deuterium, mass 0.002388 u (of proton and neutron) is converted into binding energy.

The mass defect i.e. 0.002388 u is comparable with sum of masses of the
neutron and proton (2.01594 u), the masses must decrease in nucleus considerably i.e. 0.11845 % (compared to mass in free state). In deuteron there are only proton and neutron, hence theoretically decrease in mass or mass defect 0.002388u is only at the cost of mass of proton \(M_p\) and mass of neutron \(M_n\).

The mass of proton is 1.007276 u and let decrease in mass of proton is half the mass defect (0.002388 u) i.e. 0.001194 u (which contributes towards the binding energy of deuterium). Then theoretically mass of proton in nucleus must be 1.006082 u \((1.67009 \times 10^{-27} \text{ kg})\) and then decrease in mass of proton must be 0.1185 %. Also mass of neutron is 1.008664 u and let decrease in mass of neutron is half the mass defect i.e. 0.001194 u. Then mass of neutron in nucleus must be 1.00747 u \((1.6724 \times 10^{-27} \text{ kg})\). Similarly the decrease in mass of neutron in nucleus is 0.1185%. These estimates are based upon \(\Delta E = \Delta mc^2\).

These processes are simply law of conservation of matter. According to \(E = \Delta mc^2\), the conversion factor between mass and energy is precisely \(c^2\), and in \(\Delta E = Ac^2 \Delta m\) conversion factor is \(Ac^2\), however, no such conversion factor is defined when law of conservation of mass or energy was enunciated.

### 3.0 Explanation on the basis of \(\Delta E = Ac^2 \Delta m\)

Einstein [9] initially derived light energy mass equation, \(\Delta L = \Delta mc^2\), then speculated from it \(\Delta E = \Delta mc^2\) (replacing \(L\) by \(E\)) for all energies (sound energy, heat energy, chemical energy, nuclear energy, magnetic energy, electrical energy, energy emitted in form of invisible radiations, energy emitted in cosmological and astrophysical phenomena, energy emitted in volcanic reactions, energies co-existing in various forms etc. etc.) without derivation and any mathematical or conceptual proof.

If Einstein’s derivation is further analyzed, then it is obvious that \(\Delta L = \Delta mc^2\) is derived under SPECIAL or IDEAL conditions of parameters involved (e.g. when body emits two light waves, energies of both the waves are equal, waves are emitted in exactly opposite directions i.e. 0° and 180°). But the law of inter-conversion of mass-energy holds good under all experimental conditions in weird reactions, hence constraints on the derivation are not justified.

But under general conditions (e.g. body emits two light waves of slightly different energies, the angles are different from 0° and 180° and body may emit more than two waves for different energies at different angles) then Einstein’s derivation also gives \(\Delta L \propto \Delta mc^2\).
Thus under general conditions author [4-7, 10-12] has deduced generalized mass energy equation as
\[ \Delta E = A c^2 \Delta m \] 
(5)
where A is coefficient of proportionality. Like many other co-efficients of proportionality in the existing literature the value of A depends upon the inherent characteristic conditions of the process. It can be equal, less or more than one. This perception of coefficient of proportionality is existing since days of Aristotle in physics/science. Thus according to \( \Delta E = A c^2 \Delta m \) like Einstein's equation mass is converted to energy but unlike Einstein's equation conversion factor is not always \( c^2 \). Thus according to \( \Delta E = A c^2 \Delta m \), the energy emitted can be less, equal or more than \( \Delta E = \Delta mc^2 \), as inter-conversion of mass and energy are bizarre processes right from chemical reactions to heavenly phenomena governing origin and development of universe. As far as nuclear reactions are concerned \( \Delta E = A c^2 \Delta m \) is justified with value of A as unity due to inherent characteristic conditions of the process.

4.0 \( \Delta E = A c^2 \Delta m \) implies infinitesimally small mass defect (equality of masses of nucleons) can give binding energy 2.2244MeV.

Let us consider the followings.
(i) The masses of proton and neutron are same both inside and outside the nucleus; it is universal equality of masses of nucleons which are fundamental constants in the category of atomic and nuclear constants. Thus

\[ \text{Mass of neutron in free state}(m_n) = \text{Mass of neutron inside nucleus}(m_n) \]

or

\[ \text{Mass of proton in free state}(m_p) = \text{Mass of proton inside nucleus}(m_p) \]

(ii) Universal equality of masses of nucleons implies zero mass defect (\( \Delta m=0 \)), hence zero binding energy on the basis of \( \Delta E = \Delta mc^2 \). Thus

\[ \Delta E = \Delta mc^2 = 0, c^2 = 0 \]

which implies the instability of the deuteron.

In this perception the universal equality of masses of nucleons can be understood as 'mass defect is infinitesimally small' or imperceptible. If mass defect is infinitesimally small then masses of neutron and proton are virtually equal as differences in masses are imperceptible. Thus this speculation or insight is consistent with universal equality of masses of nucleons. When this perception or speculation of 'infinitesimally small mass difference' is used in the generalized form of mass energy equation \( \Delta E = A c^2 \Delta m \), then it can explain BE of deuteron as well.
Let us perceive that the mass defect in this regard is $2.388 \times 10^{-13}$ u or $3.965 \times 10^{-40}$ kg (or may be even less), then masses of nucleons inside and outside the nucleus are equal as difference is too less to be measured. Now in this case masses of nucleons must decrease in nucleus by infinitesimally small amount i.e. $1.18455 \times 10^{-11}$% (or may be even less). Now the mass of proton ($1.007276$u) inside the nucleus will be $1.0072759999998806$u and in previous case (based upon $\Delta E = \Delta m c^2$) it was $1.006082$u. Similarly mass of neutron ($1.008662$u) inside the nucleus will be $1.0086619999998806$u and in previous case (based upon $\Delta E = \Delta m c^2$) it was $1.00747$u. Thus mass of proton and neutron will decrease in nucleus by insignificant amount, such that universal equality of masses of nucleons is obeyed. In this case mass defect is insignificant even then binding energy (2.2244MeV) can be explained with $\Delta E = A c^2 \Delta m$ (value of conversion coefficient $A$ is very high). Thus nucleus is stable.

According to the generalized form of mass-energy inter-conversion equation i.e. $\Delta E = A c^2 \Delta m$, for annihilation of infinitesimally small mass defect, large amount of energy can be emitted. It means conversion factor is much higher than $c^2$. Now applying equation $\Delta E = A c^2 \Delta m$ under these conditions ($BE= 2.2244$MeV = $3.579 \times 10^{-13}$ J, $\Delta m = 2.388 \times 10^{-13}$ u = $3.965 \times 10^{-40}$ kg), the value of $A$ can be determined as

$$A = \frac{\Delta E}{c^2 \Delta m} = \frac{[3.5634 \times 10^{-13}]}{[8.987524 \times 10^{16} \times 3.965 \times 10^{-40}]}$$

$$3.5634 \times 10^{10} / 3.5634 = 10^{10}$$

Thus in this case we have

$$\Delta E = A c^2 \Delta m = 10^{10} c^2 \Delta m$$

Thus according to generalized mass energy inter conversion equation, corresponding to small mass defect, large amount of energy is emitted in this case. The value of conversion co-efficient (‘$A$’) have resemblance with co-efficients of proportionality in C. F. von Weizsacker’s semi-empirical formula [8] for binding energy (associated with Bohr’s Liquid Drop Model).

Thus generalized equation explains both intrigues e.g. universal equality of masses of nucleons (due to imperceptible mass defect) and binding energy of nucleus 2.2244MeV (due to high value of conversion co-efficient). According to both equations i.e. $\Delta E = A c^2 \Delta m$ and $\Delta E = \Delta m c^2$ the mass is converted to energy or vice-versa, but only the difference is in magnitudes of conversion factors. The universal equality of masses of nucleons on the basis of both the equations is shown on the Table I.
4.1 The total kinetic energy (TKE) of fission fragments of U\textsuperscript{235} or Pu\textsuperscript{239} is 20-60 MeV less than predicted by \( E = \Delta mc^2 \).

The familiar fission reaction is

\[
\begin{align*}
\text{\textsuperscript{92}U}\text{\textsuperscript{235}} + \text{\textsuperscript{0}n}\text{\textsuperscript{1}} & \rightarrow \text{\textsuperscript{56}Ba}\text{\textsuperscript{141}} + \text{\textsuperscript{36}Kr}\text{\textsuperscript{92}} + 3 \text{\textsuperscript{0}n}\text{\textsuperscript{1}} + Q \text{ (202 MeV )}
\end{align*}
\]

In laboratory [13-15] it has been experimentally confirmed that using thermal neutrons the total kinetic energy of fission fragments that result from U\textsuperscript{235} or Pu\textsuperscript{239} is 20-60 MeV less than the Q value of the reaction predicted by \( E = \Delta mc^2 \). These observations are over 3 decades old. In existing physics these inconsistent observations are explained in the following ways.

(i) It is typically assumed that energy is lost in unobservable effects [13]. If so then such unobservable effects may also be applicable to those cases where \( E = \Delta mc^2 \) is regarded as to hold good.

(ii) Also attempts have been made to explain the total kinetic energy (or essentially total energy) of fission fragments by extending the successful liquid-drop model of Bohr and Wheeler [13, 16-17]. It implies the gravity of inadequacy of \( E = \Delta mc^2 \) in this case.

The same can also be explained on the basis of \( \Delta E = Ac^2 \Delta m \). Let according to \( E = \Delta mc^2 \) the total kinetic energy (or essentially total energy) of fission fragments of U\textsuperscript{235} or Pu\textsuperscript{239} is 200 MeV theoretically and experimentally observed energy is 25 MeV less i.e. 175 MeV. Then according to \( \Delta E = Ac^2 \Delta m \) the value of \( A \) is 0.875 i.e.

\[
A = \frac{\Delta E}{c^2 \Delta m}
A = 175 / 200 = 0.875
\]

So in case of fission fragments of U\textsuperscript{235} or Pu\textsuperscript{239} the value of \( A \) is less than one i.e. 0.875 as in eq.(8), hence the energy emitted is less than predicted by \( E = \Delta mc^2 \). In this case the value of \( A \) other than unity is justified.

4.2 Less efficiency of nuclear weapon

The efficiency of the nuclear weapons as well as nuclear reactors is far less than the theoretical value predicted by \( E=\Delta mc^2 \). It supports the observation that energy emitted in fission fragments of U\textsuperscript{235} or Pu\textsuperscript{239} is found 20-60 MeV less than Q value predicted by \( E=\Delta mc^2 \).

Robert Serber (member of first American team entered Hiroshima and Nagasaki in September 1945 to assess losses), has quoted [18] that the efficiency of “Little Boy” weapon [U\textsuperscript{235}, 49kg] that was used against Hiroshima was about 2% only. It is assumed that all the atoms don’t undergo fission, thus material is wasted. But no such waste material is specifically
measured quantitatively. Thus the waste material (in nuclear reactor or weapon) must be measured and corresponding energy be calculated, and it must quantitatively explain that why efficiency is less. It may require the measurements of all types of energies (may co-exist in various forms) in the processes and experimental errors. Until such experiments are specifically conducted and $E=\Delta mc^2$ is confirmed, $\Delta E \propto \Delta mc^2$ is equally feasible. In addition if some observations in future LHC experiments are found inconsistent with $E=\Delta mc^2$ then these can be explained with $\Delta E = Ac^2 \Delta m$.

4.3 $E=\Delta mc^2$ is unconfirmed chemical reactions.

When Einstein derived $E=\Delta mc^2$, chemical reactions were the most abundant sources of energy in nature. Till date $E=\Delta mc^2$ is not confirmed in the chemical reaction and the reason cited for this is that equipments are not enough sensitive [8, 19] so that it may be confirmed. However it is regarded as true in such cases without experiments which is not justified.

Consider burning of 5 kg straw or paper in controlled and sophisticated way i.e. in such a way that masses, ashes, gases and energy produced can be quantitatively measured. Thus mass annihilated will be measured along with energy emitted. Even if $10^{-6}$ gm or $10^{-3}$ mgm of matter is annihilated then energy equal to $9 \times 10^7$ J will be emitted i.e.

$$E=\Delta mc^2 = 10^{-9} \text{ kg} \times 9 \times 10^{16} \text{ m}^2/\text{s}^2 = 9 \times 10^7 \text{ J}$$

This energy i.e. $9 \times 10^7$ J can push a body of mass 1,000 kg to distance of $9 \times 10^4$ m or 90 km. It does not appear true in daily life observations in combustion of large amount of mass; hence specific and quantitative experiments are required in this regard. In nuclear physics masses smaller than $(10^{-9} \text{ kg})$ this are measured. Until $E=\Delta mc^2$ is not confirmed in chemical reactions, then scientifically $E=\Delta mc^2$ may not be regarded as quantitatively true in such cases.

The specific and quantitative experiments may take the results either in favor of $E=\Delta mc^2$ or $\Delta E = Ac^2 \Delta m$. It is equally possible that energy emitted may be less than predicted by $E=\Delta mc^2$ i.e. $E \propto \Delta mc^2$ or $\Delta E = Ac^2 \Delta m$ is feasible; it is an open possibility at present when there are no specific and quantitative experiments. This aspect will be crystal clear if specific experiments involving chemical reactions are conducted, energy emitted and mass annihilated in the process can be critically analyzed in view of $E=\Delta mc^2$ or $\Delta E = Ac^2 \Delta m$.

4.4 Conclusions

Thus it is concluded that value of conversion coefficient $A$ other than unity is justified in binding energy of deuteron, universal equality of masses of nucleons; and also in case of total kinetic energy of fission fragments of $^{235}U$ or $^{239}Pu$ which is found 20-60MeV less than
Q value predicted by $E = \Delta mc^2$. In nuclear weapons the efficiency of energy emitted is quite less i.e. nearly 2% only, further no specific experiments are conducted to understand such phenomena quantitatively. In case of the most abundant chemical reactions, $E = \Delta mc^2$ is not confirmed yet in specific experiments. Thus to understand $E = \Delta mc^2$ completely, the efficiency of nuclear weapon and energy emitted in chemical reactions must be quantitatively checked in specific experiments. These experiments will also confirm whether value of $A$ is unity or not in such cases.

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References
Table 1  Comparison of universal equality of masses of nucleons on the basis of \( \Delta E = \Delta mc^2 \) and \( \Delta E = A c^2 \Delta m \) in deuteron.

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Characteristics of ( _{1}H^2 )</th>
<th>On basis of ( E = \Delta mc^2 )</th>
<th>On basis of ( \Delta E = A c^2 \Delta m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Binding Energy (MeV)</td>
<td>2.2244</td>
<td>2.2244</td>
</tr>
<tr>
<td>2</td>
<td>Mass defect ((BE/c^2)) in amu</td>
<td>(2.388 \times 10^{-3})</td>
<td>(2.388 \times 10^{-13})</td>
</tr>
<tr>
<td>3</td>
<td>Decrease in mass per nucleon ((amu))</td>
<td>(1.194 \times 10^{-3})</td>
<td>(1.194 \times 10^{-13}) or (1.982 \times 10^{-40} \text{ kg})</td>
</tr>
<tr>
<td>4</td>
<td>(M_p) and (M_n) in nucleus</td>
<td>(M_p = 1.006082) (M_n = 1.00747)</td>
<td>Virtually same</td>
</tr>
<tr>
<td>5</td>
<td>%age Decrease in (M_p)</td>
<td>0.1185</td>
<td>1.185 \times 10^{-11}</td>
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<tr>
<td>6</td>
<td>%age Decrease in (M_n)</td>
<td>0.1183</td>
<td>1.183 \times 10^{-11}</td>
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<tr>
<td>7</td>
<td>Universal equality of masses of (M_p) and (M_n)</td>
<td>Not obeyed</td>
<td>Obeyed</td>
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