

## Traffic Flow in a 1D Cellular Automaton Model with Open Boundaries

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We have studied the open boundary cellular automaton models for the highway one-line traffic flow by using the mean field approximation and simulations. Our contribution focuses on the effect of braking probability ( $P$ ) and a maximum velocity ( $v_{\max}$ ) on the density, flow and average velocity of cars moving in the middle of the road. The phase diagram is presented for  $v_{\max} = 1$  and  $v_{\max} > 1$ . The maximal flow phase does not occur for  $v_{\max} > 1$ , in contrast with the case  $v_{\max} = 1$  where this phase appears for  $p \neq 0$ . The first-order transition arises at  $\rho^* = \rho^*$  ( $\rho^* < \rho^*$ ) for  $v_{\max} = 1$  ( $v_{\max} > 1$ ), where  $\rho^*$  and  $\rho^*$  denote, respectively, the inside rate and the outside rate. The mean field approximation gives a good results in comparison with simulations for  $v_{\max} > 1$ , while for  $v_{\max} = 1$ , the phase diagram obtained from the simulations is predicted by the mean field approximation when  $p \neq 1$ .

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### I. Introduction

Models of highway traffic flow have attracted much interest in recent years. Discrete models like cellular automata (CA) have become popular for modeling traffic because it is very simple to introduce the basic phenomena and they can be readily implemented on computers [1-4]. Much of the effort was concentrated on discrete stochastic CA models of traffic flow, first proposed by Nagel and Schreckenberg [3] and subsequently studied by other authors using a variety of techniques [4-7]. CA models exhibit a transition between a moving phase and a jamming phase in other traffic of a city as the car density is varied [8]. These models could be easily modified to deal with the effects of different kinds of realistic conditions, such as road blocks and traffic accidents [9, 10], highway junctions [11], overpasses [12], vehicle accelerations [13], quenched disorderliness [14], stochastic delay due to driver's reactions [7], and anisotropy of car distributions [15]. The comparatively low computational cost of CA models made it possible to conduct large-scale real-time simulations of urban traffic in the city of Duisburg [16] and Dallas (Forth Worth) [17].

Nagel and Schreckenberg (NS) considered the effects of acceleration and stochastic delay of vehicles with high speed [3]. In the NS models, a car can move at most  $v_{\max}$  sites in a time step, where  $v_{\max}$  is the maximum velocity. The speed at a time step depends on the gap (number

of empty sites) between successive cars. If the speed in the present time step is less than  $v_{\max}$  and the gap ahead allows, the speed increases by one unit in the next time step. If the spacing ahead is less than the speed in the present time step, then the speed is reduced to the value allowed by the spacing. The speed of a car is reduced by one unit in the next time step with a probability exhibiting a randomization in realistic traffic flows.

Fukui and Ishibashi (FI) [18] introduced another variation on the basic model, which can be understood as a generalization of the cellular automata rule 184. Rule 184 is one of the elementary CA rules investigated by Wolfram [19]. It is one of only two (symmetric) non-trivial elementary rules conserving the number of active sites [20] and, therefore, can be interpreted as a rule governing the dynamics of particles (cars). The FI models considered that cars can move by at most  $v_{\max}$  sites in one time step if they are not blocked by cars in front. More precisely, if the number of empty sites in front of a car is larger than  $v_{\max}$  at time  $t$ , then it can move forward  $v_{\max}$  ( $v_{\max} - 1$ ) sites in the next time step with probability  $1 - f$  ( $f$ ). Here, the probability  $f$  represents the degree of stochastic delay. The FI model differs from the NS model in that the increase in speed may not be gradual and stochastic delay applies to the high-speed cars, the two models are identical for  $v_{\max} = 1$ .

Our main contribution in this paper deals with the effect of boundaries on the traffic flow. The case  $v_{\max} = 1$  corresponds to the fully asymmetric exclusion model [21-23] where particles hop only in one direction. During any time interval ( $\Delta t$ ), each particle (car) has a probability  $\Delta t$  of jumping to its right-hand neighbor if this neighboring site is empty. The particles could enter the system with a given rate  $\alpha$  (i.e., with a probability  $\alpha \Delta t$ ), and could leave the system with a given rate  $\beta$  (i.e., with a probability  $\beta \Delta t$ ). This model has been solved exactly in the random sequential regime [24-26]. Several phases were found for this model, namely, a low density phase (moving phase), a high density phase (jamming phase) and a maximal current phase (maximal flow phase). The dynamic effect (i.e., sequential and parallel update) has been studied [27-29], the region of maximal current phase growth in the crossing from parallel ( $\Delta t = 1$ ) to sequential ( $\Delta t \neq 1$ ) update.

## II. Model

The NS model [3] is a probabilistic CA model of traffic flow on a one-lane highway. The road is divided into cells. Each cell can either be empty or occupied by at most one car. The state of car  $j$  ( $j = 1; \dots; N$ ) is characterized by the instantaneous velocity  $v_j$  ( $v_j = 0; 1; \dots; v_{\max}$ ). If  $d_i$  is the distance between car  $i$  and car  $i + 1$  (cars are moving to the right), velocities are updated in parallel according to the following two subrules:

$$v_i(t + 1) = \min(v_i(t) + 1; d_i(t) - 1; v_{\max}); \quad (1)$$

$$v_i(t + 1) = \max(v_i(t + 1); 0) \quad \text{with probability } p; \quad (2)$$

where  $v_i(t)$  is the velocity of car  $i$  at time  $t$ , then cars are moved according to the subrule:

$$x_i(t + 1) = x_i(t) + v_i(t + 1); \quad (3)$$

where  $x_i(t)$  is the position of car  $i$  at time  $t$ . The model contains three parameters: the maximum speed  $v_{\max}$ , which is the same for all cars, the braking probability  $p$ , and the car density  $\rho$ .

If  $p = 0$ , the NS model is deterministic and the average velocity over the whole is exactly given by:

$$\langle v \rangle = \min \left( v_{\max}, \frac{1}{\rho} \right) \quad (4)$$

This expression shows that, below a critical car density  $\rho_c = \frac{1}{v_{\max} + 1}$ , all cars move with a velocity equal to  $v_{\max}$ , while above  $\rho_c$ , the average velocity is less than  $v_{\max}$ . This transition from free-moving regime to a congested regime is a second-order phase transition.

In this paper, we shall assume that the acceleration, which is equal to 1 in the NS model, has the largest possible value ( $v_{\max}$ ) as in the FI model [18], except that these authors apply random delays only to cars whose velocity is equal to  $v_{\max}$ . That is, in our model, we just replace (1) by:

$$v_i(t+1) = \min(d_i(t) - 1; v_{\max}) \quad (5)$$

Hence, for  $v_{\max} = 1$ , the car can move one site or does not move. We denote by  $\zeta_1; \zeta_2; \dots; \zeta_L$  the occupation numbers, while  $\zeta_i = 1$  ( $\zeta_i = 0$ ) means that site  $i$  is occupied (empty), and  $L$  the road size. During each time step, each car in the system has a probability  $(1 - p)$  of jumping to the empty adjacent site on its right. Cars are injected at the left boundary with a probability  $p$  and removed on the right with a probability  $(1 - p)$ . Thus if the system has the configuration  $\zeta_1(t), \zeta_2(t); \dots; \zeta_L(t)$  at time  $t$  it will change at time  $t + 1$  to the following:

For  $1 < i < L$ ;

$$\zeta_i(t+1) = 1 \quad \text{with probability } p_i; \quad (6)$$

$$\text{where } p_i = \zeta_i(t) + (1 - \zeta_i(t))\zeta_{i-1}(t)(1 - p) - \zeta_i(t)(1 - \zeta_{i+1}(t))(1 - p);$$

$$\zeta_i(t+1) = 0 \quad \text{with probability } 1 - p_i;$$

$$\zeta_1(t+1) = 1 \quad \text{with probability } P_1;$$

$$\text{where } p_1 = \zeta_1(t) + p(1 - \zeta_1(t)) - \zeta_1(t)(1 - \zeta_2(t))(1 - p); \quad (7)$$

$$\zeta_1(t+1) = 0 \quad \text{with probability } 1 - P_1;$$

$$\zeta_L(t+1) = 1 \quad \text{with probability } p_L;$$

$$\text{where } p_L = \zeta_L(t) + (1 - \zeta_L(t))\zeta_{L-1}(t)(1 - p) - (1 - p)\zeta_L(t); \quad (8)$$

$$\zeta_L(t+1) = 0 \quad \text{with probability } 1 - p_L;$$

The dynamics of the system is then given by the following equations:

For  $1 < i < L$ ;

$$\begin{aligned} \langle \zeta_i(t+1) \rangle - \langle \zeta_i(t) \rangle &= \langle (1 - p)\zeta_{i-1}(t)(1 - \zeta_i(t)) \rangle \\ &\quad - \langle (1 - p)\zeta_i(t)(1 - \zeta_{i+1}(t)) \rangle \\ &= (1 - p)P_{i-1} - (1 - p)P_i \end{aligned} \quad (9)$$

$$\begin{aligned}
\langle \zeta_1(t+1) \rangle_i \langle \zeta_1(t) \rangle &= \textcircled{1} (1-p) \langle 1_i \zeta_1(t) \rangle \\
&\quad - \langle (1-p)\zeta_1(t)(1_i \zeta_2(t)) \rangle \\
&= \textcircled{1} (1-p)P_1(0) - \langle (1-p)P_1(10) \rangle;
\end{aligned} \tag{10}$$

$$\begin{aligned}
\langle \zeta_L(t+1) \rangle_i \langle \zeta_L(t) \rangle &= \langle (1-p)\zeta_{L-1}(t)(1_i \zeta_L(t)) \rangle \\
&\quad - \langle (1-p) \zeta_L(t) \rangle \\
&= (1-p)P_{L-1}(10) - \langle (1-p)P_L(1) \rangle;
\end{aligned} \tag{11}$$

with  $P_i(x_i x_{i+1} \dots) = \langle \pm(x_i) \pm(x_{i+1}) \dots \rangle$ . Where  $\pm(x_k) = \zeta_k$  if  $x_k = 1$  and  $\pm(x_k) = 1 - \zeta_k$  if  $x_k = 0$ .

In the case  $v_{\max} = 2$ , the car can move forward two (one) sites with a probability  $1-p$  ( $p$ ) if two sites ahead are empty, otherwise (only the adjacent cell is empty), it could move one cell or remains immovable. The equations governing this case could be written as follows:

For  $2 < i < L - 1$ ;

$$\zeta_i(t+1) = 1 \quad \text{with probability } p_i;$$

where;

$$\begin{aligned}
p_i &= \zeta_i(t) + (1 - \zeta_i(t))[\zeta_{i-1}(t)\zeta_{i+1}(t)(1-p) \\
&\quad + \zeta_{i-1}(t)(1 - \zeta_{i+1}(t))p + \zeta_{i-2}(t)(1 - \zeta_{i-1}(t))(1-p)] \\
&\quad - \zeta_i(t)[(1 - \zeta_{i+1}(t))\zeta_{i+2}(t)(1-p) + (1 - \zeta_{i+1}(t))(1 - \zeta_{i+2}(t))]; \\
\zeta_i(t+1) &= 0 \quad \text{with probability } 1 - p_i;
\end{aligned} \tag{12}$$

$$\zeta_2(t+1) = 1 \quad \text{with probability } p_2;$$

where;

$$\begin{aligned}
p_2 &= \zeta_2(t) + (1 - \zeta_2(t))[\zeta_1(t)\zeta_3(t)(1-p) \\
&\quad + \zeta_1(t)(1 - \zeta_3(t))p] - \zeta_2(t)[(1 - \zeta_3(t))\zeta_4(t)(1-p) \\
&\quad + (1 - \zeta_3(t))(1 - \zeta_4(t))];
\end{aligned} \tag{13}$$

$$\zeta_2(t+1) = 0 \quad \text{with probability } 1 - p_2;$$

$$\zeta_1(t+1) = 1 \quad \text{with probability } p_1;$$

where;

$$\begin{aligned}
p_1 &= \zeta_1(t) + (1-p)\textcircled{1} (1 - \zeta_1(t)) \\
&\quad - \zeta_1(t)[(1 - \zeta_2(t))\zeta_3(t)(1-p) + (1 - \zeta_2(t))(1 - \zeta_3(t))]; \\
\zeta_1(t+1) &= 0 \quad \text{with probability } 1 - p_1;
\end{aligned} \tag{14}$$

$\zeta_{L_i-1}(t+1) = 1$  with probability  $p_{L_i-1}$ ;

where;

$$p_{L_i-1} = \zeta_{L_i-1}(t) + (1 - \zeta_{L_i-1}(t))[\zeta_{L_i-2}(t)\zeta_L(t)(1-p) + \zeta_{L_i-2}(t)(1-\zeta_L(t))p + \zeta_{L_i-3}(t)(1-\zeta_{L_i-2}(t))(1-p)] - \zeta_{L_i-1}(t)(1-\zeta_L(t))(1-p); \quad (15)$$

$\zeta_{L_i-1}(t+1) = 0$  with probability  $1 - p_{L_i-1}$ ;

$\zeta_L(t+1) = 1$  with probability  $p_L$

where;

$$p_L = \zeta_L(t) + (1 - \zeta_L(t))[\zeta_{L-2}(t)(1 - \zeta_{L-1}(t))(1-p) + \zeta_{L-1}(t)(1-p)] - (1 - \zeta_L(t)); \quad (16)$$

$\zeta_L(t+1) = 0$  with probability  $1 - p_L$ ;

The dynamics of the system is then given in the case  $v_{\max} = 2$  as follows:

For  $2 < i < L - 1$ ;

$$\langle \zeta_i(t+1) \rangle_i \langle \zeta_i(t) \rangle = (1-p)P_{i-1}(101) + pP_{i-1}(100) + (1-p)P_{i-2}(100) - (1-p)P_i(101) - P_i(100); \quad (17)$$

$$\langle \zeta_2(t+1) \rangle_i \langle \zeta_2(t) \rangle = (1-p)P_1(101) + pP_1(100) - (1-p)P_2(101) - P_2(100); \quad (18)$$

$$\langle \zeta_1(t+1) \rangle_i \langle \zeta_1(t) \rangle = (1-p)P_1(0) - (1-p)P_1(101) - P_1(100); \quad (19)$$

$$\langle \zeta_{L_i-1}(t+1) \rangle_i \langle \zeta_{L_i-1}(t) \rangle = (1-p)P_{L_i-2}(101) + pP_{L_i-2}(100) + (1-p)P_{L_i-3}(100) - (1-p)P_{L_i-1}(10); \quad (20)$$

$$\langle \zeta_L(t+1) \rangle_i \langle \zeta_L(t) \rangle = (1-p)P_{L-2}(100) + (1-p)P_{L-1}(10) - (1-p)P_L(1); \quad (21)$$

From these equations (Eqs. (9-11) and Eqs. (17-21)), one can calculate the time evolution of any quantity of interest. The problem, however, is that the computation of the one point functions  $\langle \zeta_i \rangle$  requires the knowledge of the two point functions  $\langle \zeta_i \zeta_{i+1} \rangle$  and  $\langle \zeta_{i-1} \zeta_i \rangle$ . The problem is an N-body problem in the sense that the calculation of any correlation function requires

the knowledge of all the others, this makes the problem intractable. We would overcome this problem by neglecting the correlation (mean field theory) between cells, and the equations would be simplified when  $\langle \zeta_i \zeta_j \rangle$  is replaced by  $\langle \zeta_i \rangle \langle \zeta_j \rangle$ .

The time evolution of the average occupation of each cell, the average velocity and the flow (i.e., current) are studied in the next section using the time evolution of the dynamical equations with the mean field approximation (MFA) and using numerical simulations. In our simulations the rule described above is updated in parallel, i.e., during one updating procedure the new car positions do not influence the rest and only previous positions have to be taken into account. In order to compute the average of any parameter  $u(\langle u \rangle)$ , the values of  $u(t)$  ( $t = n\tau$ ,  $n$  is an integer) obtained in the steady state are averaged. Starting the simulations from random configurations, the system reaches a stationary state after a sufficiently large number of time steps. In all our simulations we averaged over 20 – 100 initial configurations. The parameters computed in this article are averaged over the cells  $\frac{1}{2}j - 10, \frac{1}{2}j - 9; \dots, \frac{1}{2}j + 10$ . Hence, the density is defined as the average occupation of 21 cells in the middle road. The average velocity (i.e.  $\langle v \rangle$ ) and the flow i.e.  $\bar{A}$ ) are obtained by averaging the velocities of cars and currents over 21 cells.

### III. Results and discussion

#### III-1. $v_{\max} = 1$

This case corresponds to an asymmetric exclusion model [24-29], here we alter the probability of jump  $\tau$  by  $1 - p$  and perform our results for parallel dynamics. The model exhibits three phases: a moving phase, a jamming phase and a maximal flow phase. The order parameter here is defined as the average velocity of cars instead of the density as studied in previous works [27-29]. The transition from the moving phase to the jamming phase (and vice versa) is a first order transition, while the passage from the moving phase or jamming phase to the maximal flow phase (and vice versa) is a second order transition (Fig. 1). In fact, in Figure 1(a) we note that for  $\tau = 0.3$ ,  $\langle v \rangle$  and  $\rho$  undergo a gap between two regimes, namely,  $\rho < 0.5$  (moving phase) and  $\rho > 0.5$  (jamming phase). In the case  $\tau = 0.8$ ,  $\rho$  and  $\langle v \rangle$  vary continuously in low density regime, after that  $\rho$  and  $\langle v \rangle$  take a constant values corresponding to a maximal flow regime ( $\rho = \frac{1}{2}$ ) as shown in Figure 1(b) [3,4]. The phase diagram is shown in Figure 2 for several values of  $p$ . For  $p = 0$ , the maximal flow phase exists only at  $\tau = \tau^*$ , it is a deterministic case exhibiting in the steady state the configuration 101010... (i.e.,  $\rho = \frac{1}{2}$ ). Increasing the braking probability, the maximal flow phase could be obtained for wide values of  $\tau^*$  and  $\tau$ , it occurs always at  $\rho = \frac{1}{2}$ . The equilibrium between the entering and leaving rates in one hand and the average velocity of cars on the other hand is the main factor that yields the maximal flow phase. So, having a high velocity of cars (i.e., a low value of  $p$ ) requires a high values of  $\tau^*$  and  $\tau$  to give a maximal flow ( $\rho = \frac{1}{2}$ ), otherwise, the system will exhibit jamming traffic when  $\tau^* = \tau$  (first-order transition). The MFA is in excellent agreement with our simulations for  $p \ll 1$  [24-29]. The term of braking probability (i.e.,  $1 - p$ ) arises only as a factor in the dynamical evolution of the MFA equations (Eqs. (9-11)), its contribution does not affect the steady state but it does affect the time required to reach the steady state.

#### III-2. $v_{\max} = 2$

In this case the car could move one or two cells if two cells ahead are empty, otherwise (only the adjacent cell is empty), it could move one cell or remains immovable. Looking at the

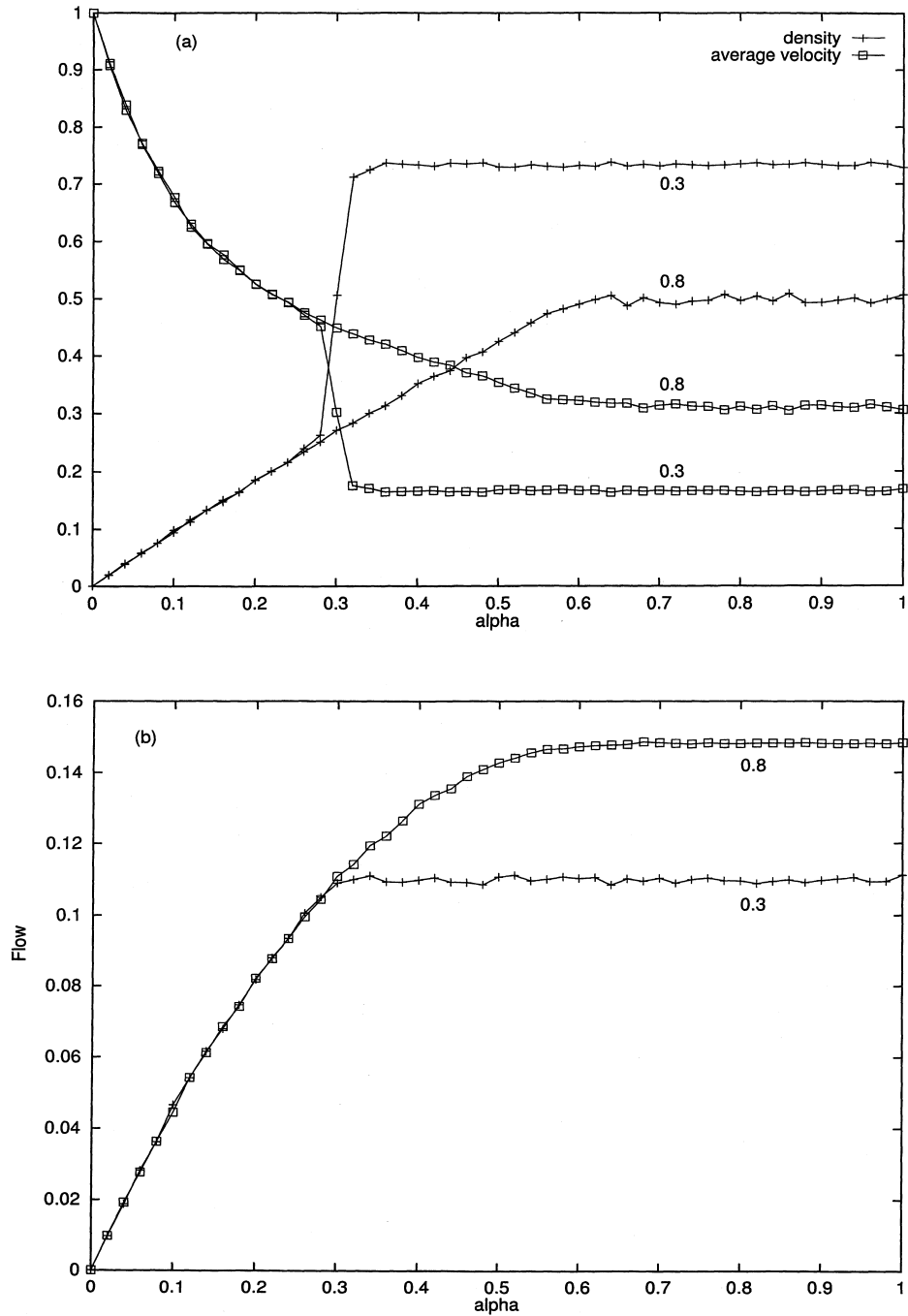


FIG. 1. (a) Variation of density ( $\rho$ ) and the average velocity ( $\langle v \rangle$ ); (b) Variation of the flow, as a function of  $\alpha$ , for  $v_{\max} = 1$  and  $p = 0.5$ , with different values of  $\beta$  indicated in the figures.

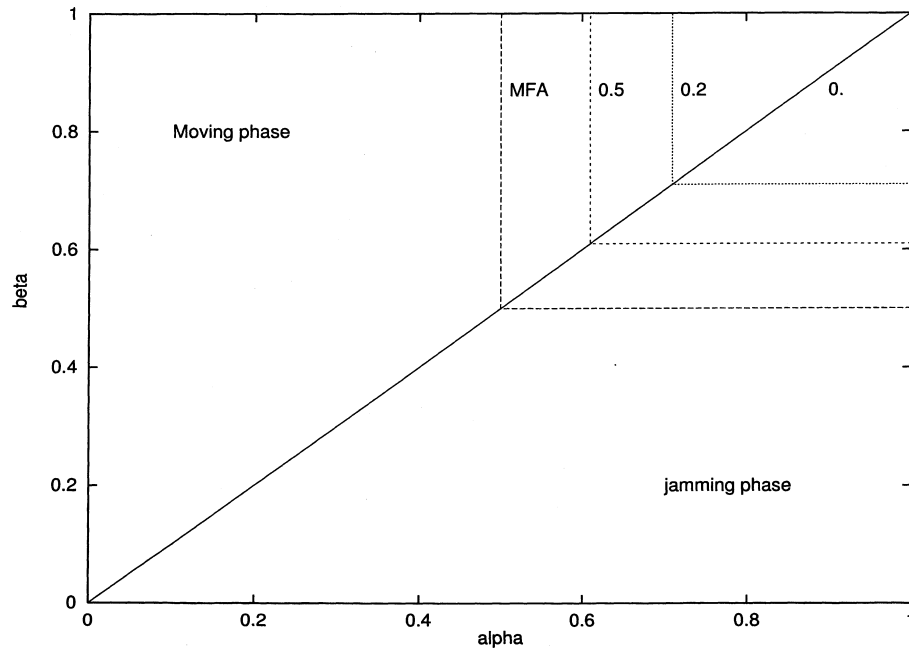


FIG. 2. Phase diagram  $(\alpha; \beta)$  in the case  $v_{\max} = 1$ , obtained from MFA (continuous line) and simulations (dashed line) for different values of  $p$  indicated in the figures.

time evolution of the MFA equations (Eqs. (17-21)), we note that the braking probability term plays a major role in the steady state. In order to check the effect of the braking probability, we show the comparison between the MFA and our simulations, respectively, in figures 3(a) and 3(b), where the average velocity varies as a function of  $\alpha$  for  $\beta = 0.7$  and different values of  $p$ . The gap between the moving phase and jamming phase occurs at the same points for both the MFA and simulations. However, in the moving phase, the average velocity obtained from the MFA equations decreases with increasing  $\alpha$ , while it remains constant by varying  $\alpha$  within our simulations. For  $p = 0$ , the transition arises at  $\alpha = \beta$ ; with increased braking probability, the transition occurs at  $\alpha < \beta$ . So, looking at the equations (19) and (21) which exhibit, respectively, the entering rate and removing rate, we note that all the terms depend on the braking probability except the term  $P_1(100)$  in the entering rate equation. Hence, the cars entering the road in the first cell could leave it independently of the braking probability when the cell ahead is empty, this fact favors entering of cars since the first cell is often empty. Therefore, increasing the braking probability the equilibrium between the entering and removing rates is destroyed and the number of cars entering the road is greater than the number of cars leaving it.

The maximal flow phase does not occur even for high braking probability (Fig. 4). For  $v_{\max} = 2$  the maximal flow phase occurs at  $\frac{1}{2} = \frac{1}{3}$  [3, 4] and corresponds to  $\hat{A} = \frac{2}{3}$ . We could check the absence of the maximal flow phase at the point  $\alpha = \beta = 1$  for  $p = 0$ , this is a deterministic case which exhibits the configuration 100010001... (i.e.,  $\langle v \rangle = 2$ ,  $\frac{1}{2} = \frac{1}{4}$  and  $\hat{A} = \frac{1}{2}$ ), it is clear that it presents one point in a moving phase. For reaching this point, we show in Figure 5 the variation of  $\frac{1}{2}$  and  $\hat{A}$  in the line  $\beta = 1$  for the deterministic case  $p = 0$ . The order

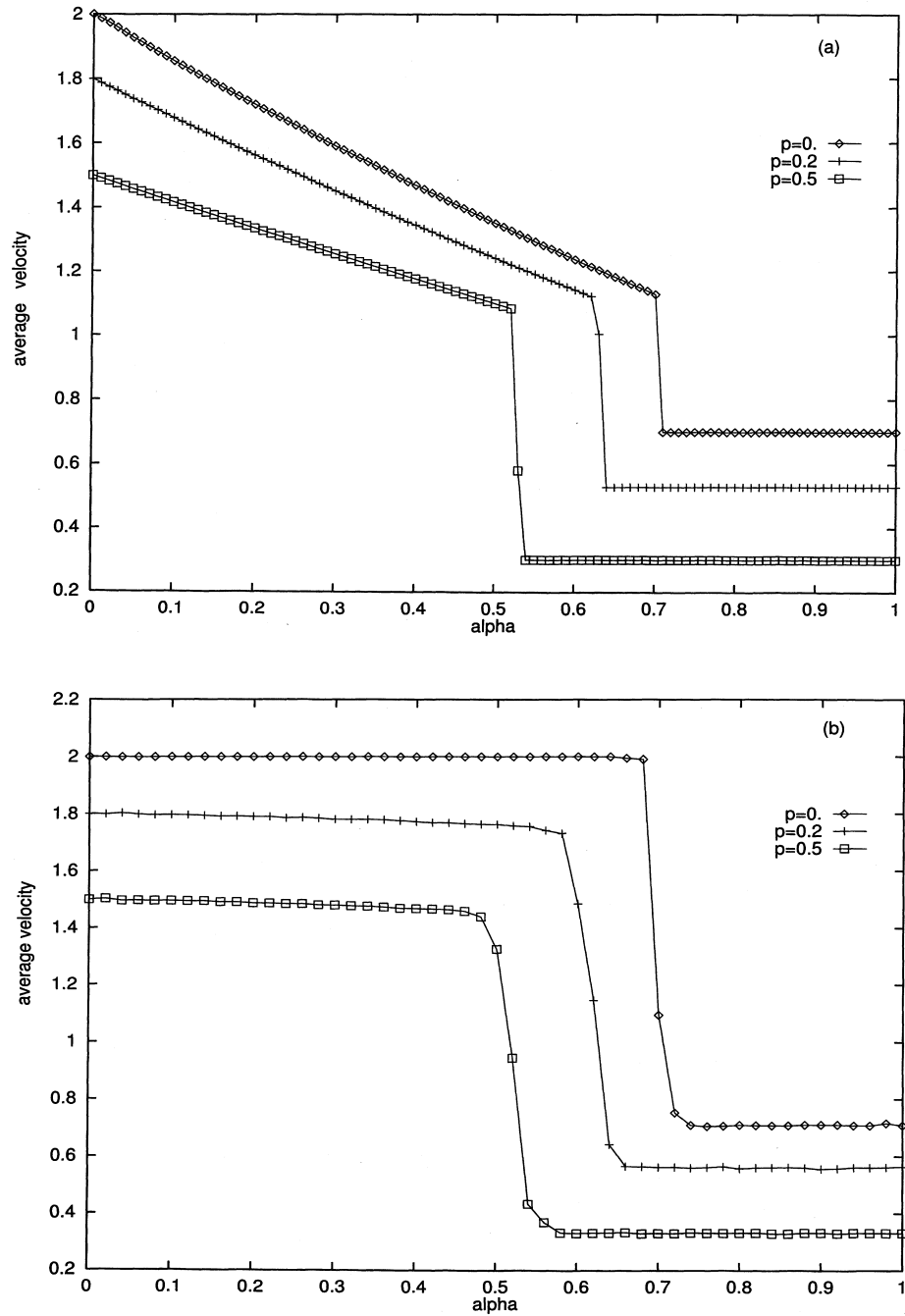


FIG. 3. Variation of  $\langle v \rangle$  as a function of  $\alpha$  ( $\tau = 0.7$ ) for various values of  $p$ , (a) MFA, (b) simulations.

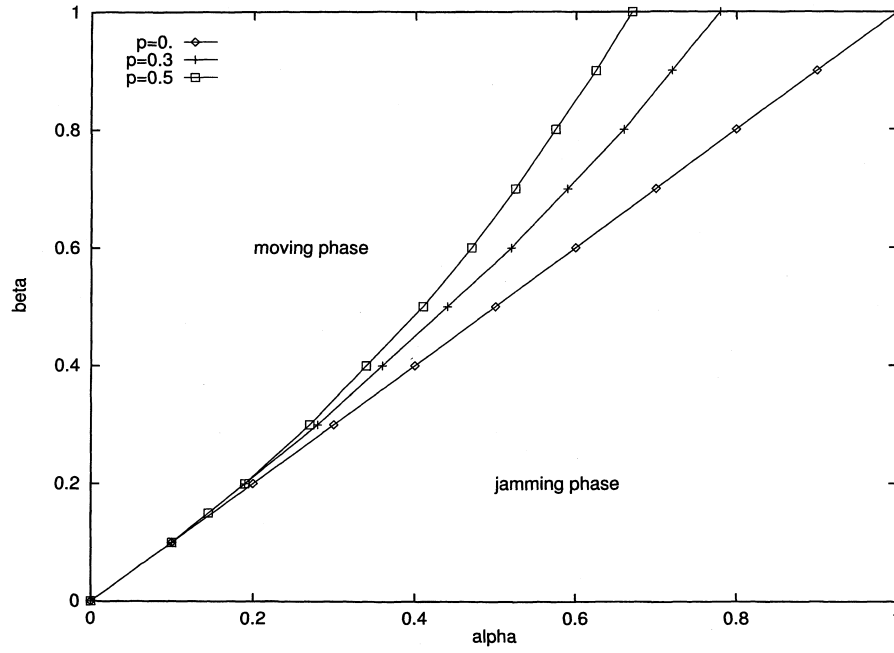


FIG. 4. Phase diagram ( $\alpha$ ;  $\beta$ ) in the case  $v_{max} = 2$ , for different values of  $p$ .

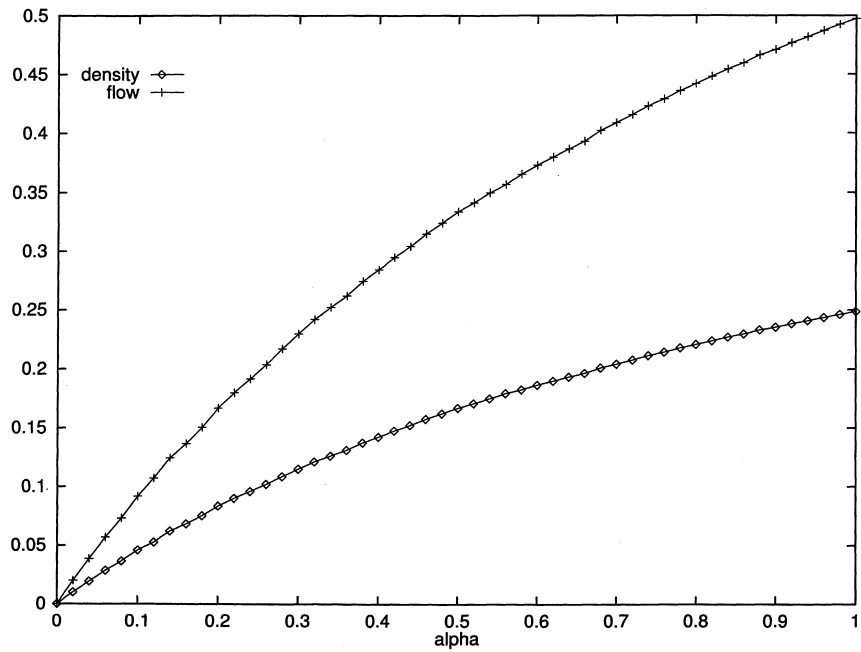


FIG. 5. Variation of  $\langle v \rangle$  as a function of  $\alpha$  for  $\beta = 1$  and  $p = 0$ .

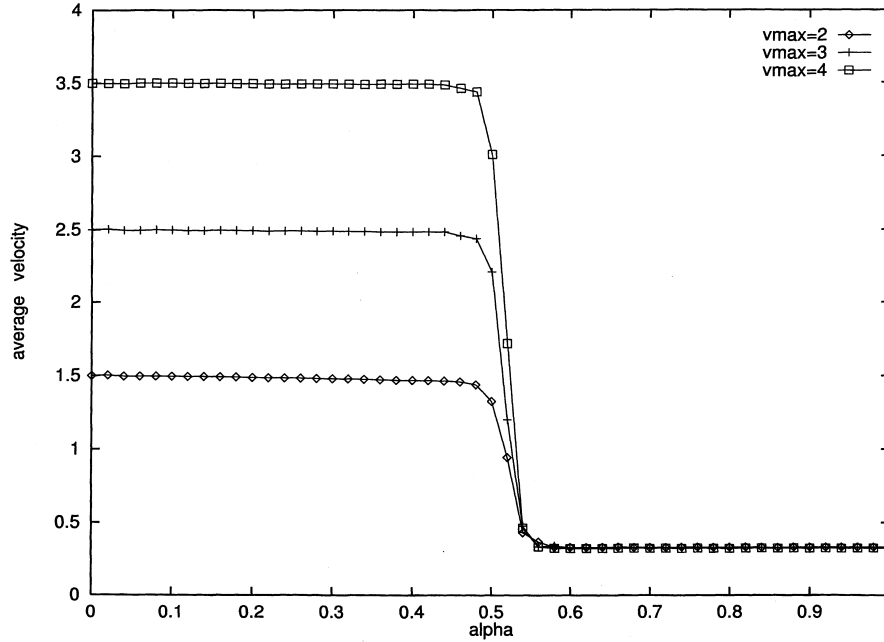


FIG. 6. Variation of  $\langle v \rangle$  as a function of  $\alpha$  for  $\tau = 0.7$  and  $p = 0.5$ , for various values of  $v_{\max}$ .

parameter  $\frac{1}{2}$  increases uniformly without undergoing any gap until reaching a maximal density (at  $\alpha = 1$ ) less than  $\frac{1}{2} = \frac{1}{3}$ . Increasing the braking probability, the jamming phase arises before reaching  $\tau = 1$ , and the density at  $\alpha = \tau = 1$  is greater than  $\frac{1}{2} = \frac{1}{2}$ . The equilibrium between the entering and removing rates on the one hand and the average velocity on the other hand is never reached, which leads to an instability (i.e., the gap between jamming and moving phases) of the system at  $\frac{1}{2} = \frac{1}{3}$ .

In figure 6 we show the dependence of  $\langle v \rangle$  upon  $\alpha$  for fixed values of  $p$  and  $\tau$  and different values of  $v_{\max}$  ( $v_{\max} > 1$ ). The first-order transition occurs at the same value of  $\alpha$ . These results confirm the arguments quoted above according to which the transition at  $\alpha < \tau$  is merely caused by the possibility of cars in the first cell to leave it independently of the braking probability.

#### IV. Conclusion

In this paper, we have established the boundary effects on the density and the flow of cars in the road by the MFA and simulations. The model studied here is based on the NS models with the maximal acceleration  $v_{\max}$  (FI models), the cars could enter the road with a probability  $\alpha(1-p)$  and removed from it with a probability  $\tau(1-p)$ . The system exhibits three phases for  $v_{\max} = 1$ , namely, a moving phase, a jamming phase and the maximal flow phase. The first-order transition from the moving phase to jamming one occurs at  $\alpha = \tau$ , the maximal flow phase appears at  $\frac{1}{2} = \frac{1}{2}$  for high values of  $\alpha$  and  $\tau$ , and the passage from the moving and jamming phases to it is a second-order transition. For  $v_{\max} = 2$ , since the high velocity of cars the maximal flow phase

disappears and the system exhibits only the first-order transition. The first order transition occurs at  $\rho^* < \rho_c$  for  $p \neq 0$  and at  $\rho^* = \rho_c$  for  $p = 0$ . The MFA phase diagram is in good agreement with one obtained by simulations for  $v_{\max} = 2$ . However, the order parameter  $\langle v \rangle$  decreases with increasing  $\rho$  from the MFA equations, while it remains constant within our simulations. For  $v_{\max} = 1$ , the MFA phase diagram coincide with one obtained by simulations for  $p \neq 1$ , otherwise, the maximal flow phase region shrinks with decreasing  $p$ , the maximal flow phase exist only at  $\rho^* = \rho_c = 1$  for  $p = 0$ .

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