On the Time-Reversal Invariance of the Fundamental Commutation Relation

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We check the time-reversal invariance of the fundamental commutation relation $[p, x] = -i\hbar$. Although it has been checked in the active picture, where the transformation affects the states but not the operators, it is a puzzling issue in the passive picture, where the transformation affects the operators but not the states. It is puzzling because $[p', x'] = [-p, x] = i\hbar$ in the passive picture. At first sight, it seems that the fundamental commutation relation is not time-reversal invariant. In fact, in both older and more recent textbooks on quantum mechanics, the time-reversal invariance of the fundamental commutation relation has never been explicitly clarified in the passive picture. We point out that the claims in standard textbooks concerning the time-reversal invariance of the fundamental commutation relation in the passive picture are misleading. We show that $[p, x]' = [p', x'] = [-p, x] = i\hbar$, if the time-reversal operator is unitary. Hence the fundamental commutation relation is not time-reversal invariant if the time-reversal operator is unitary. On the other hand, $[p, x]' = [x', p'] = [x, -p] = -i\hbar$, if the time-reversal operator is antiunitary. We conclude that the fundamental commutation relation is time-reversal invariant, provided that the time-reversal operator is antiunitary. The important fact $[p, x]' = [x', p']$, which is the origin of the difficulties, is not appreciated in standard textbooks on quantum mechanics. The present discussion provides a more fundamental justification for taking the time-reversal operator to be antiunitary, which applies to particles with arbitrary spin.

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I. INTRODUCTION

At the core of quantum mechanics lies the fundamental commutation relation

$$[p, x] = -i\hbar \,. \tag{1}$$

Relation (1) follows from the Einstein–de Broglie relations $E = h\nu$, $p = \frac{h}{\lambda}$, which are an expression of the wave-particle duality. Many puzzling features of quantum behavior result from the fundamental commutation relation (1). It is the fundamental commutation relation (1) that stands behind the Heisenberg uncertainty relation $\Delta p \Delta x \geq \frac{h}{4\pi}$. As pointed out by Dirac [1], relation (1) gives the solution to the problem of finding the quantum conditions for all those dynamical systems which have a classical analogue. For this reason, Dirac called the relation (1) "the fundamental quantum condition".

As a cornerstone of quantum mechanics, the fundamental commutation relation must be invariant under the Lorentz transformations. The aim of this paper is to check the

