Preparation of Motional Two-Mode Squeezed Pair Coherent States in an Anisotropic Trap

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We propose a scheme for generating motional two-mode squeezed pair coherent states of an ion confined in a three-dimensional anisotropic trap. In the scheme the ion is multichromatically excited by three lasers in the x-y plane, and one in the z axis. Under certain conditions, the system finally reaches a steady state, in which the vibrational motions in the x and y axes are in the two-mode squeezed pair coherent state.

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Recent advances in laser cooling and ion trapping have opened a new prospect for quantum state engineering. When an ion is confined in an electromagnetic trap it can be regarded as a particle with quantized center-of -mass motion. One can manipulate the vibrational motion by driving the ion with laser beams due to the momentum exchange between the ion and the lasers. Due to the extremely weak coupling between the vibrational modes and the external environment, it is possible to realize various nonclassical states of the vibrational motion of a trapped ion by controlling the driving fields appropriately. Over the past few years, schemes have been proposed for the generation of various nonclassical states for one-dimensional motion of a trapped ion, such as Fock states [1], squeezed states [2], Schrödinger cat states [3-7], and superpositions of squeezed states [8]. Experimental realizations of motional Fock states, squeezed states [9], and Schrödinger cat states [10] have been reported.

Recently, schemes have been presented for the preparation of two-mode motional states of a trapped ion, such as pair coherent states [11], pair cat states [12], two-mode SU(1,1) intelligent states [13], and SU(2) cat states [14]. These schemes operate in two-dimensional (2D) isotropic traps. We have proposed a scheme for the generation of motional two-mode squeezed pair coherent states in a three-dimensional (3D) isotropic ion trap [15]. In the scheme the ion is multichromatically excited by seven lasers. In this paper we propose a scheme for the generation of two-mode squeezed pair coherent states in a 3D anisotropic trap. The present scheme only requires four lasers and thus it is much more feasible experimentally than the previous one [15].

The pair coherent states [16] are defined as eigenstates of both the pair annihilation operator $\hat{a}\hat{b}$ and the number difference operator $\hat{\Delta} = \hat{a}^+\hat{a} - \hat{b}^+\hat{b}$, i.e.,

$$\hat{a}\hat{b}|\xi, q\rangle = \xi|\xi, q\rangle, \tag{1}$$

and

$$\hat{Q}|\xi;q\rangle = q|\xi;q\rangle, \tag{2}$$

where ξ is a complex number and q is a fixed integer. Without loss of generality, q is taken positive and then the pair coherent states can be expanded in terms of the two-mode Fock states

$$|\xi;q\rangle = N_q \sum_{n=0}^{\infty} \frac{\xi^n}{\sqrt{n!(n+q)!}}|n+q;n\rangle, \tag{3}$$

where N_q is a normalization factor

$$N_q = \left[\sum_{n=0}^{\infty} \frac{|\xi|^{2n}}{n!(n+q)!}\right]^{-1/2}. \tag{4}$$

Such states have nonclassical features, such as sub-Poissonian statistics, correlated photon number fluctuations, squeezing and a violation of the Cauchy-Schwarz inequalities.

Recently, Gerry [17] have studied the two-mode squeezed pair coherent states which are generated by the application of the two-mode squeeze operator to the pair coherent states, i.e.,

$$|(r,\theta);\xi;q\rangle = \hat{S}(r,\theta)|\xi;q\rangle, \tag{5}$$

where $S(r,\theta)$ is the two-mode squeeze operator

$$\hat{S}(r,\theta) = \exp\left\{r\left[\hat{a}\hat{b}e^{-2i\theta} - \hat{a}^+\hat{b}^+e^{2i\theta}\right]\right\}. \tag{6}$$

Applying such an operator to both sides of Eq. (1) we obtain

$$(\hat{a}\cosh r + \hat{b}^+e^{2i\theta}\sinh r)(\hat{b}\cosh r + \hat{a}^+e^{2i\theta}\sinh r)\,|(r,\theta);\xi;q\rangle = \xi|(r,\theta);\xi;q\rangle. \tag{7}$$

We can also rewrite Eq. (7) in the form of the eigenvalue equation

$$\begin{aligned}&\left[\hat{a}\hat{b} + \hat{a}^+\hat{b}^+e^{4i\theta}\tanh^2 r + (\hat{a}^+\hat{a} + \hat{b}^+\hat{b})\tanh r e^{2i\theta}\right]|(r,\theta);\xi;q\rangle \\ &\quad = \left(\frac{\xi}{\cosh^2 r} - \tanh r e^{2i\theta}\right)|(r,\theta);\xi;q\rangle.\end{aligned} \tag{8}$$

The two-mode squeezed pair coherent state $|(r,\theta);\xi;q\rangle$ is also the eigenstate of the number difference operator $\hat{Q}$ with eigenvalue q. It has been shown that the squeezed pair coherent states may exhibit stronger nonclassical properties than pair coherent states by the application of the two-mode squeeze operator [17]. These include the enhancement of the sub-Poissonian statistics and a violation of the Cauchy-Schwartz inequality over some ranges of r and the enhancement of squeezing. Due to the strong nonclassical nature of the two-mode squeezed pair coherent states, the generation of such states is of interest in testing quantum mechanics.

We consider an ion trapped in a three-dimensional (3D) anisotropic harmonic potential. The ion is multichromatically excited by four laser beams, with three propagating in the x-y plane and the other one along the z axis.

$$\hat H = \nu_x \hat a^+ \hat a + \nu_y \hat b^+ \hat b + \nu_z \hat c^+ \hat c) + \omega_0 \hat S_z + [D E^- (\hat x, \hat y, \hat z, t)\hat S^- + H.c.], \tag{9}$$

where $\hat a$, $\hat b$, and $\hat c$ are the annihilation operators for the vibrational motions along the X, Y, and Z axes with frequencies ν_x, ν_y, and ν_z, respectively, S_z and $S^\pm$ are the electronic flip operators for the two-level ion with transition frequency ω_0 and dipole moment D. $E^-(x, y, z, t)$ is the negative frequency part of the driving fields

$$\begin{aligned} E^-(\hat x, \hat y, t) = {} & E_1 e^{i(\omega_1 t - k_{1x}\hat x - k_{1y}\hat x + \phi_1)} + E_2 e^{i(\omega_2 t - k_{2x}\hat x - k_{2y}\hat x + \phi_2)} \\ & + E_3 e^{i(\omega_3 t - k_{3x}\hat x - k_{3y}\hat x + \phi_3)} + E_1 e^{i(\omega_4 t - k_4 \hat z + \phi_1)}, \end{aligned} \tag{10}$$

where ω_j, E_j, and ϕ_j $(j = 1, 2)$ are the frequency, amplitude, and phase of the jth driving field. k_{ju} $(u = x, y)$ is the projection of the wave-vector for the jth laser beam on the u axis. The position operators $\hat x$ and $\hat y$ can be expressed by $\hat x = \sqrt{1/(2\nu_x M)}(\hat a + \hat a^+)$, $\hat y = \sqrt{1/(2\nu_y M)}(\hat b + \hat b^+)$, and $\hat z = \sqrt{1/(2\nu_z M)}(\hat c + \hat c^+)$ with M being the mass of the trapped ion.

Choose $\omega_1 = \omega_0 - \nu_x - \nu_y$, $\omega_2 = \omega_0 + \nu_x + \nu_y$, $\omega_3 = \omega_4 = \omega_0$ and assume the trapping frequencies ν_x, ν_y, and ν_z are much larger than the other characteristic frequencies. Then, in the interaction picture, the Hamiltonian can be described by [3, 18, 19]

$$\begin{aligned} \hat H_i = \sum_{m,n=0}^{\infty} \Big[& e^{-(\eta_{1x}^2 + \eta_{1y}^2)/2} D E_1 e^{i\phi_1} \frac{(i\eta_{1x})^{2m+1}}{(m+1)!(m!)} \frac{(i\eta_{1y})^{2n+1}}{(n+1)!(n!)} \hat a^{+(m+1)} \hat a^m \hat b^{+(n+1)} \hat b^n \\ & + e^{-(\eta_{2x}^2 + \eta_{2y}^2)/2} D E_2 e^{i\phi_2} \frac{(i\eta_{2x})^{2m+1}}{(m+1)!(m!)} \frac{(i\eta_{2y})^{2n+1}}{(n+1)!(n!)} \hat a^{+(m+1)} \hat a^m \hat b^{+(n+1)} \hat b^n \\ & + e^{-(\eta_{3x}^2 + \eta_{3y}^2)/2} D E_3 e^{i\phi_3} \frac{(i\eta_{3x})^{2m}}{(m!)^2} \frac{(i\eta_{3y})^{2n}}{(n!)^2} \hat a^{+m} \hat a^m \hat b^{+n} \hat b^n \\ & + e^{-\eta_4^2/2} D E_4 e^{i\phi_4} \frac{(i\eta_4)^{2m}}{(m!)^2} \hat c\, \hat c^{+m} \hat c^m \Big] \hat S^- + H.c., \end{aligned} \tag{11}$$

where $\eta_{ju} = \sqrt{k_{ju}^2/(2M\nu_u)}$ $(j$, 1, 2, 3; $u = x, y)$ and $\eta_4 = \sqrt{k_4^2/(2M\nu_z)}$ are the respective Lamb-Dicke parameters. The damping of the vibrational motion is so weak that it can be disregarded and thus the electronic damping is the main decay process. Then the evolution of the whole system is described by the master equation for the master operator ρ [3, 8, 11-13, 15]:

$$\frac{d\rho}{dt} = -i\left[\hat H_i, \rho\right] + \frac{\Gamma}{2}\left[2\hat S^- \rho' \hat S^+ - \hat S^+ \hat S^- \rho - \rho \hat S^+ \hat S^-\right], \tag{12}$$

where Γ is the spontaneous decay rate of the excited state of the ion, and

$$\rho' = \frac{1}{8}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} dr\,ds\,dt\,W(r, s, t) e^{ik(r\hat x + s\hat y + t\hat z)} \rho e^{-ik(r\hat x + s\hat y + t\hat y)} \tag{13}$$

accounts for the changes of the vibrational energy due to spontaneous emission. $W(r,s,t)$ is the angular distribution of spontaneous emission. We here have assumed $(k_{jx}^2 + k_{jy}^2)^{1/2} \simeq k_4 \simeq k$.

In the Lamb-Dicke limit $\eta_{ju} \ll 1$ and $\eta_4 \ll 1$ the master equation (7) can be well approximated by the expansion of η_{ju} and η_4 to the second order. and ρ' can be replaced by ρ. Furthermore, small Lamb-Dicke parameters lead to $e^{-(\eta_{jx}^2+\eta_{jy}^2)/2} \simeq e^{-\eta_4^2/2} \simeq 1$. Thus we have

$$\frac{d\rho}{dt} = -i\left[\hat{H}'_I, \rho\right] + \frac{\Gamma}{2}[2\hat{S}^-\rho\hat{S}^+ - \hat{S}^+\hat{S}^-\rho - \rho\hat{S}^+\hat{S}^-], \tag{14}$$

where

$$\hat{H}'_I = [-g_1\hat{a}\hat{b} - g_2\hat{a}^+\hat{b}^+ - g_3(\hat{a}^+\hat{a} + \hat{b}^+\hat{b}) - g_4\hat{c}^+\hat{c} + h]\hat{S}^+ + H.c., \tag{15}$$

where

$$g_j = DE_j e^{-i\phi_j}\eta_{jx}\eta_{jy}, \quad j = 1,2 \tag{16}$$

$$g_3 = DE_3 e^{-i\phi_3}\eta_{3x}^2, \tag{17}$$

$$g_4 = DE_4 e^{-i\phi_4}\eta_{4z}^2, \tag{18}$$

$$h = DE_3 e^{-i\phi_3} + DE_4 e^{-i\phi_4}. \tag{19}$$

Here we have assumed $\eta_{3x} = \eta_{3y}$, which can be achieved by choosing the propagating direction of the third laser beam appropriately.

If the motion in the z axis is initially in the vacuum state $|0\rangle$ it will remain in this state, since the Hamiltonian $\hat{H}'_I$ contains no terms which can alter the motional quanta in the Z direction. In the long time limit, the ion will be populated in the ground electronic state $|g\rangle$ as a consequence of atomic spontaneous emission. In this case, the steady states solution of the master equation (15) can be assumed to be: $\rho_s = |g\rangle|\psi_{a,b}\rangle|0\rangle\langle 0|\langle\psi_{a,b}|\langle g|$, where $|\psi_{a,b}\rangle$ stands for the correlated state of the motions in the x and y axes. When the system reaches the steady state, $\frac{d\rho_s}{dt} = 0$. Since the dissipative term on the right-hand side vanishes for a steady state of the above mentioned form, the condition for the system to reach the steady state is $[\hat{H}'_I, \rho_s] = 0$. According to the methods of Refs. [3, 8, 11-13], we obtain

$$[(g_1\hat{a}\hat{b} + g_2\hat{a}^+\hat{b}^+ + g_3(\hat{a}^+\hat{a} + \hat{b}^+\hat{b})]|\psi_{a,b}\rangle = h|\psi_{a,b}\rangle. \tag{20}$$

If we choose the amplitudes and phases of the first three lasers appropriately so that $g_1 = g_2\tanh^2 r e^{4i\theta}$, $g_3 = g_1\tanh r e^{2i\theta}$, Eq. (20) is identical to (8), with the eigenvalue ξ given by

$$\frac{\xi}{\cosh^2 r} = \tanh r e^{2i\theta} + \frac{h}{g_1}. \tag{21}$$

Therefore, the values of θ and r are controllable by the three laser beams in the x-y plane and ξ by the laser beam an the Z axis. Without loss of generality, we assume that the vibrational motions in the x and y axes are initially in the two-mode Fock states $|q,0\rangle$. Since $[\hat{H}'_i, \hat{C}] = 0$, the condition that $\hat{C}|\Psi_{a,b}\rangle = q|\Psi_{a,b}\rangle$ is satisfied. Thus the vibrational motions in the X and Y axes finally reach the two-mode squeezed pair coherent state $|(r,\theta),\xi,q\rangle$.

Finally, we give a brief discussion of the requirement on the anisotropy of the trap. We note that there might be on-resonant terms in addition to the ones included in Eq. (11). However, if the ratio of the trap frequencies is chosen large enough these terms can be neglected in the Lamb-Dicke limit. For simplicity we here assume $\eta_{jx} = \eta_{jy} \simeq \eta$ and $\nu_x = l\nu_y$ with l being a positive integer [19]. Then the sum of the additional terms with the lowest order in η is

$$\Delta \hat{H}'_i \simeq \eta^{l-1}\left[\frac{1}{(l+1)!}(g_1\hat{b}^{l+1} + g_2\hat{b}^{+(l+1)}) + \frac{1}{l!}g_3(\hat{a}^+\hat{b}^l + \hat{b}^{+l}\hat{a})\right]\hat{S}^+ + H.c., \tag{22}$$

where the g_j (j = 1, 2, 3) are given by Eqs. (16) and (17). We require the coupling constant of the above terms not to be larger than the coupling constants of the terms that we have neglected in deriving $\hat{H}'_i$ of Eq. (15). Thus it is required that $l \geq 3$.

In summary, we have made a proposal for the generation of two-mode squeezed pair coherent states of the motion of an ion trapped in a 3D anisotropic potential by multichromatically driving the ion with three laser beams in the X-Y plane and one laser beam in the Z direction. Compared with the previous scheme [15], which operates in a 3D isotropic trap and needs to employ seven laser beams, the present scheme is much simpler.

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