

Letter**Localize Energy in Random Media: A New Phase State**

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This Letter reports a new wave phenomenon. We show that wave energy can be stored in random media. Associated with such energy storage is a global collective behavior. The features depicted are so general that they may be observed in many random systems.

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A major task in physics research is to search for common principles behind various natural processes. A number of greatest advances in the last century include understanding of how the manifestation of many body interactions may lead to several important ubiquitous phenomena including superconductivity, quantum Hall effect, screening effect in plasma, and the Kondo effects [1]. It becomes a daily experience that macroscopic objects arise from interaction between individuals of microscopic scale in a many body system [2].

In this Letter, we will show a new general wave phenomenon in certain many body systems. Namely, wave energy can be stored in a number of different disordered systems. When this occurs, a new phase state emerges.

Up to date, the fundamental equation describing any many body system can be more or less written as

$$L_i(@)\tilde{A}_i = F_i + \sum_{j=1; j \neq i}^N \hat{T}_{ij}[\tilde{A}_j]; \quad (1)$$

in which L is called the divisor [2], F_i denotes the external stimulation, \hat{T}_{ij} is an operator that can be of temporal, spatial or both nature, depicting the interaction between constituent bodies. In the linear approximation, the operator \hat{T} is represented by a wave propagator. For example, in the Yang-Feldman representation of field equations, \hat{T} is the usual fermionic or bosonic two-point Green's function. In Eq. (1), i is the index of the individuals in the system of total N bodies.

So far, there are two known types of divisor [2]. Type I divisor usually refers to fermionic systems and is written as

$$L(@) = i \sim \frac{@}{@t} i^{-2}(r); \quad (2)$$

Type II divisor refers to boson-like systems and is written as

$$L(\omega) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \psi(\mathbf{r}) \quad (3)$$

In these, $\psi(\mathbf{r})$ and $\psi(\mathbf{r})$ are the operators yielding the energy spectra.

Equation (1) accommodates a variety of problems of great interest. To name a few, this includes the vibration of the lattice formed by atoms, spin arrays, and electrons in lattices. Interested readers may refer to [3] for more examples. Eq. (1) can also describe multiple scattering of classical waves such as acoustic propagation in air-filled bubbles [4] and cylinders [5] and electromagnetic waves in electric dipolar systems [6]. In these systems, the interaction between constituents is mediated by waves which themselves are described by Type II equations.

Here we show some new generic features with Type II systems. Consider wave propagation in a Type II system. The propagating wave, which can be either acoustic or electromagnetic, will be interfered by scattered waves from each constituent being stimulated by the incident wave. The energy flow in the system is $\mathcal{J} \propto \text{Re}[\dot{\mathbf{A}}(\mathbf{r}) \cdot \mathbf{A}(\mathbf{r})]$. Writing the field as $\mathbf{A} = \mathbf{j}\hat{\mathbf{A}}e^{i\mu}$, the current becomes $\mathbf{j}\hat{\mathbf{A}}j^2\mathbf{r}\mu$, a version of Meissner equation. It is clear that when μ is constant while $\mathbf{j}\hat{\mathbf{A}} \neq 0$, the flow stops and the energy must be localized in space, hinting at that a phase transition occurs and modes are condensed in the real space. This implies that energy can be stored in certain spatial domains. Clearly, the constant phase μ indicates the appearance of a long range ordering in the system. We note that in the superconductivity case, the uniform phase ($\mathbf{r}\mu = 0$) and a finite super-current lead to an infinite density of carriers, indicating the condensation of electron pairs in the momentum space and resulting in the infinite conductivity.

The above simple argument naturally leads to the question of whether there are systems which can realize the above features. We found that these features can appear in a number of different systems. The previously considered acoustic propagation in bubbly water [4] is one of such systems, as well as acoustic propagation in water with air-filled cylinders [5], and the electric dipole system.

For all the systems mentioned, in the linear approximation the governing equation can be reduced to [5, 6]

$$\frac{d^2}{dt^2} \tilde{A}_i + \omega_0^2 \tilde{A}_i + \gamma \frac{d}{dt} \tilde{A}_i = F_i + \sum_{j=1}^d \sum_{j \neq i} C \frac{\tilde{A}_j}{j F_i - F_j j^{(d_i-1)=2}} \quad (4)$$

where F_i represents the external stimulation, C is the coupling constant, d denotes the dimension, and c is the wave speed. Strictly, for the 2D ($d = 2$) acoustic case, the propagator on the RHS would often be in a form of the zero-th order Hankel function of the first kind [5]. The quantity \tilde{A} refers to either the oscillation of the dipoles in electric systems and or to the vibration of the air-filled bodies in the acoustic situation, and ω_0 refers to the corresponding natural frequency, and γ the possible damping effects due to such as radiation. The parameters will depend on specific models considered. For acoustic scattering in bubbly water, for example, $C \gg a$, $\gamma \gg \omega_0$, and $\omega_0 \gg \frac{1}{a} \sqrt{\frac{\rho}{p_0}}$. Here a is the radius of air-bubbles and p_0 is the ambient pressure. In this case, \tilde{A} refers to the radial pulsating of the bubbles. The parameters can be found in [4] and references

therein. For the electric dipole systems, $C = i \frac{q^2 \omega_0}{24m}$ with q being the charge and m the mass of the charge, and \vec{A} is the electric dipole.

Eq. (4) reveals that when it is excited by the incoming wave, either of electromagnetic or of acoustic nature, the constituent will radiate or scatter waves. The radiated or scattered waves will be again re-radiated or scattered by other constituents, a process repeated to establish an infinite recursive pattern of multiple interactions. We note here that Eq. (4) can in fact also describe many other systems not being considered here.

To explore the general properties of the model in Eq. (4). We solve it numerically in the frequency domain. The generic set up is as follows. There are N bodies randomly distributed in a system, and they are excited by a point source located roughly in the middle of these bodies. We further assume that all elements in the system are identical. We write $\vec{A}_i = A_i e^{i\mu_i}$, where we dropped the time factor $e^{i\omega t}$.

For each phase μ_i , by analogy with [4] we define a phase vector such that $v_i = \cos \mu_i e_x + \sin \mu_i e_y$. The phase vectors would indicate the degree the coherence of the vibration behavior. We have done a few general numerical calculations for Eq. (4). The significant discoveries we found can be summarized as follows. For two or three dimensions, the energy is localized near the source within a range of frequencies slightly above the natural frequency for sufficiently large coupling constants and number densities of the constituents. When the energy is localized in the system, all the constituents oscillates in phase, i. e. the phase μ becomes constant, revealing a long range ordering in the system.

Here we would like to show one example. Randomly put N identical electric dipoles on the $x-y$ plane, inside a box of side length L . All the dipoles points to the positive z -axis. Assume that the averaged distance between dipoles is d ; thus the number density of the dipoles is $n \gg 1/d^2$. In the computation, the damping factor γ can be adjusted to reflect various situations. A transmitting source is put at the center of the dipole cloud. As this is the first step in our research, we simplify further that all the dipoles can only oscillate along their axes. The dipoles will oscillate vertically in response to the source and as well as the radiated waves from other dipoles, i. e. all the dipoles are coupled via the radiated waves. In the computation, all frequencies are scaled by the natural frequency of the dipole. The coupling constant C and damping γ can be varied. We study the behavior of the aforementioned phase vectors and the energy distribution in the system; it is easy to see that the energy can be represented by the oscillation amplitude of the dipoles.

Fig. 1 illustrates our findings for one arbitrary random configuration. For low frequencies, e. g. at $\omega/\omega_0 = 0.98$, the phase vectors points randomly. The distribution of the energy spreads. When increasing the frequency to certain values slightly above the natural frequency, all phase vectors tend to point to a uniform direction, indicating a new phase state. Meanwhile the energy is centered at the site of the transmitting source. This is shown by the case at $\omega/\omega_0 = 1.1$. When the frequency is increased further, the phase ordering disappears, and the energy distribution becomes extended again, as shown by the case with $\omega/\omega_0 = 10$. These features are valid for a range of the coupling constant C and the damping rate. The parameters used in the present specific example are: $C = 0.1$, $n = 0.032$, and $\gamma = 0.0001$. We take N as 900; in Fig. 1, however, only a fraction of the system is shown in order to expose the features in the most explicit way. The features tend to diminish as either the couple constant or the number density is reduced to a certain value. When the damping rate γ is increased to a large value, the phase ordering will then be degraded.

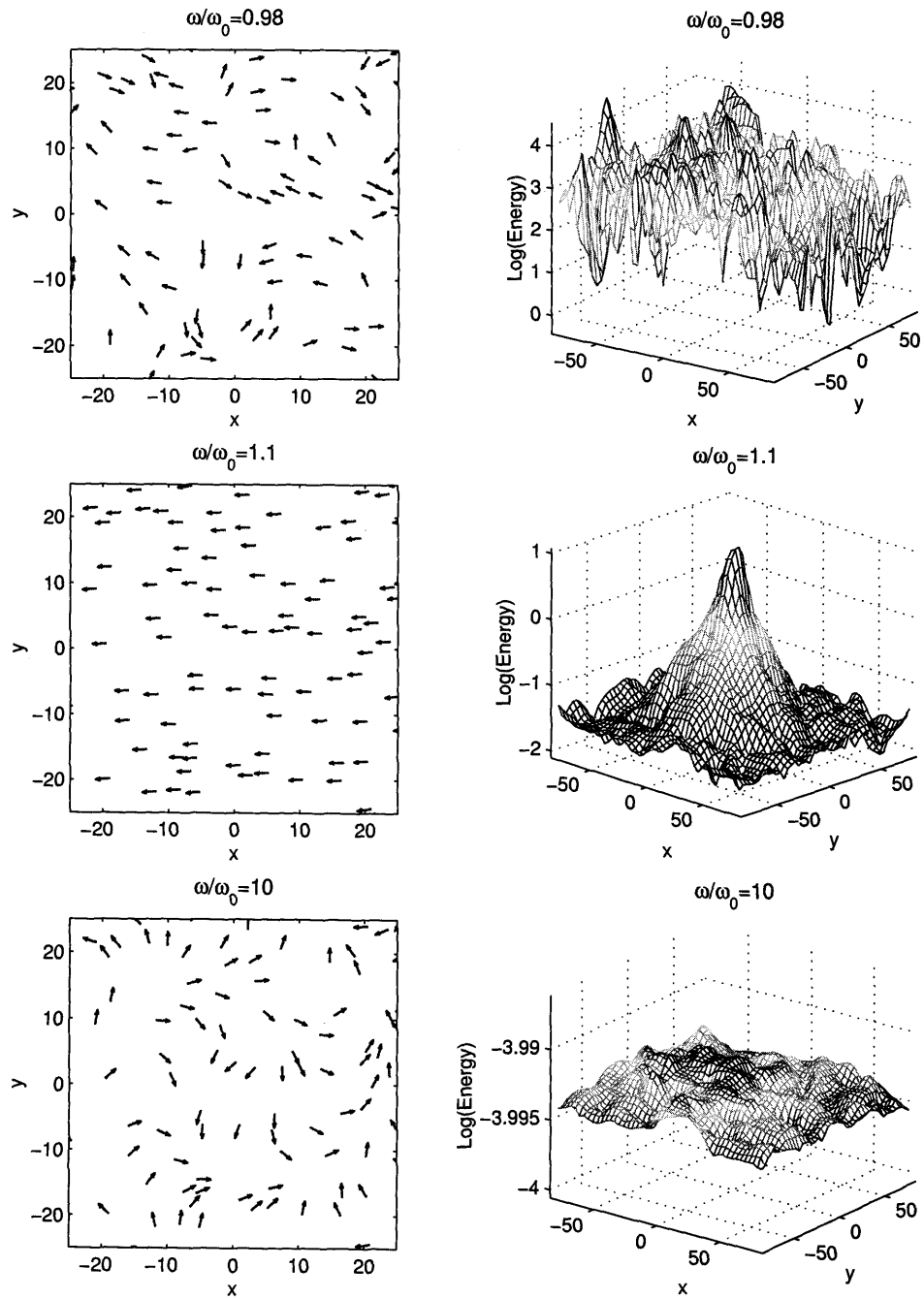


FIG. 1. Energy distribution and diagrams for the phase vectors in a 2D random configuration of electric dipoles. Right: Energy distribution; the geometrical factor ($1-r$) has been dropped out. Left: the phase diagrams for the phase vectors defined in the text.

The phase ordering in the present system has a clear physical meaning. That is, it reflects that all elements in the many body system oscillate or vibrate coherently. We stress that for simplicity we have chosen the 2D arrangement of the dipoles in a 3D system. Strictly speaking, due to radiation into the third dimension, waves cannot be stored permanently in the medium. The energy will gradually escape. We have also performed calculation for a 3D random configuration of the dipoles. Similar results hold.

In summary, we have exhibited a new phase state possibly observable in a class of many body systems. It is shown that energy can be stored coherently in random media. The discovery reported here may provide insight to the long standing problem of the Anderson localization of waves in disordered media.

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