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Letter

On Impulsive Motion, Braking and Robotry

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Newtonian dynamics is applied to impulsive motion. The third time derivative $\dddot{x}$ is introduced. By extending the concept of impulse I,

$$I = \int_0^\tau m\ddot{x}dt = m\dot{x}|_\tau - m\dot{x}|_0,$$

the concept of jumpulse J is defined as

$$\{(J_i, \tau_i)|J_i \equiv \int_{\tau_i} m\dddot{x}dt \text{ with } m\dddot{x} \geq 0 \text{ (or } m\dddot{x} \leq 0) \text{ in the entire interval } \tau_i\}.$$

The application of the concepts I and J to braking mechanisms in robotry is briefly discussed.

The merit of introducing the jumpulse J [Eq. (7) in the text] is that we can now design the size and shape of the jumpulse for a given purpose by designing the F with the needed $\frac{dF}{dt}$ within the interval τ; i.e., we are now refining our treatment of impulsive forces of the past, so that we have the proper jumpulse for different purposes in different situations. The possibility of fine braking devices may play a very important role in the development of robotry in the future.

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I. Impulse and jumpulse

The most familiar examples of impulsive motion are a base ball struck into the out field, or a tennis ball struck back to the opponent. For simplicity of presentation of the basic ideas, consider the case of the tennis ball. The incoming ball, with a velocity V_0, comes into contact with the strings of the racket. From that instant on, the ball is acted on by the combination **F** of forces, all through the intermediary of the strings, (i) arising from the return of the stretched strings to their unstretched shape, and (ii) arising from the forward "swing" of this racket. This force F on the ball vanishes at the instant the ball is not in contact with the racket. The equation of motion of the ball during the interval τ when the ball is in contact with the racket is

$$m\ddot{x} = F(t), \quad 0 \le t \le \tau, \quad F(0) = F(\tau) = 0. \tag{1}$$

The impulsive force $F(t)$, in the case of the tennis ball, can be calculated from the elastic property of the strings, but in the general case (including the golf ball, or the braking of a car) may be complicated. The gross effect of an impulsive force F can however be calculated very simply.

Thus, on integrating (1), and defining the average $\overline{F}$ in the interval τ by

$$\overline{F} = \frac{1}{\tau}\int_0^\tau F dt \tag{2}$$

and defining as the impluse I by

$$I = \overline{F}\tau \tag{3}$$

the integral of (1) is

$$m\dot{x}|_\tau - m\dot{x}|_0 = I. \tag{4}$$

On differentiating (1), we have

$$m\dddot{x} = \frac{d}{dt}m\ddot{x} = \frac{dF}{dt}. \tag{5}$$

We bring in $\dddot{x}$. On defining the average $\overline{\frac{dF}{dt}}$ in the time interval τ

$$\overline{\frac{dF}{dt}} = \frac{1}{\tau}\int_0^\tau \frac{dF}{dt}dt \tag{6}$$

with $\frac{dF}{dt}$ of the same sign in the interval, we may define the jumpulse J as the set

$$\{(J_i, \tau_i) | J_i \equiv \int_{\tau_i} m\dddot{x}dt \text{ with } m\dddot{x} \ge 0 \,(\text{or } m\dddot{x} \le 0) \text{ in the entire interval } \tau_i\}. \tag{7}$$

Here the interval $(0, \tau)$ has been subdivided into subintervals $\{\tau_i\}$ and

$$J_i = \overline{m\dddot{x}}\tau_i \tag{8}$$

which, stated in words, shows that the application of the jumpluse $J_i = \left(\overline{\frac{dF}{dt}}\right)\tau_i$ changes the $m\ddot{x}$ of the body by J_i. This is an extension of the familiar result (4) stating that the application of an impulse I changes the momentum $m\dot{x}$ by I.

II. Some properties of impulse I and jumpulse J

For a body in an impulsive motion in one dimension, the action of an impulse $I = mv_\tau - mv_0$ to a body changes its kinetic energy T by

$$\begin{aligned}\Delta T &= \frac{1}{2}mv_\tau^2 - \frac{1}{2}mv_0^2 \\ &= (mv_\tau - mv_0)\frac{1}{2}(v_\tau + v_0) \\ &= I\frac{1}{2}(v_\tau + v_0),\end{aligned} \tag{9}$$

i.e. the impulse I on the body increases the kinetic energy of the body by the product of I and the average of the initial and the final velocity of the body.

From (1)

$$m\frac{d\ddot{x}}{dt} = \dot{F}$$

we obtain

$$\frac{d}{dt}(m\ddot{x}^2) = \frac{1}{m}\frac{dF^2}{dt}. \tag{10}$$

On integrating, we have

$$\left[A - \frac{1}{2m}F^2\right]_\tau = \left[A - \frac{1}{2m}F^2\right]_0 \tag{11}$$

where

$$A = \frac{m}{2}\ddot{x}^2 \tag{12}$$

is the Appell function. Note that A is not of the dimension of energy, and the name "energy of acceleration" is misleading and incorrect. The relation (11) is not the conservation relation of energy. In impulsive motion there are usual productions of heat which is not induced in (11).

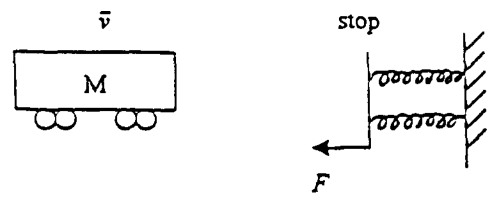
Note that, in view of Eq. (1), the integral $\int_0^\tau \dot{F}dt$ vanishes identically so that the interval $(0,\tau)$ must be subdivided into subintervals $\{\tau_i\}$ in order to define the concept of the jumpulse. The choice of a suitable set of subintervals depends on the problem and many equivalent choices may be possible for the same problem. The best solution $\{J_i, \tau_i\}$ represents an optimization problem of some sort.

III. Remarks on $\dddot{x}$ in eq. (5)

It is important to emphasize that the appearance of $\dddot{x}$ is through Newton's Law of motion (1) and is not a new physical assumption or principle [1]. The usefulness of introducing $\dddot{x}$ is as follows: From (5) we can calculate $\dddot{x}$ if we are given the impulsive force F. *This* force between the tennis ball and the racket, F, is a function of the stretching of the racket strings and the jumpulse J is determined by the elastic properties of the springs. Thus the importance of the concept of jumpulse is as follows: By a proper design of the elastic properties of the string and the lattice structure of the racket face, it is possible to construct a racket with the desired $\frac{dF}{dt}$ and give the desired jumpulse J. The co-on-sense instruction of this idea is the following: a loosely strung racket (low tension) can produce only low jumpulse and weak return of the ball, and vice versa.

IV. Impulse and jumpulse: application to braking and robotry

Consider for simplicity the following problem: a locomotive on tracks at low speed is to be braked to stop in a comparatively short track (say, 100 feet) by a strong steel stopper backed by strong heavy springs.



Upon contact of the locomotive with the stopper, the springs are compressed, and the reaction force F increases as the springs are being compressed. The force F thus has a high $\frac{dF}{dt}$, and on collision, the jumpulse $J_i = \overline{\frac{dF}{dt}}\tau_i$ on the locomotive will change its acceleration, ... and brake it to stop.

The jumpulse does not have to be limited to mechanical devices, it could be electro-magnetic so that choice of devices to create the needed jumpulse is wide.

In the case of robotry, the key problem is the design of controlled motion of parts to eliminate breakage by jarring in contacts. The design of the proper jumpulse by the design of F with the proper $\frac{dF}{dt}$, coupled with computerized motion of the moving part will no doubt be developed in the near future.

The author wishes to express his appreciation of the early discussions, in 1997-8, with Dr. Hugh Ching, of the Post-Science Institute, Berkeley, Calif., which led to the present work [2].

Editorial Note:

Due to the severe illness of the author, this paper has been prepared by Paucy W-Y. P. Hwang.

References

[1] The author is indebted to Dr. Ching for calling his attention to an article in a journal in applied physics in which x is mentioned and the name "jerk" suggested for it. There is no discussion of the concept except to say that it does not seem important and $\dddot{x}$ is passed off casually.

[2] The need to take into consideration the sign of $\dot{F}$ was pointed out by K. R. Chu while division of subintervals for solving the problem is implemented by W-Y. P. Hwang. The optimization involves additional aspects which should be addressed further.