Generation of Superpositions of SU(1,1) Coherent States for the Motion of a Trapped Ion

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A scheme is presented for the generation of superpositions of two-mode SU(1,1) coherent states for the motion of a trapped ion. In the scheme an ion is trapped in a two-dimensional harmonic potential and bichromatically driven by two laser beams along the X axis as well as two beams along the Y axis. After the ion-laser interaction, the internal state of the ion would leave the vibrational motion in a superpositon of two SU(1,1) coherent states.

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Over the past few years, much attention has been paid to the so-called Schrödinger cat states. In quantum optics, such states are described as superpositions of coherent states. Although the coherent states are the closest quantum states to the classical description of a single-mode field with definite complex amplitudes, due to the quantum interference between the coherent components, such superposition states may exhibit various nonclassical properties, such as sub-Poissonian photon statistics and squeezing [1]. Recently, such states have been realized in cavity QED [2].

There have been multi-mode generalizations of the cat states. Sanders [3] has studied entangled coherent states, also referred to as superpositions of multi-mode coherent states [4]. Under certain conditions such superposition states can exhibit various nonclassical features, such as sub-Poissonian photon number statistics, two-mode squeezing, and violations of the Cauchy-Schwarz inequalities. On the other hand, Gerry *et al.* [5] have studied the superpositons of pair coherent states [6], which are already nonclassical. More recently, they [7] have also investigated superpositions of SU(1,1) and SU(2) coherent states. The two-mode SU(1,1) coherent states are defined as

$$|\xi; q\rangle = S(r)|q; 0\rangle; \tag{1}$$

where $S(r)$ is the two-mode squeeze operator

$$S(r) = \exp\left(r a^+ b^+ - r^* a b\right); \tag{2}$$

and $|q; 0\rangle$ is a two-mode Fock state. Such states can be expanded in terms of the two-mode Fock states

$$|\xi; q\rangle = (1 - |\xi|^2)^{(1+q)/2} \sum_{n=0}^{\infty} \frac{[(n+q)!]^{1/2}\xi^n}{[n!q!]^{1/2}} |n+q; n\rangle; \tag{3}$$

where

$$\xi = \tanh\rho\, e^{i\theta}, \tag{4}$$

with ρ and θ given by the equality $r = \rho e^{i\theta}$. The superpositons of two-mode SU(1,1) coherent states studied in Ref. [7] are defined by

$$|\xi, q, \varphi\rangle = N_{1,1}[|\xi, q\rangle + e^{i\varphi}|-\xi, q\rangle], \tag{5}$$

where $N_{1,1}$ is a normalization factor. They can be also rewritten as

$$|\xi, q, \pm\rangle = N_{1,1}[S(r) + e^{i\varphi}S(-r)]|q, 0\rangle. \tag{6}$$

It has been shown that such superposition states can exhibit sub-Poissonian statistics, though SU(1,1) coherent states themeselves never have sub-Poissonian statistics [7]. On the other hand, even though the SU(1,1) coherent states show squeezing, the superposition states do not. However, the amount of sum and difference squeezing can be enhanced in the superpositions in comparison with that of the SU(1,1) coherent states. For a small parameter q the superposition states can violate the Cauchy-Schwarz inequality. The degree of nonclassical features of the superposition states depends strongly on the parameter φ. For example, such superposition states with $\varphi = 0$ violate the Cauchy-Schwarz inequality most weakly and those with $\varphi = \pi$ most strongly. The realization of such superposition states is of interest in studying the quantum interference effects between two two-mode nonclassical states. In this paper we make a proposal for the generation of such superposition states for the motion of a trapped ion.

Recent advances in laser cooling and ion trapping have opened a new prospect for quantum sate engineering. When an ion is trapped in a harmonic potential and driven by laser fields, its internal and external degrees of freedom are coupled by its interaction with laser fields. Hence, by adjusting the laser fields one can control the vibrational motion of the trapped ion. The virtue of the extremely weak coupling between the vibrational modes and the external environment provide possibilities for preparing and observing nonclassical states with a high degree of stability. Recently, proposals have been made for generating various nonclassical vibrational states of a trapped ion, squeezing states [8], even and odd coherent states [9-13], superpositions of squeezed states [14], pair coherent states [15], superpositions of pair coherent states [16], SU(1,1) intelligent states [17], and superpositions of SU(2) coherent states [18]. To date, motional Fock states, squeezing states, coherent states [19] and Schrödinger cat states [20] have also been observed.

We consider a two-level ion trapped in a two-dimensional isotropic harmonic potential and bichromatically excited by four lasers in the trap plane. Two of these lasers are tuned to the second lower and higher vibrational sideband, respectively, and propagating in a direction with an angle $\pi/4$ relative to the X axis. The other two, also so tuned, propagate in a direction with an angle $3\pi/4$ to the X axis. The Hamiltonian for this system is given by (assuming $\hbar = 1$)

$$H = \nu(a^+a + b^+b) + \omega_0 S_z + [\lambda E^{-}(x, y, t)S^{-} + H.c.], \tag{7}$$

where a and b are the annihilation operators for the vibrational motions along the X and Y axes, respectively, S_z and $S^{\pm}$ are the electronic flip operators for the two-level ion with transition frequency ω_0 and dipole matrix element λ. ν is the trap frequency in the X-Y plane. The position operators x and y are given by $x = \sqrt{1/(2\nu M)}(a + a^+)$ and $y = \sqrt{1/(2\nu M)}(b + b^+)$, with M being the mass of the trapped ion. $E^-(x, y, t)$ is the negative frequency part of the driving fields

$$\begin{aligned} E^-(x, y, t) &= E_1 e^{i[(\omega_0 - 2\nu)t - k_1 x' + \phi_1]} + E_2 e^{i[(\omega_0 + 2\nu)t - k_2 x' + \phi_2]} \\ &\quad + E_3 e^{i[\omega_0 - 2\nu)t - k_3 y' + \phi_3]} + E_4 e^{i[(\omega_0 + 2\nu)t - k_4 y' + \phi_4]}; \end{aligned} \tag{8}$$

where E_j, ϕ_j, and k_j (j = 1, 2, 3, 4) are the amplitudes, phases, and wave vectors of the driving fields, respectively. The position operators x' and y' are related to x and y by a $\pi/4$ rotation in the X-Y plane.

In the resolved sideband limit, the ion-laser interaction can be described by the nonlinear Jaynes-Cummings model [9, 21]. Then in the interaction picture the Hamiltonian can be described by

$$\begin{aligned} H_i = e^{-\eta^2/2} \Bigg\{ \sum_{n=0} \frac{(i\eta)^{2n+2}}{(n+2)!(n!)} \Big[& \Omega_1 e^{i\phi_1} a'^{+(n+2)} a'^{n} + \Omega_2 e^{i\phi_2} a'^{+n} a'^{n+2} \\ & + \Omega_3 e^{i\phi_3} b'^{+(n+2)} b'^{n} + \Omega_4 e^{i\phi_4} b'^{+n} b'^{(n+2)} \Big] \Bigg\} S^- + H.c.; \end{aligned} \tag{9}$$

where a' and b' are the annihilation operators for the trap motion in the X' and Y' directions, respectively, relative to a and b with the transformation [15]

$$\begin{pmatrix} a' \\ b' \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}. \tag{10}$$

$\eta = \sqrt{k^2/(2M\nu)}$ is the Lamb-Dicke parameter with the assumption $k_1 = k_3 \simeq k_2 = k_4 \simeq k$, and $\Omega_j = \lambda E_j$ are the Rabi frequencies of the laser fields. In the Lamb-Dicke limit, i.e., $\eta \ll 1$, the Hamiltonian (9) can be well approximated by its expansion to the second order in η. Thus we have

$$H_i = (-g_1 e^{i\phi_1} a'^{+2} - g_2 e^{i\phi_2} a'^{2} - g_3 e^{i\phi_3} b'^{+2} - g_4 e^{i\phi_4} b'^{2}) S^- + H.c.; \tag{11}$$

with $g_j = \eta^2 \Omega_j/2$.

Setting $g_1 = g_2 = g_3 = g_4 = g$, $\phi_1 = \pi/2$, $\phi_2 = -\pi/2$, $\phi_3 = -\pi/2$, $\phi_4 = \pi/2$; and using Eq. (10), we get

$$\begin{aligned} H_i &= -2ig(a^+ b^+ - ab)S^- + H.c. \\ &= -2ig(a^+ b^+ - ab)(S^+ + S^-). \end{aligned} \tag{12}$$

We define the Hermitian operator

$$O = -i(a^+ b^+ - ab); \tag{13}$$

then the time evolution operator of the system can be expressed in the form of 2×2 matrix with respect to the atomic basis

$$U(\tau) = \begin{pmatrix} \cos(2g\tau O) & -i\sin(2g\tau O) \\ -i\sin(2g\tau O) & \cos(2g\tau O) \end{pmatrix}. \tag{14}$$

Assume the ion is initially in the ground electronic state $|g\rangle$, and the motion is in the two-mode Fock state $|q,0\rangle$, which can be prepared with high efficiency [19]. Then after an interaction time τ the system is in the entangled state

$$|\psi(\tau)\rangle = \cos(2g\tau O)|q,0\rangle|g\rangle - i\sin(2g\tau O)|q,0\rangle|e\rangle. \tag{15}$$

We now detect the internal state of the ion. if we find the ion in the ground state $|g\rangle$ the vibrational motion collapses to

$$\begin{aligned} |\psi_v\rangle_g &= N_g\cos(2g\tau O)|q,0\rangle \\ &= \frac{N_g}{2}\left[e^{i(2g\tau O)}|q,0\rangle + e^{-i(2g\tau O)}|q,0\rangle\right], \end{aligned} \tag{16}$$

where N_g is a normalization factor. Substituting Eq. (13) into (16) we obtain

$$|\psi_v\rangle_g = \frac{N_g}{2}[S(r)|q,0\rangle + S(-r)|q,0\rangle], \tag{17}$$

where the operator $S(r)$ is given by Eq. (2) with $r = 2g\tau$. In this way we have obtained the motional even SU(1,1) coherent state of the ion. On the other hand, if we find the ion in the excited state $|e\rangle$ we obtain the motional odd SU(1,1) coherent state.

We briefly assess the decoherence effect. The decoherence rate for a one-phonon state is about γ_0 = 11.9 kHz [19]. The decoherence is due to some problems, such as intensity fluctuations of the laser beams and instabilities of the trap frequency and voltage amplitude. Set $q = 1$ and $|r| = 1$. The sum of mean phonon numbers for the two modes of the SU(1,1) coherent state $S(r)|1,0\rangle$ is $\overline{N} = \cosh^2 r + 3\sinh^2 r \simeq 3.5$. In this case the time scale of decoherence for the superposition state of Eq. (17) is about $T_d = 1/(2\gamma_0\overline{N}) \simeq 12\,\mu s$. Set η = 0.2, $\nu = 2\pi\times$ 500 kHz. Then, in order to generate the state of Eq. (17) we need a time $\tau = 1/(\eta^2\nu) \simeq 8\,\mu s$, shorter than T_d.

In summary, we have proposed a scheme for the generation of superpositions of SU(1,1) coherent states of the motion of a trapped ion. In the scheme the ion is trapped in a two-dimensional harmonic potential and bichromatically driven by four laser beams in the trap plane. Choosing the amplitudes and phases of the lasers appropriately, we can obtain the superposition of two SU(1,1) coherent states by the detection of the electronic state of the ion. Based on currently available techniques [19, 20], the scheme might be realizable.

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References

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[1] Y. Xia and G. Guo, Phys. Lett. **A 136**, 281 (1989); J. Janszky and An. V. Vinogradov, Phys. Rev. Lett. **64**, 2771 (1990); J. Janszky, P. Domokos and P. Adam, Phys. Rev. **A 48**, 2213 (1993); J. Janszky, P. Domokos, S. Szabo and P. Adam, *ibid*. **51**, 4191 (1995).

[2] M. Brune *et al*., Phys. Rev. Lett. **77**, 4887 (1996).

[3] B. C. Sanders, Phys. Rev. **A 45**, 6811 (1992).
[4] C. Chai, Phys. Rev. **A 46**, 7187 (1992); N. A. Ansari and V. I. Mańko, Phys. Rev. **A50**, 1942 (1994); V. V. Dodonov, V. I. Mańko and D. E. Nikonov, Phys. Rev. **A51**, 3328 (1995).
[5] C. C. Gerry and R. Grobe, Phys. Rev. **A5**, 1698 (1995).
[6] G. S. Agarwal, J. Opt. Soc. Am. **B5**, 1940 (1988).
[7] C. C. Gerry and R. Grobe, J. Mod. Opt. **44**, 41 (1997).
[8] J. I. Cirac, A. S. Parkins, R. Blatt and P. Zoller, Phys. Rev. Lett. **70**, 556 (1993).
[9] R. L. de Matos Filho and W. Vogel, Phys. Rev. Lett. **76**, 608 (1996).
[10] J. F. Poyatos, J. I. Cirac, R. Blatt and P. Zoller, Phys. Rev. **A54**, 1532 (1996).
[11] C. C. Gerry, Phys. Rev. **A55**, 2478 (1997).
[12] S. B. Zheng and G. C. Guo, European Physical Journal **D 1**, 105 (1998).
[13] S. B. Zheng, Phys. Rev. **A58**, 761 (1998).
[14] S. C. Gou, J. Steinbach and P. L. Knight, Phys. Rev. **A55**, 3719 (1997).
[15] S. C. Gou, J. Steinbach and P. L. Knight, Phys. Rev. **A54**, R1014 (1996).
[16] S. C. Gou, J. Steinbach and P. L. Knight, Phys. Rev. **A54**, 4315 (1996).
[17] C. C. Gerry, S. C. Gou and J. Steinbach, Phys. Rev. **A55**, 630 (1997).
[18] S. B. Zheng, Aust. J. Phys. **53**, 421 (2000).
[19] D. M. Meekhof *et al*., Phys. Rev. Lett. **76**, 1796 (1996).
[20] C. Monroe, D. M. Meekhof, B. E. King and D. J. Wineland, Science **272**, 1131 (1996).
[21] W. Vogel and R. L. de Matos Filho, Phys. Rev. **A 52**, 4214 (1995).