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# Resonant Final-State Interactions in Exclusive Hadronic $D$ Decays to $\eta'$ or $\eta$

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For hadronic two-body decays of charmed mesons involving $\eta$ or $\eta'$, resonance-induced final-state interactions (FSI) that mimic the $W$-exchange or the $W$-annihilation topology can play an essential role. In particular, the decays $D^0 \to \overline{K}^0\eta'$ and $D^+ \to \pi^+\eta$, which are largely suppressed in the absence of FSI, are enhanced dramatically by resonant FSI. It is stressed that the effect of resonant FSI is negligible for $\rho^+(\eta,\eta')$ final states because of the mismatch of the $G$ parity of $\rho^+(\eta,\eta')$ and the $J = 0$, $I = 1$ meson resonance. We argue that a possible gluon-mediated process in which two gluons couple directly to the gluonic component of the $\eta'$, e.g., the gluonium, rather than to the flavor-singlet $\eta_0$, can enhance both modes $D_s^+ \to \rho^+\eta'$ and $D_s^+ \to \rho^+\eta$, especially the former; that is, this new mechanism can account for the unexpectedly large branching ratio of $\rho^+\eta'$ without suppressing $\rho^+\eta$.

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## I. Introduction

We have shown recently in [1] that, in the decays of charmed mesons into the final states containing an $\eta$ or $\eta'$, final-state interactions (FSI) in the resonance formation are able to enhance $B(D^0 \to \overline{K}^0\eta')$ and $B(D^+ \to \pi^+\eta)$ by an order of magnitude. Resonance-induced coupled-channel effects will suppress $D_s^+ \to \pi^+\eta$ and enhance $D_s^+ \to \pi^+\eta'$. Contrary to $D \to P\eta^{(\prime)}$ decays, resonant FSI play only a minor role for $D^0 \to \overline{K}^{*0}\eta^{(\prime)}$ and do not contribute to $(D^+, D_s^+) \to \rho^+\eta^{(\prime)}$. We argued that it is difficult to understand the observed large decay rates of the $\rho^+\eta'$ and $\rho^+\eta$ decay modes of $D_s^+$ simultaneously. FSI are not helpful due to the absence of $D_s^+ \to PP$ decays that have much larger decay rates than $D_s^+ \to \rho^+\eta'$. $W$-annihilation and a possible production of the $\eta'$ due to gluon-mediated processes can in principle enhance $B(D_s^+ \to \rho^+\eta')$, but, unfortunately, they will also suppress $B(D_s^+ \to \rho^+\eta)$.

In [1] we have followed [2] to use the strong reaction matrix $K_0$ together with the unitarity constraint of the $S$ matrix to study the effects of resonant FSI and showed that resonance-induced FSI amount to modifying, for example, the $W$-exchange amplitude $C$ in $D^0 \to \overline{K}\pi, \overline{K}\eta, \overline{K}\eta'$ decays by [2]

$$C \to C + \left(C + \frac{1}{3}A\right)(\cos\delta e^{i\delta} - 1) \tag{1.1}$$

and leaving the other quark-diagram amplitudes intact, where $A$ is an external $W$-emission amplitude. In the present paper we will derive the above relation in a rigorous way and find that the modification due to FSI for the $W$-exchange amplitude as shown in Eq. (1.1) is too small by a factor of 2.

In the present paper we will update the previous analysis [1] by correcting the error occurred in Eq. (1.1), discussing its implication and employing the new measurement of the $D_s^+$ lifetime. Moreover, we shall show explicitly that contributions from resonant FSI to the decays $(D^+, D_s^+) \to \rho^+\eta^{(\prime)}$ should be negligible, otherwise the predicted branching ratios will become too large compared to experiment. This is ascribed to the mismatch of the $G$ parity of $\rho^+(\eta, \eta')$ and the $J = 0$, $I = 1$ meson resonance. We will also employ the mode $D^0 \to \overline{K}^0\pi^0$ as an example to demonstrate that resonance-induced FSI, which are crucial for some two-body decays involving one single isospin component, e.g. the final state containing an $\eta$ and $\eta'$, play only a minor role compared to isospin FSI for decay modes involving several different isospin components. We then turn to some possible explanation of the unexpectedly large branching ratio of $D_s^+ \to \rho^+\eta'$. Finally we discuss in detail the possible sources of theoretical uncertainties for estimating the effects of resonant FSI.

## II. Resonant final-state interactions

There are several different forms of FSI: elastic scattering and inelastic scattering such as quark exchange, resonance formation, ..., etc. Since FSI are nonperturbative in nature, in general it is notoriously difficult to calculate their effects. Nevertheless, the effect of resonance-induced coupled-channel FSI can be estimated provided that the mass and the width of the nearby resonances are known. It appears that the resonance formation of FSI via $q\bar{q}$ resonances is probably the most important one if the final state has only one single isospin component. For previous studies of the effects of resonant FSI in charm decays, see [3, 2, 1, 4, 5].

In the presence of resonances, the decay amplitude of the charmed meson $D$ decaying into two mesons $M_1M_2$ is modified by rescattering through a multiplet of resonances [6]*

$$A(D \to M_iM_j)^{FSI} = A(D \to M_iM_j) - i\frac{\Gamma}{E - m_R + i\Gamma/2} \times \sum_r c_{ij}^{(r)} \sum_{kl} c_{kl}^{(r)*} A(D \to M_kM_l), \tag{2.1}$$

where $c_{ij}^{(r)}$ are normalized coupling constants of $M_iM_j$ with the scalar resonance $r$, obeying the relations

$$\sum_{ij} c_{ij}^{(r)} c_{ij}^{(s)*} = \delta_{rs}, \qquad \sum_{ij} |c_{ij}^{(r)}|^2 = 1. \tag{2.2}$$

The presence of a resonance shows itself in a characteristic behavior of phase shifts near the resonance. For each individual resonant state $r$, there is an eigenstate of $A(D \to M_iM_j)$ with eigenvalue [6]

$$e^{2i\delta_r} = 1 - i\frac{\Gamma}{m_D - m_R + i\Gamma/2}, \tag{2.3}$$

* The same expression is also given in [3] except that the phase in Eq. (3.3) of [3] is too small by a factor of 2.

in the rest frame of the charmed meson, where $m_R$ and $\Gamma$ are the mass and the width of the resonance, respectively. Therefore, resonance-induced FSI are amenable technically in terms of the physical resonances.

**II-1.** $D^0 \to (\bar K^0, \bar K^{*0})(\eta, \eta')$ **decays**

To illustrate the effect of FSI in the resonance formation, consider the decays $D^0 \to \bar K_i P_j$ as an example. The only nearby $0^+$ scalar resonance with $(s\bar d)$ quark content in the charm mass region is $r = K_0^*(1950)$ and the states $\bar K_i P_j$ are $K^-\pi^+; \bar K^0\pi^0; \bar K^0\eta; \bar K^0\eta'$. The quark-diagram amplitudes for $D^0 \to K^-\pi^+$, $\bar K^0\pi^0$, $\bar K^0\eta_{ns}$ and $\bar K^0\eta_s$, where $\eta_{ns} = \frac{1}{\sqrt{2}}(u\bar u + d\bar d)$ and $\eta_s = s\bar s$, are given by (see Table III of [7]):

$$\begin{aligned} &A(D^0 \to (\bar K\pi)_{3/2}) = \frac{1}{\sqrt{3}}(A + B); \quad A(D^0 \to (\bar K\pi)_{1/2}) = \frac{1}{\sqrt{6}}(2A - B + 3C); \\ &A(D^0 \to \bar K^0\eta_{ns}) = \frac{1}{\sqrt{2}}(B + C); \qquad A(D^0 \to \bar K^0\eta_s) = C; \end{aligned} \tag{2.4}$$

where the subscripts 1/2 and 3/2 denote the isospin of the $\bar K\pi$ system. In Eq. (2.4), $A$ is the external $W$-emission amplitude, $B$ the internal $W$-emission amplitude and $C$ the $W$-exchange amplitude [7].† Consider the $D$-type coupling for the strong interaction $P_1P_2 \to P'$ ($P'$: scalar meson), namely $\kappa\,\mathrm{Tr}(P'\{P_1, P_2\})$ with $\kappa$ being a flavor-symmetric strong coupling [2]. Noting that $(\bar K\pi)_{3/2}$ does not couple to $(\bar K\pi)_{1/2}$, $\bar K^0\eta_{ns}$, and $\bar K^0\eta_s$ via FSI, the matrix $\chi^2$ arising from two $D$-type couplings in the $I = \frac{1}{2}$ sector has the form:

$$\chi^2 \propto \kappa^2 \begin{pmatrix} \frac{3}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{\sqrt{2}} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{\sqrt{3}}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \end{pmatrix} \tag{2.5}$$

in the basis of $(\bar K\pi)_{1/2}$, $\bar K^0\eta_{ns}$, $\bar K^0\eta_s$. Hence, the normalized matrix $\chi^2$ is given by

$$\chi^2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{6}} \\ \frac{1}{2\sqrt{3}} & \frac{1}{6} & \frac{1}{3\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{3\sqrt{2}} & \frac{1}{3} \end{pmatrix}. \tag{2.6}$$

Then it is easily seen that

$$\begin{aligned} A(D^0 \to \bar K^0\eta_s)^{FSI} &= C^{FSI} = C + (e^{2i\delta_r} - 1) \\ &\times \left[\frac{1}{\sqrt{6}}A(D^0 \to (\bar K\pi)_{1/2}) + \frac{1}{3\sqrt{2}}A(D^0 \to \bar K^0\eta_{ns}) + \frac{1}{3}A(D^0 \to \bar K^0\eta_s)\right]; \end{aligned} \tag{2.7}$$

† The quark-diagram amplitudes $A; B; C; D$ for external $W$-emission, internal $W$-emission, $W$-exchange and $W$-annihilation are sometimes denoted by $T; C; E; A$, respectively, in the literature.

TABLE I. Branching ratios (in units of %) of the charmed meson decays to an $\eta$ or $\eta'$.

| Decay | This work | | Buccella *et al.* [3] | Expt. [11] |
|---|---|---|---|---|
| | without FSI | with resonant FSI | | |
| $D^0 \to \overline{K}^0\eta$ | 0.54 | 0.62 | 0.84 | $0.70 \pm 0.10$ |
| $D^0 \to \overline{K}^0\eta'$ | 0.10 | 2.57 | 1.56 | $1.71 \pm 0.26$ |
| $D^0 \to \overline{K}^{*0}\eta$ | 0.69 | 0.81 | 0.37 | $1.9 \pm 0.5$ |
| $D^0 \to \overline{K}^{*0}\eta'$ | 0.004 | 0.05 | 0.004 | $< 0.10$ |
| $D^+ \to \pi^+\eta$ | 0.02 | 0.31 | 0.34 | $0.30 \pm 0.06$ |
| $D^+ \to \pi^+\eta'$ | 0.29 | 0.72 | 0.73 | $0.50 \pm 0.10$ |
| $D^+ \to \rho^+\eta$ | 0.19 | 0.19 | 0.013 | $< 0.7$ |
| $D^+ \to \rho^+\eta'$ | 0.08 | 0.08 | 0.12 | $< 0.5$ |
| $D_s^+ \to \pi^+\eta$ | 2.57 | 1.95 | 1.30 | $1.7 \pm 0.5$ |
| $D_s^+ \to \pi^+\eta'$ | 3.50 | 4.28 | 5.71 | $3.9 \pm 1.0$ |
| $D_s^+ \to \rho^+\eta$ | 6.27 | 6.27[a] | 7.94 | $10.8 \pm 3.1$ |
| $D_s^+ \to \rho^+\eta'$ | 4.09 | 4.09[a] | 2.55 | $10.1 \pm 2.8$ |

[a]The presence of $W$-annihilation contributions inferred from $D_s^+ \to \omega\pi^+$ will affect the branching ratios of $D_s^+ \to \rho^+\eta^{(\prime)}$; see the text.

and hence

$$C^{FSI} = C + (e^{2i\delta_r} - 1)\left(C + \frac{A}{3}\right). \qquad (2.8)$$

Therefore, resonance-induced FSI amount to modifying the $W$-exchange amplitude and leaving the other quark-diagram amplitudes $A$ and $B$ intact. Since $(e^{2i\delta_r} - 1) = 2(\cos\delta_r e^{i\delta_r} - 1)$, we see that the contribution of resonant FSI to the $W$-exchange amplitude as given in Eq. (1.1) is too small by a factor of 2.

The resonance contribution to FSI, which arises mainly from the external $W$-emission diagram for the decay $D^0 \to (\overline{K}\pi)_{1/2}$ followed by final-state $q\bar{q}$ resonance, has the same topology as the $W$-exchange quark diagram. We thus see that even if the short-distance $W$-exchange vanishes, as commonly asserted, an effective long-distance $W$-exchange still can be induced via FSI in resonance formation.

Considering the $\eta - \eta'$ mixing parameterized by

$$\eta' = \eta_8 \sin\theta + \eta_0 \cos\theta, \qquad \eta = \eta_8 \cos\theta - \eta_0 \sin\theta, \tag{2.9}$$

with $\eta_8$ and $\eta_0$ being SU(3) octet and singlet wave functions respectively, and neglecting the $W$-exchange amplitude $C$, we obtain [1]

$$\begin{aligned} A(D^0 \to \overline{K}^0\eta) &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[ a_2 X^{(D^0\eta,\overline{K}^0)} + a_1 X^{(D^0K^-,\pi^+)} \frac{e^{2i\delta_r}-1}{3\sqrt{6}} \left( -\cos\theta - 2\sqrt{2}\sin\theta \right) \right], \\ A(D^0 \to \overline{K}^0\eta') &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[ a_2 X^{(D^0\eta',\overline{K}^0)} + a_1 X^{(D^0K^-,\pi^+)} \frac{e^{2i\delta_r}-1}{3\sqrt{6}} \left( -\sin\theta + 2\sqrt{2}\cos\theta \right) \right], \end{aligned} \tag{2.10}$$

and

$$\begin{aligned} A(D^0 \to \overline{K}^{*0}\eta) = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \Bigg[ & a_2 X^{(D^0\eta,\overline{K}^{0*})} \\ & + a_1 \left( X^{(D^0K^{*-},\pi^+)} + X^{(D^0K^-,\rho^+)} \right) \frac{e^{2i\delta_{r'}}-1}{6\sqrt{6}} \left( -\cos\theta - 2\sqrt{2}\sin\theta \right) \Bigg], \\ A(D^0 \to \overline{K}^{*0}\eta') = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \Bigg[ & a_2 X^{(D^0\eta',\overline{K}^{0*})} \\ & + a_1 \left( X^{(D^0K^{*-},\pi^+)} + X^{(D^0K^-,\rho^+)} \right) \frac{e^{2i\delta_{r'}}-1}{6\sqrt{6}} \left( -\sin\theta + 2\sqrt{2}\cos\theta \right) \Bigg], \end{aligned} \tag{2.11}$$

where $X^{(DM_1,M_2)}$ denotes the factorizable amplitude with the meson $M_2$ being emitted out:

$$X^{(DM_1,M_2)} = \langle M_2 | (\bar{q}_1 q_2) | 0 \rangle \langle M_1 | (\bar{q}_3 c) | D \rangle, \tag{2.12}$$

with $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$. Explicitly,

$$\begin{aligned} X^{(D^0\eta^{(\prime)},\overline{K}^0)} &= i f_K (m_D^2 - m_{\eta^{(\prime)}}^2) F_0^{D^0\eta^{(\prime)}}(m_K^2), \\ X^{(D^0K^-,\pi^+)} &= i f_K (m_D^2 - m_K^2) F_0^{D^0K^-}(m_\pi^2), \\ X^{(D\eta^{(\prime)},K^*)} &= 2 f_{K^*} m_{K^*} F_1^{D\eta^{(\prime)}}(m_{K^*}^2)(\varepsilon \cdot p_D), \\ X^{(D^0K^-,\rho^+)} &= 2 f_\rho m_\rho F_1^{D^0K^-}(m_\rho^2)(\varepsilon \cdot p_D), \\ X^{(D^0K^{*-},\pi^+)} &= 2 f_\pi m_{K^*} A_0^{D^0K^{*-}}(m_\pi^2)(\varepsilon \cdot p_D), \end{aligned} \tag{2.13}$$

where the form factors $F_0$, $F_1$ and $A_0$ are those defined in [8].

Since $F_0^{D\eta'}(0) < F_0^{D\eta}(0)$ [1] and the available phase space for $K\eta'$ is less than that for $K\eta$, the factorization approach implies less $\eta'$ production than $\eta$ in $D^0 \to \overline{K}^{0}\eta^{(\prime)}$ decays, in disagreement with experiment (see Table I). To see how the mechanism of resonant FSI works, notice that the big parentheses in Eqs. (2.10, 2.11) reflect the coefficient of the $W$-exchange amplitude. Taking $\theta = -19.5^\circ$ as a benchmark, the wave functions of the $\eta$ and $\eta'$ have the simple expressions [9]:

$$\eta = \frac{1}{\sqrt{3}}(u\bar u + d\bar d - s\bar s), \qquad \eta' = \frac{1}{\sqrt{6}}(u\bar u + d\bar d + 2s\bar s). \tag{2.14}$$

In Eqs. (2.10, 2.11), $(\cos\theta + 2\sqrt{2}\sin\theta) = 0$ indicates that there is no intrinsic $W$-exchange diagram in $D^0 \to \overline{K}^{0(*)}\eta$, while $(2\sqrt{2}\cos\theta - \sin\theta) = 3$ shows that $D^0 \to K^{0(*)}\eta'$ contains the amplitude $3C$. Since the internal $W$-emission amplitude is color suppressed, while the contributions from resonant FSI are induced from the external $W$-emission, it is clear that the decay $D^0 \to \overline{K}^0\eta'$ receives large contributions from FSI in the resonance form, so that its decay rate is larger than that of $D^0 \to \overline{K}^0\eta$.

Using the effective coefficients

$$a_1 = 1.25, \qquad a_2 = -0.51, \tag{2.15}$$

the $\eta - \eta'$ mixing angle $\theta = -22^\circ$ [10], the mass $1945 \pm 10 \pm 20$ MeV and the width $210 \pm 34 \pm 79$ MeV for the $0^+$ resonance $K_0^*(1950)$, $m_R = 1830$ MeV and $\Gamma = 250$ MeV for the $0^-$ resonance $K(1830)$ [11], and various form factors given in [1], the calculated branching ratios are exhibited in Table I. We see that in the presence of resonant FSI, the branching ratio of $\overline{K}^{0*}\eta'$ is enhanced by an order of magnitude, while the decay rate of $\overline{K}^{0*}\eta$ is only slightly increased. The $\eta'$ enhancement for $D^0 \to \overline{K}^0\eta'$ over $D^0 \to \overline{K}^0\eta$, which cannot be accounted for in the factorization approach, can be explained in terms of resonance-induced FSI. For comparison, the theoretical predictions by Buccella *et al.*, [3] are also shown in Table I. As noted in passing, the phase in Eq. (3.3) of [3] is too small by a factor of 2.

It should be stressed that although resonant FSI can make a dramatic effect on hadronic decays of the charmed mesons containing an $\eta$ or $\eta'$, i.e. final states with one single isospin component, they are not expected to play the same essential role in the decay channels involving several different isospin components. A well known example is the decay $D^0 \to \overline{K}^0\pi^0$ with the decay amplitude:

$$\begin{aligned} A(D^0 \to \overline{K}^0\pi^0) = {} & a_2 X^{(D^0\pi^0,\overline{K}^0)} - \frac{1}{3}a_1(e^{2i\delta_r} - 1)X^{(D^0K^-,\pi^+)} \\ & + \left(a_1 X^{(D^0K^-,\pi^+)} + a_2 X^{(D^0\pi^0,\overline{K}^0)}\right)\frac{\sqrt{2}}{3}\left(e^{i(\delta_{1/2} - \delta_{3/2})} - 1\right), \end{aligned} \tag{2.16}$$

where $X^{(D^0\pi^0,\overline{K}^0)} = i f_K(m_D^2 - m_\pi^2)F_0^{D^0\pi^+}(m_K^2)/\sqrt{2}$ and $\delta_I$ are the isospin phase shifts. In naive factorization with $a_{1,2} = c_{1,2} + c_{2,1}/3$ and in the absence of any FSI, we find $B(D^0 \to \overline{K}^0\pi^0) = 0.03\%$ for $c_1(m_c) = 1.26$ and $c_2(m_c) = -0.51$, which is obviously too small compared to the experimental value $(2.12 \pm 0.21)\%$ [11]. In the large-$N_c$ limit where $a_2 = c_2$, the branching ratio is increased to 1.0%. When the resonant FSI are turned on, $B(D^0 \to \overline{K}^0\pi^0)$ is *decreased* to 0.36% ! Using the isospin phase shift difference $(\delta_{1/2} - \delta_{3/2}) = 71.4^\circ$ extracted from the isospin analysis of $D \to \bar{K}\pi$ data, the branching ratio of $D^0 \to \bar{K}^0\pi^0$ is enhanced by isospin FSI to 1.44%. It is clear that in order to understand the color non-suppression of $D^0 \to \bar{K}^0\pi^0$, one needs nonfactorizable effects to account for the non-smallness of $a_2$ and isospin FSI to generate adequate $\bar{K}^0\pi^0$ from $K^-\pi^+$.

**II-2. $D^+ \to (\pi^+, \rho^+)(\eta, \eta')$ decays**

Proceeding as before, resonance-induced coupled-channel effects among the three channels: $K^+\bar{K}^0$, $\pi^+\eta_{ns}$ and $\pi^+\eta_s$ will only modify the magnitude and phase of the $W$-annihilation amplitude $D$ and leave the other quark-diagram amplitudes unaffected [2]:

$$D \to D + \left(D + \frac{1}{3}A\right)\left(e^{2i\delta_r} - 1\right). \tag{2.17}$$

The decay amplitudes of the Cabibbo-suppressed decays $D^+ \to \pi^+\eta^{(\prime)}$ and $\rho^+\eta^{(\prime)}$ in the presence of FSI via $\bar{q}q$ resonance are [1]:

$$\begin{aligned}
A(D^+ \to \pi^+\eta) = \frac{G_F}{\sqrt{2}}V_{cd}^*V_{ud}\Bigg[& a_1 X^{(D^+\eta,\pi^+)} + a_2\left(X_d^{(D^+\pi^+,\eta)} - X_s^{(D^+\pi^+,\eta)}\right) \\
&+ \frac{1}{3\sqrt{3}}a_1 X^{(D^+\bar{K}^0,K^+)}\left(e^{2i\delta_r} - 1\right)\left(\sqrt{2}\cos\theta + 2\sin\theta\right)\Bigg], \\
A(D^+ \to \pi^+\eta') = \frac{G_F}{\sqrt{2}}V_{cd}^*V_{ud}\Bigg[& a_1 X^{(D^+\eta',\pi^+)} + a_2\left(X_d^{(D^+\pi^+,\eta')} - X_s^{(D^+\pi^+,\eta')}\right) \\
&+ \frac{1}{3\sqrt{3}}a_1 X^{(D^+\bar{K}^0,K^+)}\left(e^{2i\delta_r} - 1\right)\left(\sqrt{2}\sin\theta - 2\cos\theta\right)\Bigg],
\end{aligned} \tag{2.18}$$

and

$$\begin{aligned}
A(D^+ \to \rho^+\eta) &= \frac{G_F}{\sqrt{2}}V_{cd}^*V_{ud}\left[a_1 X^{(D^+\eta,\rho^+)} + a_2\left(X_d^{(D^+\rho^+,\eta)} - X_s^{(D^+\rho^+,\eta)}\right)\right], \\
A(D^+ \to \rho^+\eta') &= \frac{G_F}{\sqrt{2}}V_{cd}^*V_{ud}\left[a_1 X^{(D^+\eta',\rho^+)} + a_2\left(X_d^{(D^+\rho^+,\eta')} - X_s^{(D^+\rho^+,\eta')}\right)\right],
\end{aligned} \tag{2.19}$$

where

$$\begin{aligned}
X_q^{(D^+\pi^+,\eta^{(\prime)})} &= i f^q_{\eta^{(\prime)}}\left(m_D^2 - m_\pi^2\right)F_0^{D^+\pi^+}\left(m^2_{\eta^{(\prime)}}\right), \\
X_q^{(D^+\rho^+,\eta^{(\prime)})} &= 2 f^q_{\eta^{(\prime)}} m_\rho A_0^{D^+\rho^+}\left(m^2_{\eta^{(\prime)}}\right)\left(\varepsilon \cdot p_D\right),
\end{aligned} \tag{2.20}$$

and the values of the decay constants $f^q_{\eta^{(\prime)}}$ can be found in [1].

Note that since $\pi^+\pi^0$ does not couple to $\pi^+\eta^{(\prime)}$ by strong interactions, $D^+ \to \pi^+\eta^{(\prime)}$ receive contributions from resonant FSI only through the process $D^+ \to K^+\overline{K}^0 \to \pi^+\eta^{(\prime)}$. As for the decay $D^+ \to \rho^+\eta^{(\prime)}$ one may naively expect that

$$\begin{aligned} A(D^+ \to \rho^+\eta) = \frac{G_F}{\sqrt{2}}V_{cd}^*V_{ud} \Bigg[ & a_1 X^{(D^+\eta,\rho^+)} + a_2 \left( X_d^{(D^+\rho^+,\eta)} - X_s^{(D^+\rho^+,\eta)} \right) \\ & + \frac{1}{6\sqrt{3}}a_1 \left( X^{(D^+\overline{K}^{0*},K^+)} + X^{(D^+\overline{K}^0,K^{+*})} - \sqrt{2}X^{(D^+\pi^0,\rho^+)} \right. \\ & \left. - \sqrt{2}X^{(D^+\rho^0,\pi^+)} \right) \times (e^{2i\delta} - 1) \left( -\sqrt{2}\cos\theta + 2\sin\theta \right) \Bigg]; \end{aligned} \tag{2.21}$$

and likewise for the $\rho^+\eta'$ state, where use of $V_{cs}^*V_{us} \approx -V_{cd}^*V_{ud}$ has been made. Since the factorized term $X^{(D^+\pi^0,\rho^+)} \propto \langle\pi^0|(\bar{d}c)|D^+\rangle$ has a sign opposite to that of $X^{(D^+\overline{K}^0,K^{+*})}$ due to the pion wave function $\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$, there is no cancellation among various contributions to resonant FSI. Employing $\pi(1800)$ as the appropriate $0^-$ resonance with $m_R = 1795 \pm 10$ MeV and $\Gamma = 212 \pm 37$ MeV [11], we find that $\mathcal{B}(D^+ \to \rho^+\eta) = 3.4\%$ and $\mathcal{B}(D^+ \to \rho^+\eta') = 0.4\%$, which are obviously too large compared to the current experimental limit: 0.7% and 0.5%, respectively (see Table I). The point is that the $G$ parity of $\rho\eta$ and $\rho\eta'$ is even, while the $J = 0$, $I = 1$ meson resonance made from a quark-antiquark pair (i.e. $u\bar{d}$) has odd $G$ parity. This is also true for the $W$-annihilation process $c\bar{d} \to u\bar{d}$. As a consequence, the even–$G$ state $\rho\eta$ or $\rho\eta'$ does not couple to any single meson resonances, nor to the state produced by the $W$-annihilation diagram with no gluons emitted by the initial state before annihilation [12].

In the absence of FSI, the branching ratio of $D^+ \to \pi^+\eta$ is very small, of order $10^{-4}$, owing to a large cancellation between external and internal $W$-emission amplitudes, the latter being enhanced by the fact that $X_s^{(D^+\pi^+,\eta)} \approx -X_d^{(D^+\pi^+,\eta)}$. Again, owing to the large branching ratio of $D^+ \to K^+\overline{K}^0$, this mode is essentially induced by FSI through resonance. Since a nearby $0^+$ resonance $a_0$ in the charm mass region has not been observed, we employ $m_R = 1745$ MeV and $\Gamma = 250$ MeV for calculations.

## II-3. $D_s^+ \to (\pi^+, \rho^+)(\eta, \eta')$ decays

The analysis of resonant coupled-channel effects in $D_s^+ \to K^+\overline{K}^0, \pi^+\eta_{ns}, \pi^+\eta_s$ leads to the replacement of the $W$-annihilation amplitude by [2]:

$$D \to D + \left( D + \frac{1}{3}B \right) (e^{2i\delta_r} - 1); \tag{2.22}$$

where $B$ is the internal $W$-emission amplitude for $D_s^+ \to K^+\overline{K}^0$. As before, neglecting the short-distance $W$-annihilation, we then have:

$$
\begin{aligned}
A(D_s^+ \to \pi^+\eta) &= \frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\left[a_1X^{(D_s\eta,\pi^+)} + \frac{1}{3\sqrt{3}}a_2X^{(D_sK^+,\overline{K}^0)}\right.\\
&\quad\left.\times(e^{2i\delta_r}-1)\left(\sqrt{2}\cos\theta - 2\sin\theta\right)\right];\\
A(D_s^+ \to \pi^+\eta') &= \frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\left[a_1X^{(D_s\eta',\pi^+)} + \frac{1}{3\sqrt{3}}a_2X^{(D_sK^+,\overline{K}^0)}\right.\\
&\quad\left.\times(e^{2i\delta_r}-1)\left(\sqrt{2}\sin\theta + 2\cos\theta\right)\right].
\end{aligned}
\tag{2.23}
$$

Unlike $D \to \overline{K}\eta^{(\prime)}, \pi\eta^{(\prime)}$ decays, the resonant FSI here are induced from internal $W$-emission and hence play a less significant role. As noted in [1], $D_s^+ \to \pi^+\eta$ is suppressed in the presence of FSI through resonances, whereas $D_s^+ \to \pi^+\eta'$ is enhanced (see Table I). This is ascribed to the fact that the external $W$-emission amplitudes for $D_s^+ \to \pi^+\eta$ and $\pi^+\eta'$ are opposite in sign due to a relative sign difference between the form factors $F_0^{D_s\eta}$ and $F_0^{D_s\eta'}$. There are several new measurements of the $D_s^+$ lifetime [13]. We use the updated world average $\tau(D_s^+) = (4.95 \pm 0.13)\times 10^{-13}$s [11].

For reasons to be mentioned below, we shall keep the $W$-annihilation contribution in $\rho^+\eta^{(\prime)}$ decays:

$$
\begin{aligned}
A(D_s^+ \to \rho^+\eta) &= \frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\,a_1\left(X^{(D_s\eta,\rho^+)} + X^{(D_s,\eta\rho^+)}\right);\\
A(D_s^+ \to \rho^+\eta') &= \frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\,a_1\left(X^{(D_s\eta',\rho^+)} + X^{(D_s,\eta'\rho^+)}\right).
\end{aligned}
\tag{2.24}
$$

The $W$-annihilation amplitude can be related to the $D_s^+ \to \omega\pi^+$ one via SU(3) symmetry:

$$
\begin{aligned}
A(D_s^+ \to \rho^+\eta) &= \frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\,a_1X^{(D_s\eta,\rho^+)} + \left(\frac{1}{\sqrt{3}}\cos\theta - \sqrt{\frac{2}{3}}\sin\theta\right)A(D_s^+ \to \omega\pi^+);\\
A(D_s^+ \to \rho^+\eta') &= \frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}\,a_1X^{(D_s\eta',\rho^+)} + \left(\frac{1}{\sqrt{3}}\sin\theta + \sqrt{\frac{2}{3}}\cos\theta\right)A(D_s^+ \to \omega\pi^+).
\end{aligned}
\tag{2.25}
$$

The decay $D_s^+ \to \omega\pi^+$, which proceeds through $W$-annihilation topologies, has been observed recently with the branching ratio $(0.28 \pm 0.11)\%$ [14]. Unfortunately, the phase of this decay relative to $X^{(D_s^+\eta',\rho)}$ is unknown. In the extreme case that $A(D_s^+ \to \omega\pi^+)$ is real and opposite to $X^{(D_s^+\eta,\rho^+)}$ in sign, then we find $B(D_s^+ \to \rho^+\eta) = 8.3\%$, whereas $B(D_s^+ \to \rho^+\eta') = 3.6\%$, recalling that the external $W$-emission amplitudes for $\rho^+\eta$ and $\rho^+\eta'$ are opposite in sign. By contrast, if the amplitude of $\omega\pi^+$ is purely imaginary, then the branching ratios of $\rho^+\eta$ and $\rho^+\eta'$

will be 6.4% and 4.1%, respectively. Therefore, even if the former mode is accommodated, the latter is still too small compared to experiment.

If $\rho^+\eta$ and $\rho^+\eta'$ are dominated by the external $W$-emission, then it is advantageous to consider the ratios $R_{\eta^{(\prime)}} \equiv \Gamma(D_s^+ \to \rho^+\eta^{(\prime)})/\Gamma(D_s^+ \to \eta^{(\prime)}e^+\nu)$. Generalized factorization leads to the form-factor-independent predictions $R_\eta = 2.9$ and $R_{\eta'} = 3.5$, while experimentally $R_\eta = 4.4 \pm 1.2$ and $R_{\eta'} = 12.0 \pm 4.3$ [11]. The large discrepancy between theory and experiment for $R_{\eta'}$ means that there must be an additional contribution to $\rho^+\eta'$. An enhancement mechanism has been suggested in [15] that a $c\bar{s}$ pair annihilates into a $W^+$ and two gluons, then the two gluons will hadronize mostly into $\eta'$. If the two gluons couple to $\eta^{(\prime)}$ through the triangle quark loop, they will hadronize into the flavor-singlet $\eta_0$. Since

$$\eta_0 = \eta' \cos\theta - \eta \sin\theta; \tag{2.26}$$

and the $\eta - \eta'$ mixing angle $\theta$ is negative, it is evident that if $\rho^+\eta'$ is enhanced by this mechanism, $\rho^+\eta$ will be suppressed due to the destructive interference between the external $W$-emission and the gluon-mediated process. Specifically, we find that if the branching ratio of $\rho^+\eta'$ is increased to 9.5%, then $B(D_s^+ \to \rho^+\eta)$ will be decreased to 3.9%. The other possibility is that the gluonic component of $\eta'$, which can be identified with the physical state, e.g. the gluonium, couples to two gluons directly. From the wave function of the gluonium [16]

$$\text{gluonium} = -\eta \sin\theta \sin\phi - \eta' \cos\theta \sin\phi + g \cos\phi; \tag{2.27}$$

where $g$ is a glue rich particle, we see that the gluonium contribution can enhance both $\rho^+\eta'$ and $\rho^+\eta$, especially the former; that is, this new mechanism can account for the unexpectedly large branching ratio of $\rho^+\eta'$ without suppressing $\rho^+\eta$.

It is clear that a production of the $\eta'$ due to gluonium-mediated processes can in principle enhance $\rho^+\eta'$ sizeably and $\rho^+\eta$ slightly. Therefore, if the gluon-mediated process is responsible for the major production of $\eta'$ in $D_s^+ \to \rho^+\eta'$ decay, the two gluons in the intermediate state must couple to the gluonium rather than to the $\eta_0$.

Since the additional contribution to $D_s^+ \to \rho^+\eta'$ is not needed to explain the other decays involving $\eta$ and $\eta'$ (see Table I), one may wonder if the new contribution is special only to the above-mentioned decay. We conjecture that this mechanism is operative if the naive $W$-annihilation diagram is prohibited under $G$-parity consideration while allowed when gluons are emitted from the initial quark. Under this conjecture, the gluon-mediated processes are important only for the decays $(D^+; D_s^+) \to \rho^+\eta'$. It is likely that the branching ratio of $D^+ \to \rho^+\eta'$ is enhanced by a factor of 2, namely, $B(D^+ \to \rho^+\eta') = 0.16\%$, which is safely below the current experimental limit (see Table I).

## III. Theoretical uncertainties

The calculation of the effects of resonant FSI suffers from many theoretical uncertainties. It is useful to explain them below.

1. Thus far we have assumed that coupled-channel FSI are dominated by nearby resonances in the charm mass region; other types of FSI, e.g. quark exchange, are not considered in

the present work. Resonances with lower masses, e.g. the $1^-$ resonance $K^*(890)$ and the $0^+$ resonant state $K^*(1430)$ have not been included in our calculations as they are not close to the charm mass region. However, they can contribute to $W$-exchange or $W$-annihilation directly via pole diagrams [17, 18]. Recall that the determination of $a_1$ and $a_2$ from $D \to \overline{K}\pi$ decays is usually obtained by neglecting the $W$-exchange contribution. The inclusion of $K^*(1430)$ resonance will certainly affect the extraction of $a_1$ and $a_2$ [17].

2. For simplicity we have neglected $W$-exchange or $W$-annihilation contributions in our calculations. However, data analysis based on the flavor-SU(3) quark-diagram scheme indicates that $W$-exchange and $W$-annihilation are not negligible [7, 21].

3. Flavor SU(3) symmetry breaking has been introduced to the couplings $c_{ij}^{(r)}$ in the literature via phase-space corrections [3, 4]. For example, in [4] phase-space induced SU(3)-symmetry breaking is included in the coupling constants:

$$c_{ij}^{(r)} = \frac{\langle M_i M_j | H_{\rm eff} | r \rangle \sqrt{p_{ij}}}{\sqrt{\sum_{ij} \langle M_i M_j | H_{\rm eff} | r \rangle^2 p_{ij}}}, \tag{3.1}$$

where $p_{ij}$ is the momentum of the final particles in the $D$ rest frame. In our work, we assume SU(3) flavor symmetry for quark-diagram amplitudes and consider its breaking only at the decay rate level; it seems to us that it is not appropriate to have the phase-space correction in coupling constants.

4. Recently it was found in [19] that phenomenologically the $\eta - \eta'$ mixing angle is given by $\theta = -15.4^\circ$, which is somewhat smaller than the mixing angle $-22^\circ$ employed in the present paper. However, we found empirically that the latter yields a better agreement between theory and experiment than the former. For example, in the absence of resonant FSI, or equivalently $\theta = -19.5^\circ$, $B(D^0 \to \overline{K}^0\eta) = 0.54\%$. The branching ratio is increased to 0.62% when $\theta = -22^\circ$ and decreased to 0.35% at $\theta = -15.4^\circ$, recalling that experimentally $B(D^0 \to \overline{K}^0\eta) = (0.70 \pm 0.10)\%$ [11]. Of course, we cannot conclude that the magnitude of the mixing angle should be larger than $20^\circ$ in view of many simplified assumptions we have made.

5. Resonance-induced FSI are mainly governed by the width and the mass of nearby resonances, which are unfortunately not well determined. For example, a reanalysis in a $K$-matrix formalism [20] quotes $m_R = 1820 \pm 40$ MeV and $\Gamma = 250 \pm 50$ MeV for the $0^+$ resonance $K^*(1950)$. We then obtain $B(D^0 \to \overline{K}^0\eta) = 0.65\%$ and $B(D^0 \to \overline{K}^0\eta') = 3.44\%$, to be compared with 0.62% and 2.57%, respectively for $m_R = 1945$ MeV and $\Gamma = 210$ MeV. Hence, the prediction of $B(D^0 \to \overline{K}^0\eta)$ is significantly affected by the uncertainties in $m_R$ and $\Gamma$ of the resonance.

## IV. Conclusions

For hadronic decay modes $D^0 \to (\overline{K}^0, \overline{K}^{*0})(\eta, \eta')$ and $(D^+, D_s^+) \to (\pi^+, \rho^+)(\eta, \eta')$ which have only one single isospin component, resonance-induced final-state interactions that mimic the $W$-exchange or the $W$-annihilation topology can play an essential role. In particular, the decays $D^0 \to \overline{K}^0\eta'$ and $D^+ \to \pi^+\eta$, which are largely suppressed in the absence of FSI, are enhanced dramatically by resonant FSI. It is stressed that resonant FSI are negligible for $\rho^+(\eta, \eta')$ final states because of the mismatch of the $G$ parity of $\rho^+(\eta, \eta')$ and the $J = 0$, $I = 1$ meson resonance.

We have utilized the mode $D^0 \to \overline{K}^0\pi^0$ as an illustration to demonstrate that resonance-induced FSI, which are crucial for some two-body decays involving one single isospin component, e.g. the final state containing an $\eta$ and $\eta'$, play only a minor role compared to isospin FSI for decay modes involving several different isospin components.

It is difficult to understand the observed large decay rates of $D_s^+ \to \rho^+\eta'$. We argue that a possible gluon-mediated process in which the two gluons couple directly to the gluonic component of the $\eta'$, e.g. the gluonium, rather than to the flavor-singlet $\eta_0$, can enhance both decays $D_s^+ \to \rho^+\eta'$ and $D_s^+ \to \rho^+\eta$, especially the former. Since this additional contribution is not needed to explain the other decays involving the $\eta$ and $\eta'$, we conjecture that this mechanism is operative if the naive $W$-annihilation is prohibited under $G$-parity consideration while allowed when gluons are emitted from the initial quark.

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