Intermediate-Energy Positron and Electron Bremsstrahlung from Atoms

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Using a relativistic partial-wave method we calculated numerically the photon energy-angle distributions for positron and electron bremsstrahlung in the field of atoms with atomic number Z = 1, 13, and 79 for kinetic energies of incident positrons or electrons T_1 = 1.0, 2.0, and 2.5 MeV. In this intermediate-energy region, our studies indicate that the Born approximation predictions agree quite well with our partial-wave results for the shape of the photon energy-angle distribution of the positron bremsstrahlung. For high-Z elements, the Born approximation predictions are not good for the shape of the photon energy-angle distribution of electron bremsstrahlung. The Born approximation results can be improved if one multiplies by the Elwert-factor which is independent of photon angle. Our partial-wave results also indicate that the shape of the photon energy-angle distribution of positron or electron bremsstrahlung is almost independent of atomic-electron screening.

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I. Introduction

We have recently reported calculations for the bremsstrahlung cross sections, differential in photon energy and angle, for kinetic energies of incident electrons T_1 = 2 MeV to 5 MeV. Our results are obtained from a direct numerical calculation by using an exact relativistic partial-wave formulation [1]. We describe our basic process as the radiation of a single photon following the scattering of a single electron from an unpolarized isolated atom. In addition, we use a simplified model which is adequate for a wide range of atoms and the processes at incident electron kinetic energies above the keV range [2]. The target atom is described by the Kohn-Sham potential [3]. Our partial-wave results agree quite well with experiments. In this paper we wish to report comparisons of the role of positrons and electrons in positron and electron bremsstrahlung. The positron bremsstrahlung differs from electron bremsstrahlung in that the central potential is repulsive instead of attractive.

In Sec. II, we give a brief survey of bremsstrahlung theory. In Sec. III, we give and analyze our partial-wave results for the shape of the photon energy-angle distributions of positron or electron bremsstrahlung.

II. Theory

Following our previous bremsstrahlung works [1, 4], we obtain the unpolarized bremsstrahlung cross section, differential with respect to photon energy k and to photon angle θ_1,

$$
\begin{aligned}
\sigma(k;\theta_1) &= \left[\frac{\beta_1^2 k}{Z^2}\, d\sigma/dk\, d\Omega_k\right]_{\mathrm{unpol}} \\
&= \lambda_0 \sum_{\kappa_2;\, m=|m_2|} \Big\{ \Big|\sum_{\kappa_1} (-1)^{\ell_1} e^{i\delta_{\kappa_1}} A^+_+\Big|^2 + \Big|\sum_{\kappa_1} (-1)^{\ell_1} e^{i\delta_{\kappa_1}} A^+_-\Big|^2 \\
&\quad + \Big|\sum_{\kappa_1} (-1)^{\ell_1} e^{i\delta_{\kappa_1}} A^-_+\Big|^2 + \Big|\sum_{\kappa_1} (-1)^{\ell_1} e^{i\delta_{\kappa_1}} A^-_-\Big|^2 \Big\};
\end{aligned} \tag{1}
$$

where $\beta_1 = p_1/E_1$,

$$\lambda_0 = \frac{32\alpha}{Z^2 E_1}\, p_1 p_2 E_2 k^2; \tag{2}$$

$$
\begin{cases}
A^+_\pm(m) = C^\pm_{\kappa_1;m-1}\, y_{\ell_1;m-1\mp\frac{1}{2}}(\theta_1)\, R^+_{\kappa_2\kappa_1}(m); \\
A^-_\pm(m) = C^\pm_{\kappa_1;m+1}\, y_{\ell_1;m+1\mp\frac{1}{2}}(\theta_1)\, R^-_{\kappa_2\kappa_1}(m);
\end{cases} \tag{3}
$$

$$Y_{\ell m}(\theta;\phi) = y_{\ell m}(\theta)\, e^{im\phi}; \tag{4}$$

$$R^\pm_{\kappa_2\kappa_1}(m) = \sum_{n=1}^{2} Q^\pm_n(m) \sum_{\ell}{}' P^\pm_n(m)\, s_n. \tag{5}$$

The index ℓ runs from $|\ell'_2 - \ell_1|$ to $(\ell'_2 + \ell_1)$ in steps of 2 for $n = 1$, and from $|\ell_2 - \ell'_1|$ to $(\ell_2 + \ell'_1)$ in steps of 2 for n=2. Here

$$\ell' = \ell + \eta_\kappa; \quad \eta_\kappa = -\kappa/|\kappa|; \tag{6}$$

$$
\begin{cases}
Q^\pm_1(m) = \eta_{\kappa_2}(-1)^{m\mp 1/2}\, [(2\ell'_2+1)(2\ell_1+1)]^{1/2}\, C^\pm_{-\kappa_2;m} C^\mp_{\kappa_1;m\mp 1}; \\
Q^\pm_2(m) = -\eta_{\kappa_1}(-1)^{m\mp 1/2}\, [(2\ell_2+1)(2\ell'_1+1)]^{1/2}\, C^\pm_{\kappa_2;m} C^\mp_{-\kappa_1;m\mp 1};
\end{cases} \tag{7}
$$

$$
\begin{cases}
P^\pm_1(m) = (-1)^{(\ell'_2+\ell_1+\ell)/2}\, T(\ell'_2;\ell_1;\ell;m\mp\frac{1}{2}); \\
P^\pm_2(m) = (-1)^{(\ell_2+\ell'_1+\ell)/2}\, T(\ell_2;\ell'_1;\ell;m\mp\frac{1}{2});
\end{cases} \tag{8}
$$

$$T(\ell_2;\ell_1;\ell;m) = (2\ell+1)\begin{pmatrix} \ell_2 & \ell_1 & \ell \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \ell_2 & \ell_1 & \ell \\ -m & m & 0 \end{pmatrix}; \tag{9}$$

$$C^\pm_{\kappa;m} = C\left(\ell\,\tfrac{1}{2}\,j;\, m\mp\tfrac{1}{2}, \pm\tfrac{1}{2}\right); \tag{10}$$

and

$$\begin{cases} s_1 = \int_0^\infty dr\, j_\ell(kr)\, g_{\kappa_1} f_{\kappa_2}; \\ s_2 = \int_0^\infty dr\, j_\ell(kr)\, g_{\kappa_2} f_{\kappa_1}. \end{cases} \tag{11}$$

The quantity $C(\ell\frac{1}{2}j; m-s, s)$ is the Clebsch-Gordan coefficient. The radial wave functions g_κ and f_κ satisfy the radial Dirac equations

$$\begin{aligned} dg_\kappa(r)/dr &= (E+1-V)f_\kappa(r) - \kappa g_\kappa(r)/r; \\ df_\kappa(r)/dr &= -(E-1-V)g_\kappa(r) + \kappa f_\kappa(r)/r; \end{aligned} \tag{12}$$

where V is the central potential described by the target atom. For positron bremsstrahlung V is repulsive, while for electron bremsstrahlung V is attractive. Here we chose a coordinate system centered at the atomic nucleus with the z-axis along $\hat{k}$, $\hat{y}$ along $\hat{k}\times\hat{p}_1$, and $\hat{x}$ in the $(\hat{k},\hat{p}_1)$ plane. Then the directions of $\hat{p}_1$ and $\hat{p}_2$ are given by the angles θ_1, $\phi_1 = 0$ and θ_2, $\phi_2 = \phi$, respectively. Also, we obtain the unpolarized bremsstrahlung photon energy spectrum

$$\begin{aligned} \sigma(k) &= \left[\beta_1^2 \frac{k}{Z^2}\, d\sigma/dk\right]_{\text{unpol}} \\ &= \lambda_0 \sum_{\kappa_2,\kappa_1,m=|m_2|} \{[R^+_{\kappa_2\kappa_1}(m)]^2 + [R^-_{\kappa_2\kappa_1}(m)]^2\}. \end{aligned} \tag{13}$$

It is well known that many partial waves are needed to obtain accurate unpolarized photon energy-angle distributions for positron or electron bremsstrahlung $\sigma(k,\theta_1)$ in this intermediate-energy region with the partial-wave method. Equation (1), for the unpolarized photon energy-angle distribution of bremsstrahlung $\sigma(k,\theta_1)$, has the same type of form in partial-wave series as Eq. (13) for the unpolarized photon energy spectrum $\sigma(k)$. The expression given in Eq. (1) is an extension of our previous result [4] and makes the calculations easier in this intermediate-energy region. The problem of calculating the unpolarized bremsstrahlung cross sections $\sigma(k)$ and $\sigma(k,\theta_1)$ has been reduced to computing $R^\pm_{\kappa_2\kappa_1}(m)$. We used a numerical method similar to the one we used in our previous bremsstrahlung work [4]. In the soft-photon limit of the spectrum, as discussed by Low *et al.* [5], the bremsstrahlung matrix element is proportional to the matrix element for elastic scattering. This leads to

$$\begin{aligned} \sigma(k=0;\theta_1) = \beta_1^2 \frac{\alpha}{2\pi Z^2} \int_0^\pi d\theta_f\, \sin\theta_f \left(\frac{d\sigma}{d\Omega_f}\right)_{\text{elastic}} \\ \times \left[\frac{2A}{(1-\beta_1\cos\theta_1)\sqrt{a^2-b^2}} - \frac{a}{E^2(a^2-b^2)^{3/2}} - \frac{1}{E^2(1-\beta_1\cos\theta_1)^2}\right]; \end{aligned} \tag{14}$$

where $A = 1-\beta_1^2\cos^2\theta_f$, $a = 1-\beta_1\cos\theta_1\cos\theta_f$, and $b = \beta_1\sin\theta_1\sin\theta_f$. Here $(\frac{d\sigma}{d\Omega_f})_{\text{elastic}}$ is the elastic scattering angular cross section calculated numerically with the partial-wave method following the trick given by Lin [6]. Also, we have

$$\sigma(k=0) = \beta_1^2 \frac{4\alpha}{Z^2} \int_0^\pi d\theta_f\, \sin\theta_f \left(\frac{d\sigma}{d\Omega_f}\right)_{\text{elastic}} \left[\frac{A}{\sqrt{A^2-B^2}} \cosh^{-1}\frac{A}{B} - 1\right]; \tag{15}$$

where $A = 1 - \beta_1^2 \cos^2\theta_f$, $B = 1 - \beta_1^2$.

III. Results and discussion

With the partial-wave method using Eqs. (1) and (13) for $k/T_1 \neq 0$ and Eqs. (14) and (15) for $k/T_1 = 0$ we have obtained the unpolarized photon energy-angle distributions of positron and electron bremsstrahlung $\sigma(k;\theta_1)$ and the unpolarized photon energy spectrum $\sigma(k)$ for kinetic energies of incident positrons or electrons T_1 = 1.0, 2.0, and 2.5 MeV, for elements of atomic number Z = 1, 13, and 79. These calculated results are shown in Tables I-III and Figs. 1-5. Here the unpolarized bremsstrahlung cross sections are calculated numerically both with the Kohn-Sham potential and the point-Coulomb potential. Our partial-wave results for the point-Coulomb potential agree quite well with the Born approximation [7] prediction for the cases with Z = 1, T_1 = 1.0, 2.0, and 2.5 MeV, as shown in Table I and Fig. 1. The corresponding Coulomb parameters $2\pi Z\alpha E_2/p_2$ are less than 0.084 and thus the Born approximation is expected to be good. This provides a check on our numerical calculations with the partial-wave method, while for the cases Z = 13 and 79, our partial-wave results indicate that the Born approximation predictions are not good, also as expected, since the corresponding Coulomb parameters are larger than 0.62 for Z = 13 and are larger than 3.79 for Z = 79. Elwert [8] suggested a modification of the Bethe-Hetler formula which extends its usefulness to higher-Z elements. The Born approximation results can be improved if one multiplies by the Elwert factor $f_E = \frac{\nu_2}{\nu_1}\frac{1-e^{-2\pi\nu_1}}{1-e^{-2\pi\nu_2}}$, the square of the ratio of final to initial relativistic continuum-wave-function normalizations apart from corrections of order $(Z\alpha)^2$. We show these Born-Elwert (BE) predictions for the electron and positron spectrum $\sigma(k)$ in Table II. From Tables I and II, we see that for $Z = 13$, the Born-Elwert predictions agree quite well with our partial-wave results, and the BE results give almost the same results as the results calculated by the Elwert-Haug (EH) approximation [9]. A simple physical interpretation can be given. We have discussed elsewhere [10] some of the reasons for the success of the Elwert factor in improving the prediction of the bremsstrahlung energy spectrum $\sigma(k)$. The Born-Elwert prediction replaces the Born-approximation prediction of zero for the tip of the electron Bremsstrahlung spectrum $\sigma(k)$ with a finite prediction correct to lowest order in $Z\alpha$, while leaving unchangrd the (correct) Born-approximation prediction for the soft-photon limit of the spectrum. In consequence, we expect a better result throughout the spectrum, particularly for the low-Z elements. It is clear that the usefulness of the idea of normalization goes beyond the regions for which Elwert's derivation applies.

However, such an Elwert modification , since it is independent of angles, has no effect on the Born-approximation prediction for the shape functions. The shape functions are defined as the ratio of the unpolarized photon energy-angle distributions $\sigma(k;\theta_1)$ to the unpolarized photon energy spectra $\sigma(k)$ for both positron bremsstrahlung and electron bremsstrahlung, respectively. Nevertheless, the success of the Elwert-factor approach to the bremsstrahlung energy spectrum $\sigma(k)$ suggests that the Born-approximation prediction for the shape function is better than its prediction for $\sigma(k)$. In Figs. 1 and 2, we present the shape functions S for Z = 1 and 13. We see that the Born approximation predictions (the crosses) agree quite well with our partial-wave results for the shape of the photon-energy-angle distributions of positron bremsstrahlung (solid lines) and electron bremsstrahlung (dashed lines). In Figs. 3-5, we present the shape functions S for Z = 79. We see that the Born approximation predictions (the crosses) are quite good for the

TABLE I. Comparisons of the unpolarized photon energy spectrum from positron bremsstrahlung and electron bremsstrahlung for the cases with Z =1, 13, 79, T_1 = 1.0, 2.0, 2.5 MeV among our results calculated with the partial-wave method (¾$_{e+}$(k) for positron bremsstrahlung and ¾$_{ei}$ (k) for electron bremsstrahlung) for the point-Coulomb potential, the results calculated by the Born approximation ¾$_{BH}$(k), and the results calculated by the Elwert-Haug approximation (¾$_{EH+}$(k) for positron bremsstrahlung and ¾$_{EHi}$ (k) for electron bremsstrahlung). Here ¾(k) is in units of mb.

Z	T_1	k=T_1	¾$_{BH}$(k)	¾$_{EHi}$ (k)	¾$_{EH+}$(k)	¾$_{ei}$ (k)	¾$_{e+}$(k)
1	1.0	0.4	4.84	4.85	4.84	4.86	4.81
		0.6	3.01	3.02	3.00	3.03	2.99
		0.8	1.65	1.66	1.64	1.67	1.63
	2.0	0.4	5.52	5.48	5.48	5.47	5.47
		0.6	3.55	3.56	3.55	3.56	3.52
		0.8	2.03	2.04	2.02	2.05	2.02
	2.5	0.6	3.80	3.81	3.81	3.81	3.78
		0.8	2.23	2.23	2.22	2.24	2.21
		0.9	1.41	1.42	1.40	1.43	1.40
13	1.0	0.4	4.84	4.90	4.72	5.13	4.54
		0.6	3.01	3.11	2.85	3.29	2.74
		0.8	1.65	1.81	1.45	1.93	1.39
	2.0	0.4	5.52	5.50	5.41	5.66	5.23
		0.6	3.55	3.60	3.46	3.74	3.34
		0.8	2.03	2.12	1.90	2.23	1.84
	2.5	0.6	3.80	3.84	3.73	3.98	3.61
		0.8	2.23	2.30	2.11	2.41	2.06
		0.9	1.41	1.53	1.26	1.60	1.24
79	1.0	0.4	4.84	4.50	3.57	6.49	3.26
		0.6	3.01	2.94	1.74	4.77	1.64
		0.8	1.65	1.89	0.485	3.56	0.521
	2.0	0.4	5.52	4.99	4.54	6.36	4.19
		0.6	3.55	3.26	2.58	4.59	2.43
		0.8	2.03	2.01	1.02	3.27	1.09
	2.5	0.6	3.80	3.46	2.91	4.63	2.74
		0.8	2.23	2.13	1.26	3.28	1.33
		0.9	1.41	1.54	0.457	2.62	0.576

TABLE II. Comparisons of the unpolarized photon energy spectrum from positron bremsstrahlung $\sigma_{BE+}(k)$ and electron bremsstrahlung $\sigma_{BE-}(k)$ for the cases with Z = 1, 13, 79, T_1 = 1.0, 2.0, 2.5 MeV calculated by the Born-Elwert (BE) approximation and comparisons of the ratios of positron to electron bremsstrahlung cross section between $\eta_{BE} = \sigma_{BE+}(k)/\sigma_{BE-}(k)$ calculated by the Born-Elwert approximation and $\eta_{cal} = \sigma_{e^+}(k)/\sigma_{e^-}(k)$ calculated by the partial-wave method for the point-Coulomb potential. Here $\sigma(k)$ is in units of mb.

Z	T_1	k/T_1	$\sigma_{BE-}(k)$	$\sigma_{BE+}(k)$	η_{BE}	η_{cal}
1	1.0	0.4	4.85	4.83	0.997	0.990
		0.6	3.02	3.00	0.993	0.987
		0.8	1.66	1.64	0.983	0.976
	2.0	0.4	5.52	5.52	0.999	1.00
		0.6	3.56	3.55	0.997	0.989
		0.8	2.04	2.02	0.991	0.985
	2.5	0.6	3.80	3.80	0.998	0.992
		0.8	2.24	2.22	0.993	0.987
		0.9	1.42	1.40	0.985	0.979
13	1.0	0.4	4.92	4.74	0.963	0.885
		0.6	3.13	2.87	0.917	0.833
		0.8	1.82	1.46	0.799	0.720
	2.0	0.4	5.56	5.47	0.984	0.924
		0.6	3.61	3.48	0.624	0.893
		0.8	2.13	1.91	0.895	0.825
	2.5	0.6	3.85	3.74	0.972	0.907
		0.8	2.32	2.13	0.918	0.855
		0.9	1.54	1.26	0.819	0.775
79	1.0	0.4	5.11	4.06	0.794	0.502
		0.6	3.39	2.00	0.591	0.344
		0.8	2.20	0.564	0.257	0.146
	2.0	0.4	5.65	5.13	0.909	0.659
		0.6	3.75	2.97	0.792	0.529
		0.8	2.37	1.21	0.509	0.333
	2.5	0.6	3.96	3.33	0.841	0.592
		0.8	2.52	1.50	0.593	0.405
		0.9	1.84	0.548	0.297	0.220

TABLE III. Comparisons of the unpolarized photon energy spectrum from positron bremsstrahlung ¾$_{e^+}$(k) and electron bremsstrahlung ¾$_{ei}$ (k) for the cases with Z = 13, 79, T_1 = 1.0, 2.0, 2.5 MeV between our results calculated with the partial-wave method for the Kohn-Sham potential. Here ¾(k) is in units of mb.

Z	T_1	k=T_1	¾$_{ei}$ (k)	¾$_{e^+}$(k)
13	1.0	0.0	12.4	11.8
		0.4	5.07	4.51
		0.6	3.28	2.74
		0.8	1.92	1.40
	2.0	0.0	12.3	11.9
		0.4	5.58	5.16
		0.6	3.72	3.33
		0.8	2.22	1.85
	2.5	0.0	12.3	11.7
		0.6	3.95	3.60
		0.8	2.40	2.06
		0.9	1.59	1.24
79	1.0	0.0	12.1	9.07
		0.4	6.04	3.13
		0.6	4.53	1.65
		0.8	3.40	0.570
	2.0	0.0	12.0	9.73
		0.4	5.92	3.92
		0.6	4.37	2.37
		0.8	3.15	1.11
	2.5	0.0	11.9	9.90
		0.6	4.40	2.66
		0.8	3.16	1.34
		0.9	2.54	0.611

shape of the photon energy-angle distributions of positron bremsstrahlung (solid lines), but not for that of the electron bremsstrahlung (dashed lines). For Z = 79, the Elwert approximation gives only a qualitative improvement to the Born approximation.

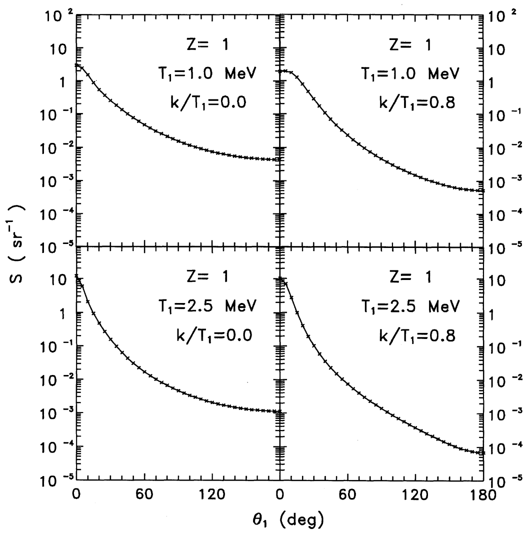

FIG. 1. Comparisons of the positron bremsstrahlung (solid lines) and electron bremsstrahlung (dashed lines) shape functions S = ¾$(k;\mu_1)$=¾(k) for Z = 1, T_1 = 1.0 and 2.5 MeV, k=T_1 = 0.0, 0.8 obtaines from our results calculated by the partial-wave method with the results calculated by the Born approximation (the crosses).

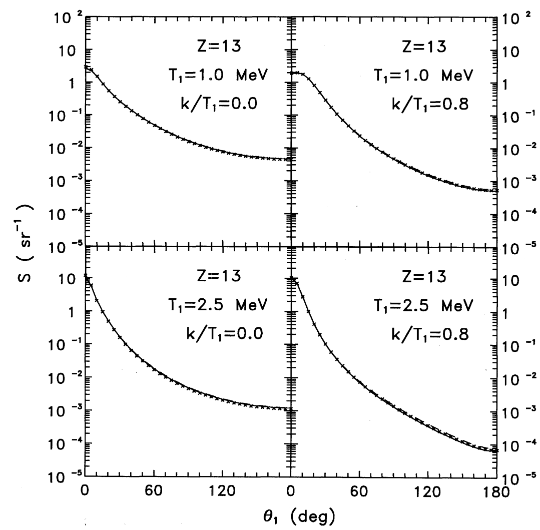

FIG. 2. Same as Fig. 1 except for Z = 13.

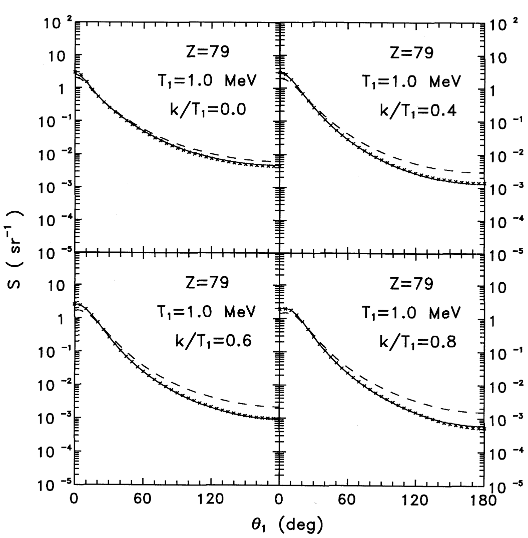

FIG. 3. Same as Fig. 1 except for Z = 79, T_1 = 1.0 MeV, k=T_1 = 0.0, 0.4, 0.6, 0.8.

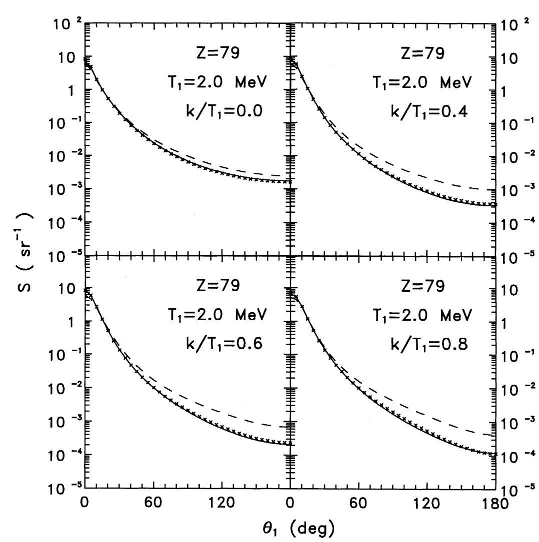

FIG. 4. Same as Fig. 1 except for Z = 79, T_1 = 2.0 MeV, k=T_1 = 0.0, 0.4, 0.6, 0.8.

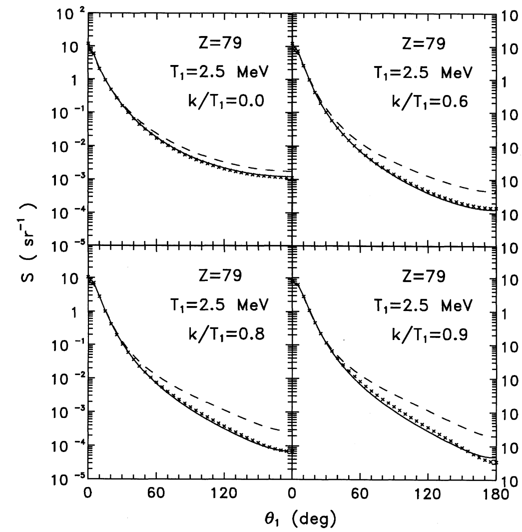

FIG. 5. Same as Fig. 1 except for Z = 79, T_1 = 2.5 MeV, k/T_1 = 0.0, 0.6, 0.8, 0.9.

It is instructive to focus some attention on the ratio of the positron to electron bremsstrahlung photon energy spectrum. In the Born-Elwert approximation, the ratio $\eta_{BE} = \sigma_{BE+}(k)/\sigma_{BE-}(k) = e^{2\pi(\xi_1-\xi_2)}$, where $\xi_1 = Z\alpha E_1/p_1$, $\xi_2 = Z\alpha E_2/p_2$. Here $\sigma_{BE+}(k)$ and $\sigma_{BE-}(k)$ are the positron and electron bremsstrahlung photon energy spectrum calculated by the Born-Elwert approximation, respectively. In Table II, we present comparisons of the ratio of positron to electron bremsstahlung cross sections between η_{BE} and $\eta_{cal} = \sigma_{e+}(k)/\sigma_{e-}(k)$ calculated with the partial-wave method for the point-Coulomb potential. From Table II, we see that for Z = 1, 13, the Born-Elwert approximations agree quite well with our partial-wave results, but not for Z = 79.

By comparing our results for the shape functions S obtained by the partial-wave method for the Kohn-Sham potential with our partial-wave results for the point-Coulomb potential, we find that the shape of the photon energy-angle distribution is almost independent of atomic-electron screening in this intermediate-energy region for both positron bremsstrahlung and electron bremsstrahlung. This suggests that the atomic-electron screening is primarily a normalization effect. In Table III, we present a comparison of the unpolarized photon energy spectrum for positron bremsstrahlung $\sigma_{e+}(k)$ and electron bremsstrahlung $\sigma_{e-}(k)$ for the Z = 13, 79, T_1 = 1.0, 2.0, 2.5 MeV cases calculated with the partial-wave method for the Kohn-Sham potential. From Tables III and I, we see that for low positron energies the atomic-electron screening increases the cross section $\sigma_{e+}(k)$. This is because the atomic electrons decrease the Coulomb repulsion of the positrons.

Acknowledgments

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