CHINESE JOURNAL OF PHYSICS VOL. 40, NO. 2 APRIL 2002

Schemes for the Realization of Multiatom Entanglement and Atomic State Teleportation with a Single Vacuum Cavity

Shi-Biao Zheng¤
Department of Electronic Science and Applied Physics, Fuzhou University, Fuzhou 350002, China

(Received December 11, 2001)

Two schemes are proposed for the generation of multiatom entangled states with a single cavity. In the schemes the cavity is always in the vacuum state and thus the cavity decay is suppressed. The scheme can be used to teleport an atomic state with a single vacuum cavity.

PACS. 03.65.Bz – Foundations, theory of measurement, miscellaneous theories.
PACS. 03.67.Lx – Quantum computation.
PACS. 42.50.Dv – Nonclassical field states; squeezed, antibunched, and sub-Poissonian states; operational definitions of the phase of the field; phase measurements.

Entanglement is one of the most striking features of quantum mechanics, which can not be explained in classical terms. Recently, much interest has been paid to the generation of entangled states of two particles, which provide for the possibility of testing local hidden theory against quantum mechanics [1]. Furthermore, such entangled states are useful in quantum information processing, e.g., quantum cryptography [2] and quantum teleportation [3]. Recently, two-particle entangled states have been realized within a cavity [4] and an ion trap [5].

On the other hand, much interest has been paid to entangled states involving three or more particles, referred to as Greenberger-Horne-Zeilinger (GHZ) states [6]. With such states, a set of measurements is sufficient for demolishing local hidden theories, in contrast with the case using two-particle states where the contradictions with locality is of a statistical nature [1]. Mermin [7] has derived a Bell inequality for states and has shown that quantum mechanics violates this inequality by an amount that grows exponentially with the number of particles. Apart from fundamental tests of quantum mechanics, GHZ states have applications in quantum information processing such as cryptographic conference, multiparticle generalization of superdense coding [8], reducing communication complexity [9], and quantum secret sharing [10]. Recently, three-photon GHZ states have been observed [11]. Entanglement of four trapped ions has also been demonstrated [12], using the technique proposed by M¿lmer and S¿rensen [13]. In the context of cavity QED, many schemes have been presented for the generation of multi-atom GHZ states [14]. A three-particle entangled state has been succesfully produced within a cavity [15]. We have proposed a scheme for the generation of entangled states for n atoms using n-1 cavities [16]. The first cavity is allowed to exchange energy with the environment, and the other n-2 cavities are always in vacuum states and thus the cavity decay is suppressed. In this contribution, we propose two alternative schemes for the generation of multiatom entangled states within the framework of microwave cavity QED. This time we only require one cavity always in the vacuum state and thus the procedure is greatly simplified. We note that our schemes can also be used to teleport an unknown atomic state with a single vacuum cavity.

We first consider two identical two-level atoms simultaneously interacting with a single-mode cavity field. The Hamiltonian for the system is given by

$$H = H_0 + H_i, \tag{1}$$

where

$$H_0 = \omega a^+ a + \omega_0 \sum_{j=1,2} S_{z,j}, \tag{2}$$

$$H_i = g \sum_{j=1,2} (a^+ S_j^- + a S_j^+), \tag{3}$$

where $S_{z,j} = \frac{1}{2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, $S_j^+ = |e_j\rangle\langle g_j|$ and $S_j^- = |g_j\rangle\langle e_j|$, with $|e_j\rangle$ and $|g_j\rangle$ ($j = 1$, 2) being the excited and ground states of the jth atom, a^+ and a are the creation and annihilation operators for the cavity mode, ω_0 is the atomic transition frequency, ω is the cavity frequency, and g is the atom-cavity coupling strength. In the case $\delta = \omega_0 - \omega \gg g$, there is no energy exchange between the atomic system and the cavity. The energy conserving transitions are between $|e_1 g_2 n\rangle$ and $|g_1 e_2 n\rangle$. The Rabi frequency λ for the transitions between these states, mediated by $|g_1 g_2 n+1\rangle$ and $|e_1 e_2 n-1\rangle$ is given by [16]

$$\begin{aligned} \lambda &= \frac{\langle e_1 g_2 n|H_i|g_1 g_2 n+1\rangle\langle g_1 g_2 n+1|H_i|g_1 e_2 n\rangle}{\delta} \\ &\quad + \frac{\langle e_1 g_2 n|H_i|e_1 e_2 n-1\rangle\langle e_1 e_2 n-1|H_i|g_1 e_2 n\rangle}{-\delta} \\ &= \frac{g^2}{\delta}. \end{aligned} \tag{4}$$

Since the two transition paths interfere destructively the Rabi frequency is independent of the photon-number of the cavity mode. Then the effective Hamiltonian is

$$H_e = \lambda\left[\sum_{j=1,2} (|e_j\rangle\langle e_j| a a^+ - |g_j\rangle\langle g_j| a^+ a) + (S_1^+ S_2^- + S_1^- S_2^+)\right], \tag{5}$$

where $\lambda = \frac{g^2}{\delta}$. The first and second terms describe the photon-number dependent Stark shifts, and the third and fourth terms describe the dipole coupling between the two atoms induced by the cavity mode. Assume the cavity field is initially in the vacuum state; the Hamiltonian reduces to

$$H = \lambda\left[\sum_{j=1,2} |e_j\rangle\langle e_j| + (S_1^+ S_2^- + S_1^- S_2^+)\right]. \tag{6}$$

We send three ladder-type three-level atoms across a vacuum cavity sequentially. The highest, mediate, and lowest levels are denoted by $|i\rangle$, $|e\rangle$, and $|g\rangle$, respectively. The transition frequency between the states $|e\rangle$ and $|i\rangle$ is highly detuned from the cavity frequency and thus the state $|i\rangle$ is not affected during the atom-cavity interaction. Set L to be the length of the cavity, v to be the atomic velocity. Let the second atom enters the cavity $L/2v$ later than the first atom, and $L/2v$ sooner than the third atom. In this case the first atom exits the cavity when the third atom enters the cavity and thus there is no interaction between these two atoms. The effective Hamiltonian for the first and second atoms is given by Eq. (6). Assume that the three atoms are initially in the states $|g_1\rangle$, $\frac{1}{\sqrt{2}}(|i_2\rangle + |e_2\rangle)$, and $|e_3\rangle$, respectively. After the first atom exits the cavity the first and second atoms are in the entangled state

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\{|g_1\rangle|i_2\rangle + e^{-i\lambda t}[\cos(\lambda t)|g_1\rangle|e_2\rangle - i\sin(\lambda t)|e_1\rangle|g_2\rangle]\}, \tag{7}$$

where $t = L/2v$. Choose the atomic velocity appropriately so that $\lambda t = \pi/2$. Then we obtain a two-atom maximally entangled state, i.e., Einstein-Podolsky-Rosen (EPR) pair [17]

$$\frac{1}{\sqrt{2}}(|g_1\rangle|i_2\rangle - |e_1\rangle|g_2\rangle). \tag{8}$$

After the second atom exits the cavity the whole system is in the state

$$\begin{aligned} &\frac{1}{\sqrt{2}}e^{-i\lambda t}[|g_1\rangle|i_2\rangle|e_3\rangle + \cos(\lambda t)|e_1\rangle|g_2\rangle|e_3\rangle - i\sin(\lambda t)|e_1\rangle|e_2\rangle|g_3\rangle] \\ &= -\frac{1}{\sqrt{2}}(i|g_1\rangle|i_2\rangle|e_3\rangle + |e_1\rangle|e_2\rangle|g_3\rangle). \end{aligned} \tag{9}$$

In this way we obtain a three-atom maximally entangled state.

We now describe the second scheme. We send n atoms across the cavity sequentially. This time we do not use the state $|i\rangle$. Let the $(j+1)$th atom enter the cavity $L/2v$ later than the jth atom. In this case there is no interaction between the jth atom and $(j+2)$th atom. Assume that the first atom is initially in the state $\frac{1}{\sqrt{2}}(|g_1\rangle + i|e_1\rangle)$ and the jth atom is in the state $\frac{1}{\sqrt{2}}(|g_j\rangle + |e_j\rangle)$ $(j > 1)$. When the first atom arrives at the center of the cavity it is in the state

$$\frac{1}{\sqrt{2}}(|g_1\rangle + ie^{-i\lambda t}|e_1\rangle). \tag{10}$$

Choose the atomic velocity appropriately so that $\lambda t = \pi/2$. Then we have

$$\frac{1}{\sqrt{2}}(|g_1\rangle + |e_1\rangle). \tag{11}$$

When the first atom enters the cavity the first and second atoms are in the state

$$\frac{1}{2}[|g_1\rangle(|g_2\rangle + |e_2\rangle) + |e_1\rangle(|g_2\rangle + |e_2\rangle)]. \tag{12}$$

When the first atom exits the cavity we obtain the transformation

$$\begin{aligned} |g_1\rangle(|g_2\rangle + |e_2\rangle) &\longrightarrow |g_1\rangle|g_2\rangle + e^{-i\lambda t}[\cos(\lambda t)|g_1\rangle|e_2\rangle - i\sin(\lambda t)|e_1\rangle|g_2\rangle] \\ &= (|g_1\rangle - |e_1\rangle)|g_2\rangle, \end{aligned} \tag{13}$$

$$
\begin{aligned}
|e_1\rangle(|g_2\rangle + |e_2\rangle) &\longrightarrow e^{-i\lambda t}[\cos(\lambda t)|e_1\rangle|g_2\rangle - i\sin(\lambda t)|g_1\rangle|e_2\rangle] + e^{-2i\lambda t}|e_1\rangle|e_2\rangle \\
&= -(|g_1\rangle + |e_1\rangle)|e_2\rangle .
\end{aligned} \tag{14}
$$

In this way the first and second atoms are prepared in the maximally entangled state

$$\frac{1}{2}[(|g_1\rangle - |e_1\rangle)|g_2\rangle - (|g_1\rangle + |e_1\rangle)|e_2\rangle]. \tag{15}$$

When the third atom enters the cavity atoms 1, 2, and 3 are in the state

$$\frac{1}{2\sqrt{2}}[(|g_1\rangle - |e_1\rangle)|g_2\rangle(|g_3\rangle + |e_3\rangle) - (|g_1\rangle + |e_1\rangle)|e_2\rangle(|g_3\rangle + |e_3\rangle)]. \tag{16}$$

When the second atom exits the cavity we have

$$|g_2\rangle(|g_3\rangle + |e_3\rangle) \longrightarrow (|g_2\rangle - |e_2\rangle)|g_3\rangle, \tag{17}$$

$$|e_2\rangle(|g_3\rangle + |e_3\rangle) \longrightarrow -(|g_2\rangle + |e_2\rangle)|e_3\rangle. \tag{18}$$

Thus the state of the three atoms are

$$\frac{1}{2\sqrt{2}}[(|g_1\rangle - |e_1\rangle)(|g_2\rangle - |e_2\rangle)|g_3\rangle + (|g_1\rangle + |e_1\rangle)(|g_2\rangle + |e_2\rangle)|e_3\rangle]. \tag{19}$$

In this way we obtain a three-atom GHZ state. When the $(n-1)$th atom exits the cavity the n atoms are in the maximally entangled state

$$
\begin{aligned}
&\frac{1}{\sqrt{2^n}}[(|g_1\rangle - |e_1\rangle)(|g_2\rangle - |e_2\rangle)\ldots(|g_{n-1}\rangle - |e_{n-1}\rangle)|g_n\rangle \\
&+(-1)^{n-1}(|g_1\rangle + |e_1\rangle)(|g_2\rangle + |e_2\rangle)\ldots(|g_{n-1}\rangle + |e_{n-1}\rangle)|e_n\rangle].
\end{aligned} \tag{20}
$$

We now make a comparison between the above mentioned two schemes. The first scheme can not be used to generate maximally entangled states for more than three atoms while the second scheme can. The first scheme only requires that the second atom be initially prepared in a superpositon state while the second scheme requires that all atoms be initially prepared in superposition states.

We now turn to the problem of atomic state teleportation. Quantum teleportation has been demonstrated using optical systems [18] and NMR [19]. A scheme has been proposed to teleport the internal state of a trapped ion [20]. In the optical regime, Bose *et al.* [21] have proposed a novel scheme for teleportation of an atomic state via cavity decay. The scheme requires the ability to trap a single three-level atom in a cavity. In the context of microwave cavity QED, several schemes have been proposed for the teleportation of an unknown atomic state [22]. In these schemes the cavities act as memories, which store the information of an atom and then transfer it to another atom after the conditional dynamics. Thus these schemes require that the cavities be initially cooled to the vacuum states and have a very high quality factor, which is experimentally problematic. We have proposed two alternative schemes, insensitive to the cavity decay, for realizing atomic state teleportation [16, 23]. However, these two schemes require two cavities. We note that the atomic state teleportation can also be achieved using only one cavity always in the vacuum state.

Assume that atom 1, which is to be teleported, is initially in a superposition state

$$|\phi_1\rangle = c_e|e_1\rangle + c_g|g_1\rangle; \tag{21}$$

where c_e and c_g are unknown coefficients. Atoms 2 and 3 are initially in the state $|e_2\rangle|g_3\rangle$. Atom 2 enters the cavity $L/2v$ later than atom 3, but $L/2v$ sooner than atom 1. When atom 3 exits the cavity, atoms 2 and 3 are in the entangled state

$$|\psi'(t)\rangle = \cos(\lambda t)|e_2\rangle|g_3\rangle - i\sin(gt)|g_2\rangle|e_3\rangle. \tag{22}$$

With the choice of $\lambda t = \pi/4$; we obtain the maximally two-atom entangled state

$$\frac{1}{\sqrt{2}}(|e_2\rangle|g_3\rangle - i|g_2\rangle|e_3\rangle); \tag{23}$$

where we have discarded the common phase factor $e^{-i\pi/4}$.

The state for the whole system can be expanded as

$$\begin{aligned} |\psi_{1,2,3}\rangle = \frac{1}{2}[&|\Phi^+\rangle(c_e|e_3\rangle + c_g|g_3\rangle) + |\Phi^-\rangle(c_e|e_3\rangle - c_g|g_3\rangle) \\ &+|\Psi^+\rangle(c_e|g_3\rangle - c_g|e_3\rangle) + |\Psi^-\rangle(c_e|g_3\rangle + c_g|e_3\rangle)]; \end{aligned} \tag{24}$$

where $|\Phi^\pm\rangle$ and $|\Psi^\pm\rangle$ are the Bell states [24]

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(-i|e_1\rangle|g_2\rangle \pm |g_1\rangle|e_2\rangle); \tag{25}$$

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle|e_2\rangle \pm i|g_1\rangle|g_2\rangle). \tag{26}$$

When atom 2 exits the cavity we obtain the evolution of $|\Phi^\pm\rangle$

$$|\Phi^\pm\rangle \to \left\{ \begin{array}{l} -i|e_1\rangle|g_2\rangle \\ -|g_1\rangle|e_2\rangle \end{array} \right. . \tag{27}$$

Again, the common phase factor $e^{-i\pi/4}$ is discarded. On the other hand $|\Psi^\pm\rangle$ involve two terms $|e_1\rangle|e_2\rangle$ and $|g_1\rangle|g_2\rangle$; which do not undergo transitions but impart phase shifts during the interaction.

Then the sender (Alice) performs a joint measurement on atoms 1 and 2 separately. The outcomes $|e_1\rangle|g_2\rangle$ and $|g_1\rangle|e_2\rangle$ correspond to $|\Phi^+\rangle$ and $|\Phi^-\rangle$, respectively. If Alice obtains outcome $|e_1\rangle|g_2\rangle$, she can tell the receiver (Bob) that atom 3 has been prepared in the initial state of atom 1. While if Alice obtains $|g_1\rangle|e_2\rangle$ she tells Bob to perform the following transformation on atom 3

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{28}$$

In this way Bob can exactly convert atom 3 into a replica of the original state of atom 1. On the other hand, Alice can not distinguish $|\Psi^{+}\rangle$ from $|\Psi^{-}\rangle$. Thus, if she obtains outcome $|e_1\rangle|e_2\rangle$ or $|g_1\rangle|g_2\rangle$ the procedure fails. Therefore, the present scheme is a probabilistic one with the probability of success being 50%.

In summary, we have proposed two schemes to prepare multiatom maximally entangled states with only one empty cavity. In the schemes several appropriately prepared atoms are sent through an empty cavity one by one. If the distances between two adjacent atoms are chosen appropiately we can obtain a maximally entangled state for these atoms. Futhermore, we note that the teleportation of atomic states can also be achieved using only one vacuum cavity.

This work was supported by the National Natural Science Foundation of China under Grant No. 60008003, Fujian Provincial Natural Science Foundation of China under Grant Nos. K20004 and F0110027, and Funds from Fuzhou University.

References

*E-mail: sbzheng@pub5.fz.fj.cn

[1] J. S. Bell, Physic **1**, 195 (1965).

[2] A. Ekert, Phys. Rev. Lett. **67**, 661 (1991).

[3] C. H. Bennett *et al*., Phys. Rev. Lett. **70**, 1895 (1993).

[4] E. Hagley *et al*., Phys. Rev. Lett. **79**, 1 (1997); S. Osnaghi *et al.*, Phys. Rev. Lett. **87**, 037902 (2001).

[5] Q. A. Turchette *et al*., Phys. Rev. Lett. **81**, 3631 (1998); M. A. Rowe *et al*., Nature **409**, 791 (2001).

[6] D. M. Greenberger, M. A. Horne, and A. Zeilinger, in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, ed. M. Kafatos (Kluwer, Dordrecht, 1989); D. M. Greenberger *et al.*, Am. J. Phys. **58**, 1131 (1990).

[7] N. D. Mermin, Phys. Rev. Lett. **65**, 1838 (1990).

[8] S. Bose, V. Vedral, and P. L. Knight, Phys. Rev. A **57**, 822 (1998).

[9] R. Cleve and H. Buhrman, Phys. Rev. A **56**, 1201 (1997).

[10] M. Hillery, V. Buzek, and A. Berthiaume, Phys. Rev. A **59**, 1829 (1999).

[11] D. Bouwmeester *et al*., Phys. Rev. Lett. **82**, 1345 (1999); J-W. Pan *et al*., Nature **403**, 515 (2000); J-W. Pan *et al*., Phys. Rev. Lett. **86**, 4435 (2001).

[12] C. A. Sackett *et al*., Nature **404**, 256 (2000).

[13] K. Mølmer and A. Sørensen, Phys. Rev. Lett. **82**, 1835 (1999).

[14] J. I. Cirac and P. Zoller, Phys. Rev. A **50**, R2799 (1994); C. C. Gerry, Phys. Rev. A **53**, 2857 (1996); S. Haroche, Ann. (N.Y.) Acad. Sci. **755**, 73 (1995); C. C. Gerry, Phys. Rev. A **53**, 4591 (1996); S-B. Zheng and G-C. Guo, J. Mod. Opt. **44**, 963 (1997); S-B. Zheng, J. Opt. B: Quantum Semiclass. Opt. **1**, 534 (1999).

[15] A. Rauschenbeutel *et al*., Science **288**, 2024 (2000).

[16] S-B. Zheng and G-C. Guo, Phys. Rev. A (2001).

[17] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[18] D. Bouwmeester *et al*., Nature (London) **390**, 575 (1997); D. Boschi *et al*., Phys. Rev. Lett. **80**, 1121 (1998); A. Furusawa *et al*., Science **282**, 706 (1998).

[19] M. A. Nielsen, E. Knill, and R. Laflamme, Nature **396**, 52 (1998).

[20] E. Solano, C. L. Cesar, R. L. de Matos Filho and N. Zagury, Eur. Phys. J. D **13**, 121 (2001).

[21] S. Bose, P. L. Knight, M. B. Plenio, and V. Vedral, Phys. Rev. Lett. **83**, 5158 (1999).

[22] L. Davidovich *et al.*, Phys. Rev. A **50**, R895 (1994); J. I. Cirac and A. S. Parkins, Phys. Rev. A **50**, R4441 (1994); M. H. Y. Moussa, Phys. Rev. A **54**, 4661 (1996); S-B. Zheng and G-C. Guo, Phys. Lett. A **232**, 171 (1997); S-B. Zheng, Opt. Commun. **167**, 111 (1999).

[23] S-B. Zheng and G-C. Guo, Phys. Rev. Lett. **85**, 2392 (2000).

[24] S-L. Braunstein, A. Mann and M. Revzen, Phys. Rev. Lett. **68**, 3259 (1992).