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# Energy Associated with the Bianchi Type VI0 Universe

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We calculate the energy of a model of the universe based on the Bianchi type VI0 metric using the energy-momentum complexes of Landau and Lifshitz and of Papapetrou. The energy due to the matter plus field is equal to zero. This result is the same as that which we obtained for the Bianchi type VI0 universe using the energy-momentum complexes of Tolman, Bergmann and Møller.

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## I. Introduction

One of the most interesting and intricate problems of relativity is energy-momentum localization. The different attempts at constructing an energy-momentum density have not led to a generally accepted expression. Instead, there are various energy-momentum complexes including those of Einstein [1], Tolman [2], Landau and Lifshitz [3], Papapetrou [4], Bergmann [5], Weinberg [6], and Møller [7]. Cooperstock [8] gave his opinion that the energy and momentum are confined to the regions of non-vanishing energy-momentum tensor of the matter and all non-gravitational fields. Although the energy-momentum complexes are coordinate dependent they can give a reasonable result if calculations are carried out in Cartesian coordinates. Some interesting results obtained recently lead to the conclusion that different energy-momentum complexes give the same energy distribution for a given space-time [9-16].

Tryon [17] assumed that the net energy of the Universe may be equal to zero. Rosen [18] computed, in the Einstein prescription, the energy of a closed homogeneous isotropic universe described by a Friedmann-Robertson-Walker (FRW) metric. The total energy is zero. Johri *et al*. [19] using the Landau and Lifshitz energy-momentum complex, found that the total energy of a FRW spatially closed universe is zero at all times, irrespective of the equations of state of the cosmic fluid. Also, the total energy enclosed within any finite volume of the spatially flat FRW universe is zero at all times. Banerjee and Sen [20] calculated, in the Einstein prescription the total energy density of the Bianchi type I solutions. The total energy density was found to be zero everywhere. Xulu [21], using the prescriptions of Landau and Lifshitz, Papapetrou and Weinberg, found that the energy of the Universe, in the case of the Bianchi type I model, is zero. The author obtained the energy distribution of a Bianchi type VI0 universe, using the energy-momentum complexes of Tolman, Bergmann and Møller [22]. The energy due to the matter plus field is equal to zero.

The purpose of this paper is to compute the energy of a model of the universe based on the Bianchi type VI0 metric, using the energy-momentum complexes of Landau and Lifshitz and of Papapetrou. We use the geometrized units $(G = 1, c = 1)$ and follow the convention that Latin indices run from 0 to 3.

## II. Energy of the bianchi type VI0 universe

It is well known that for the homogeneous space-times there is the Bianchi cassification. A class of perfect fluid space-times of Bianchi type VI0 was found by Dunn and Tupper (1976), which includes the dust solution found by Ellis and MacCallum (1969). Another case of a Bianchi VI0 solution corresponding to the Einstein-Maxwell hypersurface homogeneous solution, was given by Tariq and Tupper (1975). The vacuum solution given by Ellis and MacCallum (1969) can be used to generate another Bianchi VI0 perfect fluid solution given by Wainwright *et al*. (1979). In the case of the metric considered below, for the class of perfect fluid space-times Bianchi VI0 given by Dunn and Tupper (1976) and for the Einstein-Maxwell hypersurface homogeneous solution given by Tariq and Tupper (1975), we have $B(t) = C(t)$.

We consider the line element that describes a model of universe based on the Bianchi type VI0 metric

$$ds2 = dt^2 - A^2(t)dx^2 - B^2(t)e^{-2x}dy^2 - C^2(t)e^{2x}dz^2. \tag{1}$$

The symmetric energy-momentum complex of Landau and Lifshitz [3] is given by

$$L^{ij} = \frac{1}{16\pi}S^{ijkl}{}_{;kl}, \tag{2}$$

where

$$S^{ijkl} = -g(g^{ij}g^{kl} - g^{ik}g^{jl}). \tag{3}$$

$L^{00}$ and $L^{0\alpha}$ are the energy and momentum densities.

Integrating $L^{ik}$ over the three-space gives the energy and momentum components

$$P^i = \iiint L^{i0}dx^1dx^2dx^3. \tag{4}$$

$P^0$ is the energy and $P^\alpha$ are the momentum components. For the metric given by (1) we obtain

$$\begin{aligned} L^{00} &= 0, \\ L^{\alpha 0} &= 0. \end{aligned} \tag{5}$$

From (5) it follows that all of the components of $P^i$ vanish.

In the Papapetrou prescription [4], the energy-momentum complex is given by

$$\Sigma^{ik} = \frac{1}{16\pi}N^{iklm}{}_{;lm}, \tag{6}$$

where

$$N^{iklm} = \sqrt{-g}(g^{ik}\eta^{lm} - g^{il}\eta^{km} + g^{lm}\eta^{ik} - g^{lk}\eta^{im}), \tag{7}$$

with

$$N^{ikl} = N^{iklm}{}_{;m} \tag{8}$$

and, also, we have

$$\eta^{ik} = \mathrm{diag}(1, -1, -1, -1). \tag{9}$$

In this case $\Sigma_{00}$ and $\Sigma_{\alpha 0}$ are the energy and, respectively, the momentum densities. Also, the symmetric energy-momentum complex of Papapetrou satisfies the local conservation laws

$$\frac{\partial \Sigma^{ik}}{\partial x^k} = 0. \tag{10}$$

The energy and momentum are given by

$$P^i = \iiint \Sigma^{i0} dx^1 dx^2 dx^3. \tag{11}$$

Using (1) and (6) we have

$$\Sigma^{00} = 0, \quad \Sigma^{\alpha 0} = 0. \tag{12}$$

From (12) we find that all the components of $P^i$ vanish.

In the both prescriptions we obtain that the total energy due to the matter and field is zero.

## III. Discussions

Bondi [23] maintaines that a nonlocalizable form of energy is not admissible in relativity, so its location can in principle be found. Some interesting results which have been found recently show that several energy-momentum complexes can give the same acceptable result for a given space-time. Also, in his recent paper Virbhadra [24] emphasized that, though the energy-momentum complexes are non-tensors under general coordinate transformations, the local conservation laws with them hold in all coordinate systems. Chang, Nester and Chen [25] showed that the energy-momentum complexes are actually quasilocal and legitimate expressions for the energy-momentum.

We obtain in the prescriptions of Landau and Lifshitz and that of Papapetrou the same result for the total energy of a Bianchi type VI0 universe. The total energy due to the matter plus field is zero. Also, the energy distribution is the same as that which we obtained [22] using the energy-momentum complexes of Tolman, Bergmann and Møller. This result is one more piece of evidence supporting the idea of the localization of energy in general relativity.

The total energy density vanishes everywhere because the energy contributions from the matter and gravitational field inside an arbitrary two-surface in the case of the anisotropic model of universe based on the Bianchi type VI0 metric, cancel each other.

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