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Propagation of Electromagnetic Field From a Pulsed Electric Dipole in a Dielectric Medium

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The transient electromagnetic fields of a vertical electric dipole antenna with an impulsive current in a dielectric medium are expressed in an analytical form. It is found that the first part of the electromagnetic fields of the excited electromagnetic pulse is an impulsive wave, which propagates with the speed $1/\sqrt{\mu\varepsilon}$ and decays exponentially. The other part builds up gradually and propagates slowly, and attenuates with a much lower rate than exponential decay.

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I. Introduction

Modification of propagated electromagnetic waves and their distortion greatly affects radio navigation, identification of targets by means of radar and generally other telecommunication systems. The sinusoidal electromagnetic wave undergoes a strong attenuation of an exponential decay nature when it propagates in a dielectric medium. However, transient electromagnetic fields excited by a pulsed antenna may attenuate rapidly and may propagate over a moderate distance from the antenna because it contains a wide band of low frequency components. The propagation of a transient electromagnetic wave in a dielectric medium has been studied by several authors [1-5].

In considering our problem, the exact solutions for the transient electromagnetic fields excited by a pulsed electric dipole in a dielectric medium are derived. The transient field responses due to a step-function current antenna are then calculated under the assumption that the displacement currents are insignificant for reasonable values of time after the initial antenna pulse.

Consequently, by convolution, the propagation of electromagnetic pulses excited by an antenna with currents of arbitrary waveforms can be evaluated.

II. Description of the configuration and method of solution

The geometry of the problem is illustrated in Fig. 1. To specify the position of the source in the configuration, we employ the coordinates (r,θ,ϕ) as a spherical coordinate system with origin O and three mutually perpendicular base vectors (e_r, e_θ, e_ϕ) of unit length each. The source is a vertical electric dipole antenna immersed in a dielectric medium of infinite extent and excited by

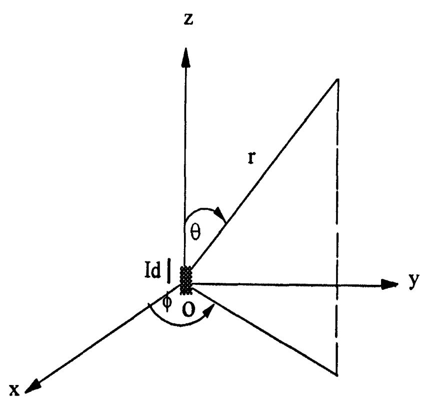

FIG. 1. Pulsed Electric Dipole and Coordinate Systems.

an impulsive current. The electric permittivity is ε, the magnetic permeability μ is taken equal to that of the free space μ_0 everywhere, and the conductivity σ is assumed to be zero.

The electric dipole antenna, located at the origin O and oriented vertically in the z-direction, is small in length, $d\ell$, carying a current If(t). I is the amplitude and $f(t)$ is the temporal function of the antenna current. Without loss of generality, we can restrict attention to situations where $f(t) = 0$ for $t < 0$. The corresponding frequency domain representation for the current is then

$$F(s) = \int_0^\infty f(t)\exp(-st)dt = \mathcal{L}\{f(t)\} \tag{1}$$

where $s = i\omega$ in terms of the angular frequency ω, where the operator $\mathcal{L}$ denotes the Laplace transform.

In the frequency domain, Maxwell's fundamental electromagnetic equations are written as

$$\text{curl}\,\vec{E}(\vec{r}; s) = -s\mu\vec{H}(\vec{r}; s); \tag{2}$$

$$\text{curl}\,\vec{H}(\vec{r}; s) = s\varepsilon\vec{E}(\vec{r}; s) + \vec{J}(\vec{r}; s); \tag{3}$$

where $\vec{E}$ and $\vec{H}$ are the electric and magnetic field intensity vectors, respectively and $\vec{J}$ is the impressed current density vector of the dipole antenna which can be expressed as

$$\vec{J}(\vec{r}; s) = Id\ell\,\delta(\vec{r})F(s)\vec{e}_z; \tag{4}$$

$\vec{e}_z$ is the unit vector in the z-direction.

Each component of the electric and magnetic fields is related to a Hertz vector $\vec{\Pi}$. In the frequency domain, these well-known relations are given by [2]

$$\vec{E}(\vec{r}; s) = -\gamma^2\vec{\Pi}(\vec{r}; s) + \text{grad div}\,\vec{\Pi}(\vec{r}; s); \tag{5}$$

$$\vec{H}(\vec{r}; s) = s\varepsilon\,\text{curl}\,\vec{\Pi}(\vec{r}; s); \tag{6}$$

where

$$\gamma^2 = \mu \varepsilon s^2. \tag{7}$$

The vector electromagnetic wave equation is

$$(\nabla^2 - \gamma^2)\,\vec{\Pi}(\vec{r}; s) = -\frac{\vec{J}}{s\varepsilon}(\vec{r}; s). \tag{8}$$

A solution of this equation is given by Stratton [6]

$$\vec{\Pi}(\vec{r}; s) = \frac{1}{4\pi s\varepsilon} \int_V \vec{J}(\vec{r}\,'; s) \frac{e^{-\gamma R}}{R} dV' \tag{9}$$

where $\vec{\Pi}(\vec{r}; s)$ is evaluated at the observation point $(r; \theta; \phi)$ and $\vec{J}(\vec{r}\,'; s)$ at the source point $(r'; \theta'; \phi')$. The distance R is equal to $R = |\vec{r} - \vec{r}\,'|$, and the integration takes place over all the $\vec{r}\,'$ space. (The medium is infinite and homogenous).

As the dipole antenna is situated at the origin of the spherical coordinate system $(r; \theta; \phi)$, as indicated in Fig. 1; and oriented in the z-direction, on applying of equation (9) it is found that the Hertz vector has only a z-component which is given by

$$\vec{\Pi}(\vec{r}; s) = \Pi_z \vec{e}_z = \frac{I d\ell F(s)}{4\pi s\varepsilon} \frac{e^{-\gamma(s) r}}{r} \vec{e}_z. \tag{10}$$

The operations (5) and (6) are applied to obtain the field components [2]:

$$E_\theta(\vec{r}; s) = \frac{I d\ell F(s)}{4\pi \varepsilon s r^3} \{1 + \gamma(s) r + \gamma^2(s) r^2\} e^{-\gamma(s) r} \sin\theta; \tag{11}$$

$$E_r(\vec{r}; s) = \frac{I d\ell F(s)}{2\pi \varepsilon s r^3} \{1 + \gamma(s) r\} e^{-\gamma(s) r} \cos\theta; \tag{12}$$

$$H_\phi(\vec{r}; s) = \frac{I d\ell F(s)}{4\pi r^2} \{1 + \gamma(s) r\} e^{-\gamma(s) r} \sin\theta. \tag{13}$$

Now choosing $F(s) = 1$, that is to say taking $f(t) = \delta(t)$, then equation (10) becomes,

$$\vec{\Pi}(\vec{r}; s) = \frac{I d\ell}{4\pi s\varepsilon} \frac{e^{-\gamma(s) r}}{r} \vec{e}_z = \Pi_z(\vec{r}; s) \vec{e}_z. \tag{14}$$

Using the tables of Laplace transfonns [7, 8], we have

$$L^{-1}\left\{e^{-as}\right\} = \delta(t - a); \tag{15}$$

where L^{-1} denotes the inverse Laplace transform. This leads to

$$\Pi_z(\vec{r}; t) = \frac{I d\ell}{4\pi r s\varepsilon} \delta\left(t - \frac{r}{v}\right); \tag{16}$$

where $\delta\left[t - \frac{r}{v}\right]$ is the Dirac-δ function and $v = \frac{1}{\sqrt{\mu\varepsilon}}$. Applying Maxwell's equations in the time domain, we can find the magnetic fields

$$H_\phi(\vec{r};t) = \frac{Id\ell\sin\theta}{4\pi r^2}\left[\delta\left(t - \frac{r}{v}\right) + \frac{r}{v}\delta'\left(t - \frac{r}{v}\right)\right]; \tag{17}$$

where $\delta'(x) = \frac{d}{dx}\delta(x)$. To obtain the electric field, let's define

$$G_1(r;s) \equiv [1 + \gamma(s)r]e^{-\gamma(s)r}; \tag{18}$$

and then compare $G_1(r;s)$ with $H_\phi(\vec{r};s)$. In view of equations (13), (17) and (18), we find the inverse Laplace transform of $G_1(r;s)$ to be

$$L^{-1}\{G_1(r;s)\} = g_1(r;t) = \delta\left(t - \frac{r}{v}\right) + \frac{r}{v}\delta'\left(t - \frac{r}{v}\right): \tag{19}$$

Then the time domain electric field can be obtained if we rewrite $E_r(\vec{r};s)$ as

$$E_r(\vec{r};s) = \frac{Id\ell\cos\theta}{2\pi r^3}G_2(r;s)G_1(r;s); \tag{20}$$

where $G_2(r;s) = \frac{1}{s\varepsilon}$. Applying the convolution theorem of the Laplace transform [7],

$$L^{-1}\{G_2(r;s)G_1(r;s)\} = \int_0^t g_2(r;t)g_1(r;t-\tau)d\tau = g_1(r;t) * g_2(r;t); \tag{21}$$

to equation (20) leads to

$$E_r(\vec{r};t) = \frac{Id\ell\cos\theta}{2\pi\varepsilon r^3}\left[\frac{r}{v}\delta\left(t - \frac{r}{v}\right) + u\left(t - \frac{r}{v}\right)\right]; \tag{22}$$

where $u(t - \frac{r}{v})$ is the Heaviside step function. Similarly, $E_\theta(\vec{r};s)$ can be rewritten as

$$E_\theta(\vec{r};s) = s\prod_z(r;s)\sin\theta + \frac{Id\ell\sin\theta}{4\pi r^3}G_1(r;s)G_2(r;s): \tag{23}$$

Then, we have

$$E_\theta(\vec{r};t) = \frac{Id\ell\sin\theta}{4\pi\varepsilon r^3}\left[\frac{r}{v}\delta\left(t - \frac{r}{v}\right) + \frac{r^2}{v^2}\delta'\left(t - \frac{r}{v}\right) + u\left(t - \frac{r}{v}\right)\right]: \tag{24}$$

Equations (17), (22) and (24) gives us the exact time domain solutions for an electromagnetic pulse excited by a dipole antenna with an impulsive current in a dielectric medium. These forms are considered to be original and new. These findings may lead to some useful applications in an environment such as the atmosphere.

III. Conclusions

The exact time domain solutions for an electromagnetic field excited by an electric dipole antenna with an impulsive current in a dielectric medium have been derived. It is found that electromagnetic fields of the excited electromagnetic pulse can be divided into two parts. The first part is an impulse wave, which propagates with the speed $v = 1/\sqrt{\mu\varepsilon}$ and decays exponentially. The other part builds up gradually and propagates slowly, and attenuates with a much lower rate than the exponential decay. These findings may lead to some useful applications in an environment such as the atmosphere.

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