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A New Kerr-Newman-De Sitter Exact Solution of the Poincaré Gauge Theory of Gravitation

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We present a new exact solution of the Poincaré gauge theory of gravitation with the Kerr-Newman-De Sitter metric and nonzero torsion. The metric and torsion of the solution are characterized by four arbitrary parameters: mass m, angular momentum j, electric charge q, and effective cosmological constant L. All three irreducible components of the torsion are nonzero. In particular the totally antisymmetric torsion (the only part of the torsion which is coupled directly to spin-i fermions) is nonvanishing. The solution is obtained from a previously known one with the help of a special discrete transformation.

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I. Introduction

The Poincaré gauge theory of gravitation (PGT) is an interesting alternative to the standard general relativity theory [1,2]. The PGT appears naturally as a gauge theory of the Poincaré group, the fundamental group of spacetime symmetries, and under rather weak conditions the PGT is compatible with all modern experimental data [3].

One of the most attractive features of the PGT is that the theory treats the spin of matter fields on an equal footing with the energy-momentum tensor [1]. The new gravitational field variable which corresponds to the spin source is the torsion of spacetime. Therefore, it is especially interesting to investigate axially symmetric black hole exact solutions with nonzero torsion. Such solutions should describe, in the framework of the PGT, the external gravitational field of compact astrophysical objects with rotation and possible spin. In this paper we present a new axially symmetric solution of the PGT with nontrivial torsion. The new solution was obtained with the help of the computer algebra program GRG [4].

II. Poincaré gauge theory

The two independent gravitational potentials of the Poincaré gauge theory of gravitation are the tetrad h^a_μ and the Lorentzian connection $\omega_{ab\mu} = \omega_{[ab]\mu}$ [1,2] but we shall work with another *equivalent* set of dynamic variables: the metric $g_{\mu\nu}$ and the torsion

$$Q^{\alpha}{}_{\beta\gamma} = \Gamma^{\alpha}{}_{\beta\gamma} - \Gamma^{\alpha}{}_{\gamma\beta}.$$

The curvature tensor

$$R^{\alpha}{}_{\beta\mu\nu} = \partial_{\mu}\Gamma^{\alpha}{}_{\beta\nu} - \partial_{\nu}\Gamma^{\alpha}{}_{\beta\mu} \ \mathbf{t}\ \Gamma^{\alpha}{}_{\gamma\mu}\Gamma^{\gamma}{}_{\beta\nu} - \Gamma^{\alpha}{}_{\gamma\nu}\Gamma^{\gamma}{}_{\beta\mu},$$

in spaces with torsion, can be algebraically decomposed into six irreducible curvature components [2,5]

$$R_{\alpha\beta\gamma\delta} = C_{\alpha\beta\gamma\delta} + T_{\alpha\beta\gamma\delta} + S_{\alpha\beta\gamma\delta} \ \mathrm{t}\ A_{\alpha\beta\gamma\delta} \ \mathrm{t}\ B_{\alpha\beta\gamma\delta} + D_{\alpha\beta\gamma\delta}.$$

Here the first three irreducible parts are well known from Riemannian geometry. The $C_{\alpha\beta\gamma\delta}$ is the Weyl tensor and

$$\begin{aligned}
&R_{\mu\nu} = R^{\pi}{}_{\mu\pi\nu}, \quad R = R^{\pi}{}_{\pi}, \quad C_{\alpha\beta} = R_{(\alpha\beta)} - \frac{1}{4}g_{\alpha\beta}R,\\
&T_{\alpha\beta\gamma\delta} = \frac{1}{2}(g_{\alpha\gamma}C_{\beta\delta} + g_{\beta\delta}C_{\alpha\gamma} - g_{\beta\gamma}C_{\alpha\delta} - g_{\alpha\delta}C_{\beta\gamma}),\\
&S_{\alpha\beta\gamma\delta} = \frac{1}{12}R(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}).
\end{aligned} \tag{2.1}$$

The other three irreducible pieces are specific to Riemann-Cartan geometry

$$\begin{aligned}
&A_{\alpha\beta} = R_{[\alpha\beta]},\\
&A_{\alpha\beta\gamma\delta} = \frac{1}{2}(g_{\alpha\gamma}A_{\beta\delta} + g_{\beta\delta}A_{\alpha\gamma} - g_{\beta\gamma}A_{\alpha\delta} - g_{\alpha\delta}A_{\beta\gamma}),\\
&D_{\alpha\beta} = R^{*\pi}{}_{\alpha\pi\beta}, \quad R^{*}_{\alpha\beta\gamma\delta} = \frac{1}{2}\mathcal{E}_{\gamma\delta}{}^{\mu\nu}R_{\alpha\beta\mu\nu},\\
&B_{\alpha\beta} = D_{(\alpha\beta)} - \frac{1}{4}g_{\alpha\beta}E, \quad E = D^{\mu}{}_{\mu},\\
&B_{\alpha\beta\gamma\delta} = \frac{1}{2}(\mathcal{E}_{\alpha\beta\gamma}{}^{\pi}B_{\pi\delta} - \mathcal{E}_{\alpha\beta\delta}{}^{\pi}B_{\pi\gamma}), \quad D_{\alpha\beta\gamma\delta} = -\frac{1}{12}E\mathcal{E}_{\alpha\beta\gamma\delta},
\end{aligned} \tag{2.2}$$

where $\mathcal{E}^{\alpha\beta\gamma\delta}$ is the totally antisymmetric tensor with $\mathcal{E}^{0123} = [-\det(g_{\mu\nu})]^{-1/2}$.

Similarly the torsion can be decomposed into three irreducible parts

$$\begin{aligned}
&Q_{\alpha\beta\gamma} = C_{\alpha\beta\gamma} + T_{\alpha\beta\gamma} + P_{\alpha\beta\gamma},\\
&T_{\alpha\beta\gamma} = \frac{1}{3}(g_{\alpha\gamma}Q_{\beta} - g_{\alpha\beta}Q_{\gamma}), \qquad P_{\alpha\beta\gamma} = \frac{1}{3}\mathcal{E}_{\alpha\beta\gamma\mu}P^{\mu},
\end{aligned} \tag{2.3}$$

where Q_{α} and P_{α} are the torsion trace and pseudotrace respectively

$$Q_{\alpha} = Q^{\nu}{}_{\alpha\nu}, \ P_{\alpha} = Q^{*\nu}{}_{\alpha\nu}, \ Q^{*}_{\alpha\beta\gamma} = \frac{1}{2}\mathcal{E}_{\beta\gamma}{}^{\mu\nu}Q_{\alpha\mu\nu}. \tag{2.4}$$

We restrict ourselves to the most general quadratic in curvature and torsion Lagrangian [2,6]

$$\mathcal{L}_{\rm g} = \frac{1}{16\pi G}\left[R - 2\Lambda + R_{\alpha\beta\gamma\delta}\overline{R}^{\alpha\beta\gamma\delta} + Q_{\alpha\beta\gamma}\overline{Q}^{\alpha\beta\gamma}\right], \tag{2.5}$$

where the constants G and A are parameters of the theory and $\overline{R}_{\alpha\beta\gamma\delta}$ and $\overline{Q}_{\alpha\beta\gamma}$ are the linear combinations of the curvature and torsion irreducible components taken with the arbitrary constants **L,** and M_i:

$$\begin{aligned}\overline{R}_{\alpha\beta\gamma\delta} &= L_1 C_{\alpha\beta\gamma\delta} + L_2 T_{\alpha\beta\gamma\delta} + L_3 S_{\alpha\beta\gamma\delta} + L_4 A_{\alpha\beta\gamma\delta} + L_5 B_{\alpha\beta\gamma\delta} + L_6 D_{\alpha\beta\gamma\delta},\\ \overline{Q}_{\alpha\beta\gamma} &= M_1 C_{\alpha\beta\gamma} + M_2 T_{\alpha\beta\gamma} + M_3 P_{\alpha\beta\gamma}.\end{aligned} \tag{2.6}$$

The Lagrangian (2.5) includes, in addition to the gravitational constant G, nine parameters: the cosmological constant A, three "torsion" parameters M_i, and five "curvature" parameters L_i (due to the Gauss-Bonnet invariant only five of the six are really independent, see e.g. [5]). The correspondence between our parametrization of the PGT Lagrangian and the notations of other authors can be found in [5]. Later on in this paper we will also use the following combinations of the PGT parameters

$$\Lambda_1 = \frac{1}{2}(L_1 + L_2), \qquad \Lambda_3 = \frac{1}{2}(L_2 + L_3)$$

The PGT equations can be obtained by varying the Lagrangian (2.5) with respect to the tetrad and Lorenzian connection or equivalently with respect to the metric and torsion:

$$-\mathcal{D}^{\pi}\overline{Q}_{(\alpha\beta)\pi} - \overline{Q}^{\pi\kappa}{}_{(\alpha}Q_{\pi\kappa|\beta)} - \widetilde{R}^{\pi\kappa\tau}{}_{(\alpha}R_{\pi\kappa\tau|\beta)} + g_{\alpha\beta}\widetilde{\mathcal{L}}_{\rm g} = -16\pi G T_{\alpha\beta}, \tag{2.7}$$

$$\mathcal{D}^{\pi}\widetilde{R}_{\alpha\beta\mu\pi} + \overline{Q}_{[\alpha\beta]\mu} = -16\pi G S_{\alpha\beta\mu}. \tag{2.8}$$

Here $T_{\alpha\beta}$ and $S_{\alpha\beta\gamma}$ are the energy-momentum and spin tensors of the matter fields respectively, $\widetilde{R}_{\alpha\beta\gamma\delta} = g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma} + \overline{R}_{\alpha\beta\gamma\delta}$, and $\widetilde{\mathcal{L}}_{\rm g} = 16\pi G\mathcal{L}_{\rm g}$. The equations are quite complicated and we refer the reader to [5] where they can be found in various representations.

III. Kerr-newman solutions with torsion

In this paper we are concerned with axially symmetric exact solutions of the vacuum or electro-vacuum PGT equations whose metric is the well known Kerr-Newman spacetime with cosmological constant characterized by four parameters: the mass m, the charge q, the angular momentum j, and the effective cosmological constant **L.** It is important to emphasize that such a metric with zero torsion is a solution of the PGT equations for an arbitrary Lagrangian (2.5) [6]. We are interested here in solutions with *nonzero* torsion.

The first solution of this type was obtained by Chen, Chern, Hsu, Yeung, and Chen [7]. This spacetime, characterized by two parameters m, and j and one additional constant (the torsion's magnitude), is a solution of the PGT equations for a three-parametric family of Lagrangians (zero cosmological constant, one torsion parameter, and two curvature parameters [8]).

Soon after this Baekler, Giirses, Hehl, and McCrea (BGHM) discovered another solution which is valid for a much wider seven-parametric class of Lagrangians. The solution includes the full set of parameters m, q, j, L and even the additional NUT parameter n[9,10]. The teleparallel (zero curvature) solution [11] recently discovered by Toma turns out to be a special case of this solution [12]. Another teleparallel solution was discovered by Baekler [13].

We are going to present here the BGHM solution [10] explicitly. We make only one simplification, setting the NUT parameter to zero: $n = 0$. In this case the metric in the coordinates t, r, ϑ, ϕ is given by the orthnormal tetrad

$$\begin{aligned} ds^2 &= -\theta^0 \otimes \theta^0 + \theta^1 \otimes \theta^1 + \theta^2 \otimes \theta^2 + \theta^3 \otimes \theta^3, \\ \theta^0 &= \sqrt{\Delta/\Sigma}(dt + j \sin^2 \vartheta d\varphi), \qquad \theta^1 = \sqrt{\Sigma/\Delta} dr, \\ \theta^2 &= \sqrt{\Sigma/F} d\vartheta, \qquad \theta^3 = \sqrt{F/\Sigma} \sin \vartheta [j dt + (r^2 + j^2) d\varphi], \end{aligned} \tag{3.1}$$

where the structural functions are

$$\begin{aligned} F &= 1 + \frac{1}{3} L j^2 \cos^2 \vartheta, \qquad \Sigma = r^2 + j^2 \cos^2 \vartheta, \\ A &= r^2 + j^2 + q^2 - 2mr - \frac{1}{3} L r^2 (r^2 + j^2). \end{aligned} \tag{3.2}$$

The torsion 2-form $\Theta^a = \frac{1}{2} Q^a{}_{bc} \theta^b \mathrm{A} \theta^c$ of the BGHM solution are

$$\begin{aligned} \Theta^0 = \Theta^1 = {} & \sqrt{\Sigma/\Delta}(V_1 \theta^0 \mathrm{A}\, \theta^1 + 2V_4 \theta^2 \mathrm{A}\, \theta^3) \\ & + \Sigma/\Delta[-V_2(\theta^0 A\, \theta^2 - \theta^1 A\, \theta^2) - V_3(\theta^0 A\, \theta^3 - \theta^1 A\, \theta^3)], \end{aligned}$$

$$\begin{aligned} \Theta^2 &= \sqrt{\Sigma/\Delta}[-V_5(\theta^0 A\, \theta^2 - \theta^1 \mathrm{A}\, \theta^2) - V_4(\theta^0 \mathrm{A}\, \theta^3 - \theta^1 \mathrm{A}\, \theta^3)], \\ \Theta^3 &= \sqrt{\Sigma/\Delta}[V_4(\theta^0 \wedge \theta^2 - \theta^1 \mathrm{A}\, \theta^2) - V_5(\theta^0 \mathrm{A}\, \theta^3 - \theta^1 \mathrm{A}\, \theta^3)], \end{aligned} \tag{3.3}$$

where

$$\begin{aligned} V_1 &= \Sigma^{-2}[r(Q - \frac{1}{2} q^2) - mJ^2], \qquad V_2 = -\Sigma^{-2} j Q J \sqrt{F/\Sigma} \sin \vartheta, \\ V_3 &= \Sigma^{-2} j Q r \sqrt{F/\Sigma} \sin \vartheta, \qquad V_4 = \Sigma^{-2} Q J, \qquad V_5 = \Sigma^{-2} Q r, \\ \mathrm{Q} &= \mathrm{mr} - \frac{1}{2} q^2, \qquad J = j \cos 19. \end{aligned} \tag{3.4}$$

Notice that the totally antisymmetric part of the torsion (3.3) vanishes since $\Theta^a \mathrm{A}\, \theta_a = 0$.

The Riemann-Cartan curvature of the spacetime with the metric (3.1) and torsion (3.3) has only two nonzero irreducible parts: the scalar curvature

$$R = 4L, \tag{3.5}$$

and the traceless part of the Ricci tensor C_{ab}, which in the spinorial representation (see [5] for more details) has only one nonzero component,

$$C_{2\dot{2}} = \frac{2L(2mr - q^2)}{-3(r^2 + j^2 + q^2) + 6mr + Lr^2(r^2 + j^2)} = -\frac{4LQ}{3\Delta} \tag{3.6}$$

The connection between tensorial and spinorial components is given by the relation $C_{2\dot{2}} = 2C_{00}$.

Finally, the electromagnetic 2-form $\Phi = \frac{1}{2}F_{ab}\theta^a \wedge \theta^b$ of the solution (3.1), (3.3) is

$$\Phi = \sqrt{(1 + \frac{2}{3}L\Lambda_3)/G}\ \frac{q}{\Sigma^2}[(r^2 - J^2)\,\theta^0 \wedge \theta^1 + 2rJ\,\theta^2 \wedge \theta^3]. \tag{3.7}$$

The spacetime (3.1), (3.3) is a solution of the PGT equations if the parameters of the Lagrangian (2.5) satisfy the constraints

$$L = \Lambda, \quad M_1 = -2 - \frac{4}{3}L\Lambda_3, \quad M_2 = 4 + \frac{8}{3}L\Lambda_3. \tag{3.8}$$

IV. New solution

The BGHM solution presented above belongs to the so called double-duality ansatz [14]. Recently we demonstrated that in many cases it is possible to construct new double-dual PGT solutions starting from known ones with the help of some transformations [15]. The transformation depends on several *arbitrary* functions and maps the torsion of the original solution to the new torsion keeping the metric unchanged.

It is possible to apply the transformation to the BGHM solution (3.1), (3.3) and obtain a new geometry with the same metric and new torsion which depends on several arbitrary functions. This new geometry is always a solution of the PGT equations but for quite a special class of the PGT Lagrangians. The reason for this is that the new configuration has in general a larger number of nonvanishing irreducible components of the curvature and each nonzero irreducible piece requires one additional constraint on the PGT parameters [15]. This situation can be improved if one of the irreducible curvatures R or C_{ab} vanishes. This happens only if $L = 0$ and the curvature of the BGHM solution (3.5), (3.6) vanishes identically. This teleparallel case is degenerate and belongs to a large class of teleparallel solutions parametrized by arbitrary Lorenzian rotations (six arbitrary functions) [15]. In particular the teleparallel case of the BGHM solution and the teleparallel solution of Baekler [13] belong to this class as special cases.

Fortunately, there exists a special case of the transformation which allows one to minimize the additional constraints on the parameters of the Lagrangian. This special transformation is discrete and does not depend on any arbitrary functions or parameters. The form of the transformation is very simple: if $\omega_{ab} = \omega_{ab\mu}dx^\mu$ is the connection 1-form of the BGHM solution in the orthonormal tetrad (3.1) then the connection of the new solution ω'_{ab} in the same tetrad is given by the relation

$$\begin{aligned} &\omega'_{01} = \omega_{01}, \quad \omega'_{23} = \omega_{23}, \\ &\omega'_{02} = -\omega_{02}, \quad \omega'_{03} = -\omega_{03}, \quad \omega'_{12} = -\omega_{12}, \quad \omega'_{13} = -\omega_{13}. \end{aligned} \tag{4.1}$$

The corresponding torsion is

$$\begin{aligned}
\Theta^0 &= \sqrt{\Sigma/\Delta}(V_1\theta^0 \wedge \theta^1 - 2V_4\theta^2 \wedge \theta^3) \\
&\quad + \Sigma/\Delta[V_2(\theta^0 \wedge \theta^2 - \theta^1 \wedge \theta^2) + V_3(\theta^0 \wedge \theta^3 - \theta^1 \wedge \theta^3)] \\
&\quad + \Sigma/Q(2V_2\theta^0 \wedge \theta^2 + 2V_3\theta^1 \wedge \theta^3) - 4\sqrt{\Delta/\Sigma}(J/\Sigma)\theta^2 \wedge \theta^3, \\
\Theta^1 &= \sqrt{\Sigma/\Delta}(V_1\theta^0 \wedge \theta^1 - 2V_4\theta^2 \wedge \theta^3) \\
&\quad + \Sigma/\Delta[V_2(\theta^0 \wedge \theta^2 - \theta^1 \wedge \theta^2) + V_3(\theta^0 \wedge \theta^3 - \theta^1 \wedge \theta^3)] \\
&\quad + \Sigma/Q(2V_2\theta^1 \wedge \theta^2 + 2V_3\theta^0 \wedge \theta^3), \\
\Theta^2 &= \sqrt{\Sigma/\Delta}[V_5(\theta^0 \wedge \theta^2 - \theta^1 \wedge \theta^2) - 3V_4(\theta^0 \wedge \theta^3 - \theta^1 \wedge \theta^3)] \\
&\quad + \sqrt{\Sigma\Delta}/Q(-2V_5\theta^1 \wedge \theta^2 - 2V_4\theta^0 \wedge \theta^3), \\
\Theta^3 &= \sqrt{\Sigma/\Delta}[3V_4(\theta^0 \wedge \theta^2 - \theta^1 \wedge \theta^2) + V_5(\theta^0 \wedge \theta^3 - \theta^1 \wedge \theta^3)] \\
&\quad + \sqrt{\Sigma\Delta}/Q(-2V_5\theta^1 \wedge \theta^3 + 2V_4\theta^0 \wedge \theta^2) - 4(\Sigma/\Delta)V_3\theta^0 \wedge \theta^1.
\end{aligned} \tag{4.2}$$

All three irreducible pieces of the torsion (4.2) are nonzero.

In spite of the rather complicated structure of the torsion the curvature of the new spacetime (3.1), (3.2) is remarkable simple. The scalar curvature and the traceless part of the Ricci tensor are

$$\boldsymbol{R} = -\frac{4}{3}L, \tag{4.3}$$

$$C_{2\dot{2}} = \frac{-2L(2mr - q^2)}{-3(r^2 + j^2 + q^2) + 6mr + Lr^2(r^2 + j^2)} = \frac{4LQ}{3\Delta}. \tag{4.4}$$

The only additional irreducible piece which acquires a nonzero value is the Weyl tensor. The corresponding Weyl spinor has one nonzero component

$$C_2 = -\frac{2}{9}L. \tag{4.5}$$

The Weyl tensor C_{abcd} can be recovered with the help of the relation $C_{0101} = 2C_2$. The electromagnetic 2-form of the new solution reads

$$\Phi = \sqrt{(1 - \frac{2}{9}L\Lambda_3)/G}\,\frac{q}{\Sigma^2}[(r^2 - J^2)\theta^0 \wedge \theta^1 + 2rJ\,\theta^2 \wedge \theta^3]. \tag{4.6}$$

Finally, the new geometry (3.1), (4.2) together with (4.6) is a solution of the PGT equations if the effective cosmological constant $\boldsymbol{L}$ and the parameters of the PGT Lagrangian satisfy the relations:

$$\begin{aligned}
&\Lambda = \boldsymbol{L} - \frac{8}{27}L^2\Lambda_3, \quad \Lambda_1 = 0, \\
&M_1 = -2 + \frac{4}{9}L\Lambda_3, \quad M_2 = 4 - \frac{8}{9}L\Lambda_3, \quad M_3 = 1 - \frac{2}{9}L\Lambda_3.
\end{aligned} \tag{4.7}$$

One can see that, in comparison with the BGHM solution, we have two extra constraints on the parameters of the Lagrangian. This fact can be easily understood. In general each additional nonzero irreducible piece of curvature or torsion of the double-dual solution results in one additional constraint on the parameters of the theory the [15]. In our case two extra nonzero irreducible components, the Weyl tensor C_{abcd} and the totally antisymmetric torsion P_{abc}, corresponds to two additional constraints on Λ_1 and M_3 respectively.

The fact that the new solution has a nonzero antisymmetric torsion part is especially interesting since only this part of the torsion is coupled to spin-$\frac{1}{2}$ particles. Therefore one would expect that metric produced by a spin-\$ fermion source should have nonzero antisymmetric torsion. Notice that the antisymmetric part of the torsion for the BGHM solution vanishes. In fact the physical meaning of the new solution presented here as well as the previously known ones requires special investigation.

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