

Wave Localization in Two Dimensions

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(Received April 25, 2001)

It has been the dominant view that all waves are localized in two dimensions for any given amount of disorder. Here, I point out that this is not always the case. By a scaling analysis, it is shown that wave localization in two and three dimensional random systems are similar. Extended states are possible in two dimensions.

PACS. 43.25.+y – Nonlinear acoustics, macrosonics.

PACS. 71.55.Jv – Disordered structures; amorphous and glassy solids.

I. Introduction

In 1979, Abrahams *et al.* published a paper [1] which has set the doctrine guiding nearly all later investigations of the phenomenon of wave localization. In this seminal paper, by means of a scaling argument, the authors concluded that all waves are localized in one and two dimensions for any amount of disorder. This has been the prevailing view over the past twenty years, particularly in the studies of localization of classical waves.

Our recent rigorous numerical results, however, show that waves are not always localized in 2D random systems [2, 3]. The work was done with acoustic propagation in water containing many parallel, randomly placed, air-filled cylinders, which could be any gas enclosed within a thin unimportant shell. It was found that, while the transmitted waves are indeed trapped inside the random medium for a range of frequencies for a sufficient number of cylinders, they are extended outside the localized regime.

An immediate criticism of this observation might be that the apparent wave propagation is an artifact of the finite size effect. Although this criticism has already been refuted by two arguments discussed in [2, 3], the fact that the results are numerical is discomfoting. While leaving the details to a fuller paper, here I present a new scaling analysis to resolve the contradiction between the observation and the previous assertion. I believe that the evident conflict is due to the difference in the ways that the wave localization is inferred or interpreted. To be explicit, I first repeat the theory in [1], followed by a new scaling analysis.

According to [1], in the propagating state, the resistance follows the ohmic behavior

$$R \gg L^{2i-d}; \quad (1)$$

where d is the dimension. For a localized state, i.e. large R , the resistance grows exponentially,

$$R \gg e^{L-L_1}; \quad (2)$$

where L_1 is the localization length, which may differ for different dimensions. A scaling function is defined as

$$\bar{\tau} = \frac{\partial \ln R}{\partial \ln L} \tag{3}$$

Taking Eqs. (1) and (2) into (3), we obtain the asymptotic behavior

$$\bar{\tau} \gg \begin{cases} \ln R; & \text{as } R \rightarrow 1 \text{ (Localized)} \\ 2j - d; & \text{as } R \rightarrow 0 \text{ (Ohmic)} \end{cases} \tag{4}$$

From the asymptotic behavior in Eq. (4), we can sketch the universal curves in $d = 1, 2, 3$ dimensions. The central assumption here is continuity: once the wave is localized, an increasing sample size would always mean more localization. The generic behavior of $\bar{\tau}$ is plotted in Fig. 1. It is clear that in the 3D case, the curve crosses the horizontal axis, yielding an unstable fixed point. Above this point, the waves become more and more localized as the sample size increases. Below the critical point, propagation tends to follow the Ohmic behavior as the sample size is enlarged. This fixed point is called the mobility edge which separates the localized and extended states. For the one and two dimensional cases, there is no such fixed point. As the sample size increases, all states move towards the localization regime, as illustrated in Fig. 1. This leads to the conclusion that all waves are localized in one and two dimensions.

It is not difficult to see that the above scaling theory works for situations when both the transmitter and the receiver, used to measure the wave transmission from which the localization is inferred, are located outside the scattering medium. It is my opinion that the reduction in the wave transmission does not guarantee that the wave actually can be *trapped* in the medium once

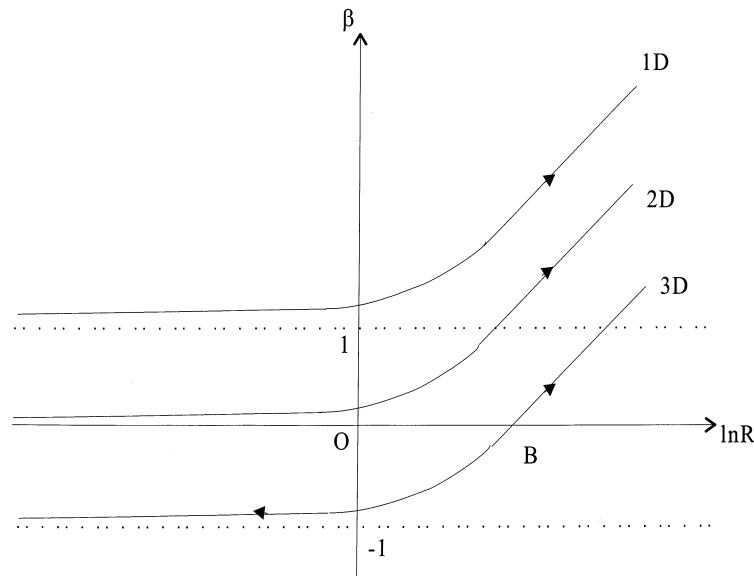


FIG. 1. The scaling function versus $\ln R$ from Eq. (4).

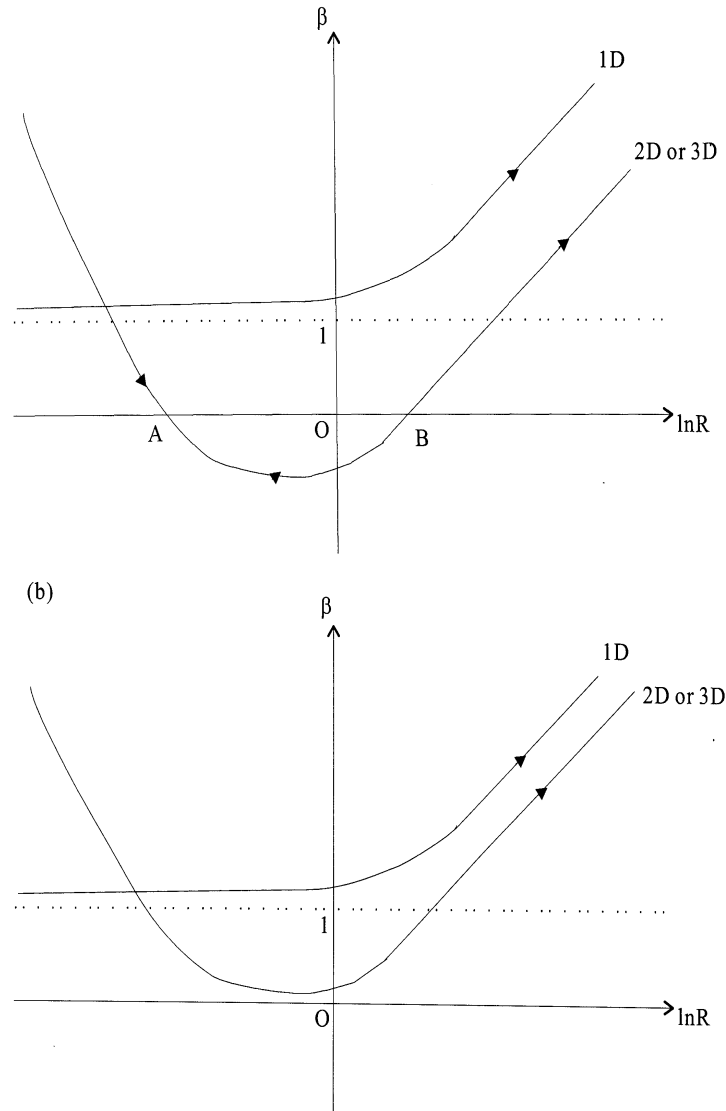


FIG. 2. Plots of β vs $\ln(R)$ for 1, 2, and 3 dimensions.

the source is moved into the randomly scattering medium. In other words, it is necessary to differentiate the situation where the wave is blocked from transmission, from the situation where the wave actually can be localized in the medium. I believe that the latter case is in fact what the concept of wave localization is meant to be [4].

Taking the view that wave localization refers to a situation where transmitted waves in a scattering medium are *trapped* in space and remain confined in the vicinity of the initial site until dissipated, we need to consider the cylindrical and spherical geometries for the 2D and 3D scaling. In line with the above discussion, for small resistance R the medium is assumed to follow the

ohmic behavior. This leads to

$$R \gg \begin{cases} \ln(L=L_0); & \text{for 2D} \\ \frac{1}{L_0} \text{ i } \frac{1}{L}; & \text{for 3D:} \end{cases} ; \quad (5)$$

where L_0 refers to the microscopic size [1]. In the other limit, where the resistance is large, exponential wave localization is expected. By taking into account the geometric factors, the resistance thus grows as

$$R \gg L^{d_i - 1} e^{L=L_1}; \quad (6)$$

The asymptotic behavior for the scaling function in both 2D and 3D becomes

$$- \gg \begin{cases} e^{i \ln(R)}; & \text{for } \ln(R) \ll 1 \\ \ln(R); & \text{for } \ln(R) \gg 1 \end{cases} ; \quad (7)$$

This indicates that the wave localization behavior in 2D and 3D should be similar.

The new scaling function is plotted in Fig. 2. There are two possibilities for two and three dimensions. In the first instance, shown in Fig. 2(a), there are two fixed points: A and B. It is clear that A and B are, respectively, stable and unstable fixed points. Point B separates the localization state and the extended state. When $\ln(R)$ is greater than B, the increasing sample size leads to an infinite resistance, thus the waves become localized *inside* the medium. When $\ln(R)$ is below point B, increasing the sample size leads to the fixed point A, where the increasing L will no longer affect the resistance. On first sight, this feature seems awkward. After closer inspection, it can be understood as follows. For extended states, when there is no absorption, by energy conservation the total wave transmission, a function of the resistance, will not change along the propagation path. The transmission will thus not vary as the sample size changes. This picture has been supported by the previous simulations [5]. Therefore, as in 3D, a transition between the localized and extended states is possible for two dimensions. The observation in [2] thus has an explanation. The second possibility is shown in Fig. 2(b). There is no fixed point. In this case, all waves in two and three dimensions are localized as in the 1D situation. This is expected to occur when the amount of disorder is exceedingly large [2, 6].

This work received support from the National Science Council. Stimulating discussions with Profs. Zhao-Qing Zhang (HKUST) and T. K. Lee (Academia Sinica) were greatly appreciated. K. X. Wang is thanked for preparing the figures.

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