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Dynamics of the Multiphoton Process of the Electrons Scattering by Atoms in a Laser Field

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The electron-atom scattering in the pressence of an intense radiation field has been investigated by solving the time-dependent Schrödinger equation in momentum space. It is found that the dynamics of the multiphoton process during the scattering can be well understood according to this formulation. Some interesting points regarding the multiphoton process are discussed.

I. INTRODUCTION

The scattering of electrons by atoms in the presence of a radiation field has attracted many experimental and theoretical investigations." When the electromagnetic field is strong, such as provided by a laser, many photons may be emitted or absorbed during the scattering process. Kroll and Watson' analyzed this multiphoton process to obtain a detailed form for the differential elastic scattering cross section in the classical limit. Rosenberg[2] extended the standard time-independent scattering theory to take into account the presence of an intense external field, and some techniques in the intermediate and strong-coupling approximations were developed for the scattering problem. Shakeshaft[8] formulated a method of coupled integral equations to calculate the cross section for stimulated emission .or absorption of photons and successful application for the case of a separable potential is obtained. Recently, we formulated a time-dependent method[9] to treat the scattering of electron from a potential in the laser field and applied it to a one-dimensional system. The result is interesting in that the multiphoton absorption process is manifested with many peaks in the transition probability function. In this paper, we extend this method to solve the time-dependent Schrödinger equation in momentum space, which facilitates the extraction of the rapidly varying part of the wavefunction, in orders to investigate the dynamics of the multiphoton process of the electron-atom scattering in a laser field.

II. FORMULATION

Consider an electron moving in the radiation field of a vector potential $\vec{A}(t)$ and scattering by the atomic potential V; the time-dependent Schrödinger equation is

$$i\hbar \frac{d}{dt} | \Psi(t) > = [\frac{p^2}{2m} + Ve^{-\epsilon|t|} + H,(t)] \; | \Psi(t) > \tag{1}$$

in which H,(t) is the interaction between the electron and the radiation field

$$H,(t) \; = \; -\frac{e}{mc}\vec{A}(t) \cdot \vec{p} + \frac{e^2}{2mc^2} A^2 (t) \tag{2}$$

As $\vec{A}$ is considered to be spatially homogeneous over any microscopic region, the A^2 term is eliminated by the transformation[7]

$$|\Psi(t) > = \exp \; [-\frac{1}{\hbar} \int^t \frac{e^2}{2mc^2} A^2 (t')dt'] | \psi(t)> \tag{3}$$

Equation (1) becomes

$$i\hbar \frac{d}{dt} | \psi(t)> = [\frac{p^2}{2m} + Ve^{-\epsilon|t|} - \frac{e}{mc}\vec{A}\cdot\vec{p}] \, | \psi(t)> \tag{4}$$

Initially, at time $t \rightarrow -\infty$, the electron is far from the atom, i.e., free of the potential; thus V can be replaced by $Ve^{-\epsilon|t|}$ in which ϵ is positive but infinitesimal. Let $|\vec{k}>$ denote the eigenvector of $\vec{p}$ with momentum eigenvalue $\hbar\vec{k}$ normalized so that

$$<\vec{r}|\vec{k}> = (2\pi)^{-3/2} \, e^{i\vec{k}\cdot\vec{r}} \tag{5}$$

Initially when the potential V is turned off completely, the electron with initial momentum $\vec{k}_i$ is in the state Ix;;,(t)> which satisfies the Schrddinger equation

$$H_0 \, | \chi_{\vec{k}}(t) > \; = \; i\hbar \frac{\partial}{\partial t} \, | \chi_{\vec{k}} (t) > \tag{6}$$

.in which $H_0 = \frac{p^2}{2m} - \frac{e}{mc} \vec{p} \cdot \vec{A}$

The solution of Eq. (6) is readily obtained as follows

$$| \chi_{\vec{k}} (t)> \; = \; e^{-iE_i t/\hbar} \; e^{-i\theta_{\vec{k}}(t)} \, |\vec{k}> \tag{7}$$

in which E_i is the initial energy and

$$\theta_{\vec{k}} (t) = \; \frac{1}{\hbar} \int_0^t dt' \, [\frac{\hbar^2 k^2}{2m} - E_i - \frac{e}{mc} \vec{p} \cdot A(t')] \tag{8}$$

The solution of Eq. (4) is expressed as

$$|\psi(t)\rangle = |\chi_{\vec{k}_i}(t)\rangle + |\phi(t)\rangle \tag{9}$$

We expand $|\phi(t)\rangle$ in terms of the complete set of functions $|\chi_{\vec{k}}(t)\rangle$ as

$$|\phi(t)\rangle = \int d\vec{k}\, a_{\vec{k}}(t)\, |\chi_{\vec{k}}(t)\rangle \tag{10}$$

with the boundary condition $|\psi(t)\rangle \to |\chi_{k_i}(t)\rangle$ as $t \to -\infty$, i.e., $|\phi(-\infty)\rangle = 0$. Substituting Eqs. (9) and (10) into eq. (4), we obtain an inhomogeneous integrodifferential equation for the coefficient $a_{\vec{k}}(t)$:

$$i\hbar \frac{d}{dt} a_{\vec{k}}(t) = e^{i\theta_{\vec{k}}(t)} [e^{-i\theta_{\vec{k}_i}(t)} \langle \vec{k} | V e^{-\epsilon|t|} | \vec{k}_i \rangle + b_{\vec{k}}(t)] \tag{11}$$

in which

$$b_{\vec{k}}(t) = e^{iE_i(t)/\hbar} \langle \vec{k} | V e^{-\epsilon|t|} | \phi(t) \rangle = \int d\vec{k}'\, e^{-i\theta_{\vec{k}'}(t)} a_{\vec{k}'}(t) \langle \vec{k} | V e^{-\epsilon|t|} | \vec{k}' \rangle \tag{12}$$

with the boundary condition $a_{\vec{k}}(-\infty) = b_{\vec{k}}(-\infty) = 0$. It has been shown[9,10] that, because of the phase factor $\exp[i\theta_{\vec{k}}(t)]$ on the right-hand side of Eq. (11), the function $a_{\vec{k}}(t)$ varies rapidly with both $\vec{k}$ and t. On the other hand, $b_{\vec{k}}(t)$ varies relatively slowly with $\vec{k}$ and t. Consequently, we can interpolate $b_{\vec{k}}(t)$. We discuss several interesting points.

(A) Substituting Eq. (12) into Eq. (11) and making use of Eq. (7), we obtain

$$i\hbar \frac{d}{dt} a_{\vec{k}}(t) = e^{i\theta_{\vec{k}}(t)} [e^{iE_i t/\hbar} \langle k | V e^{-\epsilon|t|} | \chi_{\vec{k}_i}(t) \rangle + e^{iE_i t/\hbar} \langle \vec{k} | V e^{-\epsilon|t|} | \phi(t) \rangle]$$

Using Eq. (9), we find

$$i\hbar \frac{d}{dt} a_{\vec{k}}(t) = e^{-ig(\vec{k},t)} \langle \vec{k} | V e^{-\epsilon|t|} | \psi(t) \rangle \tag{13}$$

in which $g(\vec{k},t) = -E_i t/\hbar - \theta_{\vec{k}}(t)$. Formally integrating Eq. (13) over t, we have

$$a_{\vec{k}}(t) = -\frac{1}{\hbar} \int_{-}^{t} dt'\, e^{-ig(\vec{k},t')} \langle \vec{k} | V e^{-\epsilon|t'|} | \psi(t') \rangle \tag{14}$$

Substituting Eq. (14) into Eq. (10) and using Eq. (7), we find

$$\phi(t) = -\frac{1}{\hbar} \int d\vec{k} \int_{-\infty}^{t} dt'\, |\vec{k}\rangle \langle \vec{k} | V e^{-\epsilon|t'|} | \psi(t') \rangle e^{ig(\vec{k},t) - ig(\vec{k},t')}$$

Substituting into Eq. (9), finally we obtain

$$|\psi(t)\rangle = |\chi_{k_i}(t)\rangle - \frac{1}{\hbar}\int d\vec{k} \int_{-\infty}^{t} dt' \, |\vec{k}\rangle\langle\vec{k}|Ve^{-\epsilon|t'|}|\psi(t')\rangle e^{ig(\vec{k},t)-ig(\vec{k},t')} \qquad (15)$$

Therefore, the differential cross section for the electron to absorb n photons is[7]

$$\frac{d\sigma}{d\Omega} = \left[\frac{4\pi^2 m}{\hbar^2}\right]^2 \frac{k_n}{k_i} \left|\frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt \langle\chi_{\vec{k}_n}(t)|V|\psi(t)\rangle\right|^2 \qquad (16)$$

in which ω is the angular frequency of the radiation field and

$$\frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2 k_i^2}{2m} + n\hbar\omega \qquad (17)$$

The results of Eq. (15) and (16) are consistent with those obtained in the previous work.[8]

(B) As we have pointed out previously, $b_{\vec{k}}(t)$ is a slowly varying function of $\vec{k}$ and t. If we temporarily ignore it, then Eq. (11) becomes

$$i\hbar\frac{d}{dt}a_{\vec{k}}(t) = e^{i\theta_{\vec{k}}(t)-i\theta_{\vec{k}_i}(t)}\langle\vec{k}|Ve^{-\epsilon|t|}|\vec{k}_i\rangle \qquad (18)$$

For a linearly polarized field

$$\vec{A}(t) = \vec{a}\cos\omega t \qquad (19)$$

we have

$$\begin{aligned}\theta_{\vec{k}}(t) &= \frac{1}{\hbar}\int_0^t dt' \left[\frac{\hbar^2 k^2}{2m} - E_i - \frac{e\hbar}{mc}\vec{k}\cdot\vec{a}\cos\omega t'\right] \\ &= \frac{1}{\hbar}\left(\frac{\hbar^2 k^2}{2m} - E_i\right)t - \frac{e\hbar}{mc\omega}(\vec{k}\cdot\vec{a})\sin wt\end{aligned} \qquad (20)$$

Substituting into Eq. (18), we find

$$i\hbar\frac{d}{dt}a_{\vec{k}}(t) = e^{\frac{i}{\hbar}\left(\frac{\hbar^2 k^2}{2m}-\frac{\hbar^2 k_i^2}{2m}\right)t} e^{-i\frac{e}{mc\omega}(\vec{k}-\vec{k}_i)\cdot\vec{a}\sin wt} \langle\vec{k}|Ve^{-\epsilon|t|}|\vec{k}_i\rangle \qquad (21)$$

Using the Fourier-Bessel expansion

$$e^{-i\xi_k \sin\omega t} = \sum_{n=-\infty}^{+\infty} J_n(\xi_k)e^{-in\omega t} \qquad (22)$$

with

$$\xi_k = \frac{e}{mc\omega}(\vec{k}-\vec{k}_i)\cdot\vec{a} \qquad (23)$$

and integrating Eq. (21) from $t = -\infty$ to $t = +\infty$, we obtain

$$a_k(\infty) = \sum_n J_n(\xi_k)<\vec{k}|V|\vec{k}_i> \{ P[\frac{1}{E_n(k)}] + i\pi\delta[E_n(k)] \} \tag{24}$$

in which

$$E_n(k) = \frac{\hbar^2 k^2}{2m} - \frac{\hbar^2 k_i^2}{2m} - n\hbar\omega \tag{25}$$

Clearly the delta function $\delta[E_n(k)]$ on the right-hand side of Eq. (24) expresses the conservation of energy such that the outgoing electron with momentum $\vec{k}$ would have energy

$$E(k) = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 k_i^2}{2m} + n\hbar\omega = E_i + n\hbar\omega \tag{26}$$

which is the same as Eq. (17). This result indicates that during the scattering process the electron can absorb n photons (for n positive), or emit n photons (for n negative). The principal part $P[1/E_n(k)]$ arises from turning on the field during scattering.

(C) The above result is obtained by neglecting the slowly varying part of the function $b_{\vec{k}}(t)$. Of course, the correct treatment must also take into account the effect of $b_{\vec{k}}(t)$. In so doing, we have to solve the integro-differential equations (11) and (12). An explicit or an implicit method has been proposed[9,10] to solve these equations. We intend not to solve them numerically in this paper, but wish to study the dynamics of the multiphoton process during the scattering. Thus we rewrite Eq. (11) as

$$i\hbar\frac{d}{dt}a_{\vec{k}}(t) = e^{i\theta_{\vec{k}}(t)-i\theta_{\vec{k}_i}(t)}[<\vec{k}|Ve^{-\epsilon|t|}|\vec{k}_i> + b_{\vec{k}}(t)e^{i\theta_{\vec{k}_i}(t)}]$$

Using Eq. (12) for $b_k(t)$, we have

$$i\hbar\frac{d}{dt}a_{\vec{k}}(t) = e^{i\theta_{\vec{k}}(t)-i\theta_{\vec{k}_i}(t)}[<\vec{k}|Ve^{-\epsilon|t|}|\vec{k}_i>$$
$$+ \int dk'\, e^{i\theta_{\vec{k}_i}(t)-i\theta_{\vec{k}'}(t)} a_{\vec{k}'}(t)<\vec{k}|Ve^{-\epsilon|t|}|\vec{k}'>]$$

Substituting Eq. (2) into the above equation, and using the expansion of Eq. (22), we find

$$i\hbar\frac{d}{dt}a_{\vec{k}}(t) = \sum_n J_n(\xi_k)e^{\frac{i}{\hbar}[E(k)-E_i-n\hbar\omega]t}<\vec{k}|Ve^{-\epsilon|t|}|\vec{k}_i>$$
$$+\int dk' \sum_m\sum_n J_n(\xi_k)e^{\frac{i}{\hbar}[E(k)-E(k')-(n-m)\hbar\omega]t}$$
$$\times\ J_m(\xi_{k'})a_{\vec{k}'}(t)<\vec{k}|Ve^{-\epsilon|t|}|\vec{k}'>$$

Therefore

$$a_{\vec{k}}(t) = -\frac{i}{\hbar}\int_{-\infty}^{t} dt' \Big\{ \sum_n J_n(\xi_k)\, e^{\frac{i}{\hbar}[E(k)-E_i-n\hbar\omega]t'} <\vec{k}|Ve^{-\epsilon|t'|}|\vec{k}_i> + \int d\vec{k}' \sum_m\sum_n J_n(\xi_k)\, e^{\frac{i}{\hbar}[E(k)-E(k')-(n-m)\hbar\omega]t'} \times J_m(\xi_{k'}) a_{k'}(t') <\vec{k}|Ve^{-\epsilon|t'|}|\vec{k}'> \Big\} \tag{27}$$

Eq. (27) gives the expression to describe the scattering amplitude $a_{\vec{k}}(t)$ of the electron scattered from the atom at time t. The first term on the right-hand side of Eq. (27) is just the result described in part (B), which expresses the conservation of energy. The general behavior of the scattering process comes mainly from the second term on the right-hand side of Eq. (27). One sees that $a_{\vec{k}'}(t')$ represents the scattering amplitude of the electron with momentum $\hbar\vec{k}'$, at the time t', having absorbed m photons for which $J_m(\xi_{k'})$ is the amplitude for this process and, by conservation of energy, the electron energy is given by $E(k') = E_i + m\hbar\omega$. Then it reabsorbs (n-m) photons (due to the propagator exp $\{\frac{i}{\hbar}(E(k)-E(k')-(n-m)\hbar\omega]t\}$) with amplitude $J_n(\xi_k)$. The net result is that the electron has absorbed n photons at time t and is represented by the scattering amplitude $a_k(t)$. The energy of the outgoing electron is given by $E(k) = E(k') + (n-m)\hbar\omega = E_i + n\hbar\omega$, which again recovers the result of Eq. (17). Therefore, Eq. (27) gives the general formulation to describe electron-atom scattering in a field of intense radiation; the dynamics of the multiphoton process during the scattering is clearly manifested in this formulation.

III. CONCLUSION

Based on the method of solving the time-dependent Schrodinger equation in momentum space, which facilitates the extraction of the rapidly varying part of the wavefunction, we have obtained the general formulation to describe the scattering of on electron by on atom in the presence of a field of intense radiation. We found that the dynamics of multiphoton process during the scattering can be well understood by this formulation. We hope to apply in furture this method to specific problems.

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REFERENCES

1. J. I. Gersten and M. H. Mittleman, Phys. Rev. A13, 123 (1976).

2. L. Rosenberg, Phys. Rev. A16, 1941 (1977); ibid. A23, 2283 (1981).
3. F. W. Byson, Jr. and C. J. Joachain, J. Phys. B17, L295 (1984).
4. A. Dubois, A. Maquet, and S. Jetzke, Phys. Rev. A34, 1888 (1986).
5. P. Francken and C. J. Joachain, Phys. Rev. A35, 1590 (1987).
6. K. Unnikrishman, Phys. Rev. A38, 2317 (1988).
7. N. M. Kroll and K. M. Watson, Phys. Rev. A8, 804 (1973).
8. R. Shakeshaft, Phys. Rev. A28, 667 (1983).
9. R. Shakeshaft and C. S. Han, Phys. Rev. A38, 2163 (1988).
10. R. Shakeshaft and M. Dorr, Z. Phys. D8, 255 (1988).