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Dyon Charge Conjugation In Weinberg-Salam's Theory*

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We show that there exists a discrete symmetry transformation in the bosonic Weinberg-Salam's theory. The discrete operation is identified as the dyon charge conjugation (DCC). We also demonstrate that the parallel component of the non-Abelian gauge field combined with the Abelian gauge field does not contribute to the total magnetic charge of the system, if the electromagnetic field tensor is formulated to be odd under DCC.

1. INTRODUCTION

AFTER the first attempt of analyzing the magnetic monopoles by Wu and Yang within the framework of non-Abelian gauge theory[1], a tremendous amount of activities has been going on in this area of investigation. A specific model of O(3) gauge symmetry, also known as Georgi-Glashow's model[2] was introduced later by G.'t Hooft and Polyakov[3] to demonstrate the connection between the monopole structure and the classical soliton nature of non-Abelian gauge fields. It was found that the quantization of magnetic charge of monopole is a direct consequence of the Kronecker index theorem[4]. The calculation of the total flux for various charge states of the magnetic monopole corresponds to the integration of the topological charge for different homotopic classes in mapping the triplet Higgs field on the unit sphere in field space onto the infinitive shpere in the three dimensional configuration space. As to the Weinberg-Salam's theory of $SU(2) \times U(1)$ gauge symmetry[5], the Higgs vacuum is defined over the surface of a four dimensional sphere in field space. The topological monopole solution does not exist in this model for the reason that the second homotopic group of $SU(2)$ is trivially zero, namely $\pi_2(SU(2))=0$. An alternative approach has also been taken toward this problem from the viewpoint of non-topological solution, and an unit isovector was artificially introduced in order to simulate the role played by the triplet Higgs field. But unfortunatelly, it lead to no conclusive results[6,7].

The origin of the difficulties in analyzing the magnetic properties in Weinberg-Salam's theory was gradually understood after the discovery of the existence of non-Abelian strings. The

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electromagnetic field tensor in Weinberg-Salam's model contains a neutral component of a non-Abelian vector gauge field in addition to the usual Abelian gauge part. The total magnetic flux obtained by surface integration over the magnetic field constructed form the Weinberg-Salam's electroweak theory comes only from the non-Abelian gauge field. The Abelian gauge field contributes no net flux just like the sourceless magnetic system of a Dirac monopole plus the string[8,9]. It was observed further, that the flux originated from the $SU(2)$ part of gauge field is, by no means, of spherical symmetry. The explicit calculation in the Bogomol'ny condition[10] shows that the non-Abelian gauge potential could be decomposed into the components which are perpendicular and parallel to $\phi^{+}\tau\phi$ with completely arbitrary strength in such division[11]. The parallel part turned out to be a non-Abelian string starting from the origin to spatial infinitive, while the transverse part forms a genuine monopole with the field free from singularity.

A recent investigation on the topology of the bosonic Weinberg-Salam's theory has found that the system has a classical static, finite energy solution of field equation. Yet the solution is unstable. The instability arises from the fact that the loops in the configuration space, beginning and ending at vacuum, are non-contractible. Energy of the system is defined to be the infimum points on the loops, analogous to a case of quantum pendulum satisfying the Mathieu equation[12]. Since the presence of non-Abelian string will destroy the condition of finitiness in energy, therefore the arbitrariness of the non-Abelian string strength seems totally impossible.

The purpose of the work is to investigate the magnetic properties of a pure bosonic $SU(2) \times U(1)$ gauge theory of Weinberg-Salam's model, which contains a non-Abelian triplet gauge potential $W_\mu^a(x)$, an Abelian gauge potential $B_\mu(x)$ and the Higgs doublet $\phi(x)$. There exist two ways of defining the discrete symmetry transformation that leave the Lagrangian invariant. One of them corresponds to dyon charge conjugation which changes a dyon into an anti-dyon or, in the case of pure magnetic system, a monopole into an anti-monopole. We shall explore the dyon charge conjugation properties of the gauge field and Higgs field from which a electromagnetic field tensor with correct discrete transformation property will be constructed. It is explicitly demonstrated that the magnetic fields in such contruction are free from the arbitrainess of the non-Abelian string strength.

For sake of clarity, we shall put the lengthy derivation in various appendices at the end of the text.

2. DISCRETE SYMMETRY TRANSFORMATION

In the relativistic quantum field theory, combined discrete transformation of PCT has been proved to be a good symmetry of the theory, known as Lüdus-Pauli theorem[13]. For the case of strong interaction, as well as that of electromagnetic interaction, the symmetry of P, C and T are preserved separately. There is no demand for the consideration of extra discre symmetry beyond P, C and T far as the system of pure hadrons and leptons is concerned. The question that could be addressed to ourselves is whether there exists an additional symmetry in the theory of non-Abelian gauge field that admits dyons or magnetic monopoles[3,14]. In order to answer the question, we choose a pure bosonic Weinberg-Salam's model of the following Lagrangian

$$\mathcal{L}=-\frac{1}{2}TrW_{\mu\nu}W^{\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}+(D_\mu\phi)^{+}D^\mu\phi-V(\phi^{+}\phi), \tag{2-1}$$

where

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$$W_{\mu\nu}=\partial_\mu W_\nu-\partial_\nu W_\mu-ig[W_\mu,\ W_\nu] \tag{2-2}$$

and

$$W_\mu=T^a W^a_\mu \tag{2-3}$$

are 2x2 matrices gauge field tensor and gauge potential respectively. T^a's $(a=1,2,3)$ are the generators of SlJ(2) gague group.

$$B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu \tag{2-4}$$

is the usual Abelian gauge field tunsor, and the covariant derivatuie of ϕ is defined as

$$D_\mu\phi=(\partial_\mu-igW_\mu-\frac{i}{2}g'B_\mu)\phi. \tag{2-5}$$

Let us define a discrete transformation $\mathcal{D}$ as follows,

$$\mathcal{D}:\begin{cases} W_\mu\to\tilde{W}_\mu=DW^*_\mu D^{-1}, & (2\text{-}6)\\ B_\mu\to\tilde{B}_\mu=\gamma B^*_\mu, & (2\text{-}7)\\ \phi\to\tilde{\phi}D\phi^*, & (2\text{-}8)\end{cases}$$

where D is a 2×2 constant matrix and γ is a pure phase factor. The corresponding transformation for the field tensor $W_{\mu\nu}$ and $D_\mu\phi$ are

$$\mathcal{D}:\begin{cases} W_{\mu\nu}\to\tilde{W}_{\mu\nu}=DW^*_{\mu\nu}D^{-1}, & (2\text{-}9)\\ D_\mu\phi\to\widetilde{D_\mu\phi}=D(D_\mu\phi)^*. & (2\text{-}10)\end{cases}$$

The explicit expression of eq. (2-10) can be obtained from the definition of eq. (2-6) to (2-8), they are

$$\tilde{W}_{\mu\nu}=\partial_\mu\tilde{W}_\nu-\partial_\nu\tilde{W}_\mu+ig[\tilde{W}_\mu,\tilde{W}_\nu], \tag{2-11}$$

$$\widetilde{D_\mu\phi}==(\partial_\mu+ig\tilde{W}_\mu+\frac{i}{2}g'\gamma^{-1}\tilde{B}_\mu)\tilde{\phi}. \tag{2-12}$$

If the matrix D is chosen to meet the following requirements,

$$D^+=D^{-1}, \tag{2-13}$$

$$DT^{a*}D^{-1}=-T \tag{2-14}$$

there exist two cases of discrete symmetry transformation defined in eq. (2-6) to (2-8), which leave the Lagrangian eq. (2-l) invariant. We shall explicitly show in Appendix A that if phase factor takes the value $\gamma=+1$ or $\gamma=-1$, the symmetry is realized by suitable change of the sign in coupling constants g and g'.

Case a:

If $\gamma=1$ and $g\to-g, g'\to+g'$, we immediately recognize $\mathcal{L}$ is invariant by $W_\mu\to\tilde{W}_\mu, B_\mu\to\tilde{B}_\mu$ and $\phi\to\tilde{\phi}$, namely

$$\mathcal{L}(W_\mu,\ B_\mu,\ \phi;\ g,\ g')=\mathcal{L}(\tilde{W}_\mu,\ \tilde{B}_\mu,\ \tilde{\phi};\ -g,\ +g') \tag{2-15}$$

Case b:

If $\gamma=-1$ and $g\to-g, g'\to-g'$. The Lagrangian $\mathcal{L}$ again remains unchanged if $\tilde{W}_\mu\to\tilde{W}_\mu$, $B_\mu\to\tilde{B}_\mu$ and $\phi\to\tilde{\phi}$, or

$$\mathcal{L}(W_\mu,\ B_\mu,\ \phi;\ g,\ g'=\mathcal{L}(\tilde{W}_\mu,\ \tilde{B}_\mu,\ \tilde{\phi};\ -g,\ -g'). \tag{2-16}$$

We are now able to provide the answer to our previous question. Besides the discrete symmetry transformation of parity, charge conjugation and time reversal, the bosonic Weinberg-Salam's theory also exhibits an additional symmetry of Q-transformation. What is more important is the physical significance of such transformation, which will be discussed in the following secton.

3. DYON CHARGE CONJUGATION

The transformation of P, C or T could be understood intuitively through the geometric nterpretation of their corresponding equations of motion both on the classical level and on the quantum level. In the case of electro-weak unified theory, the physical interpretation of the extra discrete symmetry from the field equations deserves special attention because of the complexities in the electro-weak mixings. It has been recently shown that there exists a discrete symmetry of magnetic charge conjugation in Georgi-Glashow's model with Higgs doublets[15]. One might contemplate that the new discrete symmetry in bosonic Weinberg-Salam's theory could be of relevance to the charges which characterized either the electromagnetic or the weak interaction. Therefore we are compelled to study the field equations only after the spon taneous breaking of $SU(2)\times U(1)$ gauge symmetry to the usual electromagnetic $U(1)_{EM}$ gauge symmetry.

Consider the potential $V(\phi^+\phi)=\lambda(\phi^+\phi-c^2)^2$, the non-vanishing vacuum expectation value of ϕ, chosen as

$$\langle\phi\rangle_0=\begin{pmatrix}0\\ c\end{pmatrix} \tag{3-1}$$

allows one to generate, in addition to the mass of Higgs field, the masses of three gauge field out of W_μ^a and B_μ, and leaving one gauge field to remain massless. Since $\langle\phi\rangle_0$ corresponds to zero signevalue of operator $T_3+\frac{1}{2}I$, i. e.

$$\left(T_3+\frac{1}{2}I\right)\langle\phi\rangle_0=0, \tag{3-2}$$

one can immediately verify that the Higgs vacuum is invariant under the $U(1)_{EM}$ gauge transformation if the electromagnetic gauge potential A_μ is defined as

$$A_\mu=\frac{1}{\sqrt{g^2+g'^2}}(g'W_\mu^3+gB_\mu) \tag{3-3}$$

The neutral partner of the three massive gauge potential is found to be (see Appendix B),

$$Z_\mu=\frac{1}{\sqrt{g^2+g'^2}}(gW_\mu^3-g'B_\mu). \tag{3-4}$$

Let us now examine the two cases of discrete symmetry defined in previous section. Since all the gauge potentials W_μ^a and B_μ are real and D could be taken as $-i\sigma_2$. Therefore in the case that $\gamma=1$, one has $\tilde{W}_\mu^a=-W_\mu^a$, $\tilde{B}_\mu=B_\mu$. As $g\to-g$ and $g'\to g'$ the discrete operations of eq. (3-3) and (3-4) become

$$A_\mu\to\tilde{A}_\mu=-A_\mu, \tag{3-5}$$

$$Z_\mu\to\tilde{Z}_\mu=Z_\mu. \tag{3-6}$$

For the second case, that $\gamma=-1$, the coupling constants g, g', as well as the gauge potential W_μ^a and B_μ, simultaneously reverse their signs under the discrete transformation, and hence both the massive neutral gauge potential Z_μ and the EM massless potential A_μ, remain unchanged, i.e.

$$A_\mu\to\tilde{A}_\mu=A_\mu, \tag{3-7}$$

$$Z_\mu\to\tilde{Z}_\mu=Z_\mu \tag{3-8}$$

The trivial properties of eq. (3-7) and (3-8) cause no change in physics as far as the equations of motion are concerned. It corresponds merely the redefination of all fields in the bosonic electro-weak system. While the properities of eq. (3-5) and (3-6), on the other hand, imply that the first case of discrete transformation leaves the field equations of weak interaction invariant,

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but changes the field equation of electromagnetic interaction into that of the anti-particle. For the bosonic Weinbery-Salam's theory, this transformation corresponds to the conversion of a dyon into an anti-dyon, or a monopole into an anti-monopole in the special solution of pure magnetic system. Therefore we shall call such such discrete symmetry operation the dyon charge conjugation.

Before ending this section, we now come to investigate the gauge symmetry of the conjugate fields. Since the gauge potentials W_μ, B_μ and Higgs fields ϕ satisfy the following transformation law of $SU(2)\times U(1)$, i.e.

$$W_\mu \to W'_\mu = \Omega W_\mu \Omega^+ - \frac{i}{g}(\partial_\mu \Omega)\Omega^+, \tag{3-9}$$

$$B_\mu \to B'_\mu = S B_\mu S^+ - \frac{i}{g'}(\partial_\mu S)S^+, \tag{3-10}$$

and

$$\phi \to \phi' = \Omega\phi, \tag{3-11}$$

where

$$\Omega = e^{i\vec{\Lambda}\cdot\vec{T}} \tag{3-12}$$

is the $SU(2)$ matrix and

$$S = e^{i\theta(x)} \tag{3-13}$$

is the Abelian gauge transformation function. With $\Lambda(x)$ and $\theta(x)$ being real, one concludes that

$$D\Omega^* D^+ = \Omega, \tag{3-14}$$

$$(\partial_\mu S)S^+ = -S(\partial_\mu S^+), \tag{3-15}$$

which allow one to derive the corresponding transformation law for the conjugate fields

$$\tilde{W}'_\mu = \Omega \tilde{W}_\mu \Omega^+ - \frac{i}{g}(\partial_\mu \Omega)\Omega^+, \tag{3-16}$$

$$\tilde{B}'_\mu = S\tilde{B}_\mu S^+ - \frac{i}{g}(\partial_\mu S)S^+, \tag{3-17}$$

$$\tilde{\phi}' = \Omega\tilde{\phi}. \tag{3-18}$$

The same transformation matrix Ω and functions S appear in eq. (3-9) to (3-11) and eq. (3-16) to (3-18), plus the unitarity condition of matrix D could also provide us another verification of the invariance of $\mathcal{L}$ in eq. (2-l) under the dyon charge conjugation.

4. THE ELECTROMAGNETIC FIELD TENSOR WITHOUT NON-ABELIN STRING

This section will be devoted to the investigation of the magnetic properties of the bosonic Weinberg-Salam's theory. Since the theory admits the dyon solution, the electric properties, in fact, can be simultaneously analyzed. Nevertheless, we would only concentrated our attention in its magnetic nature for the reason that, on the classical level, neither the electric charge of a dyon could be quantized, nor its explicit calculation could be independent of the ansatz to the particular *model* of interest" ".

Let us consider the electromagnetic field tenson $F_{\mu\nu}$ of the dyon system. The fact that ϕ is not of self dyon charge conjugation implies the Higgs fields are not electrically and magnetically neutral. In addition to the usual covariant properties and gauge symmetry we shall construct $F_{\mu\nu}$ to satisfy the extra symmetry properties of dyon charge conjugation, namely

$$\tilde{F}_{\mu\nu} = -F_{\mu\nu}, \tag{4-1}$$

where $\tilde{F}_{\mu\nu}$ is defined by interchanging the fields with their corresponding conjugate fields from which $F_{\mu\nu}$ is built, i.e. if $F_{\mu\nu} = F_{\mu\nu}(\phi, \tilde{\phi}, W_{\mu\nu}, \tilde{W}_{\mu\nu}, B_{\mu\nu}, \tilde{B}_{\mu\nu})$

$$\tilde{F}_{\mu\nu}=F_{\mu\nu}(\tilde{\phi}, \phi, \tilde{W}_{\mu\nu}, W_{\mu\nu}, \tilde{B}_{\mu\nu}, B_{\mu\nu}). \quad (4\text{-}2)$$

The explicit expression of $F_{\mu\nu}$ can then be cast into the following form,

$$F_{\mu\nu}=\cos\vartheta B_{\mu\nu}+\sin\vartheta\Big\{\frac{1}{2|\phi|^2}(\phi^+W_{\mu\nu}\phi-\tilde{\phi}^+W_{\mu\nu}\tilde{\phi}-\tilde{\phi}^+\tilde{W}_{\mu\nu}\tilde{\phi}+\phi^+\tilde{W}_{\mu\nu}\phi)$$
$$+\frac{1}{ig|\phi|^2}[\phi^+D_\mu\phi\phi^+D_\nu\phi-\phi^+D_\mu\tilde{\phi}\tilde{\phi}^+D_\nu\phi+\tilde{\phi}^+D_\mu\phi\phi^+D_\nu\tilde{\phi}-\tilde{\phi}^+D_\mu\tilde{\phi}\tilde{\phi}^+D_\nu\tilde{\phi}$$
$$-(\mu\leftrightarrow\nu)]\Big\}, \quad (4\text{-}3)$$

where ϑ is the Weinberg-Salam ' s mixing angle, which is related to the coupling constents g and g' by

$$\sin\vartheta=g'/\sqrt{g^2+g'^2}, \quad (4\text{-}4)$$
$$\cos\vartheta=g/\sqrt{g^2+g'^2}. \quad (4\text{-}5)$$

From the condition of eq. (2-13), one could verify that

$$\tilde{\phi}^+\tilde{\phi}=\phi^+\phi=|\phi|^2, \quad (4\text{-}6)$$

therefore in the region of Higgs vacuum, the electromagnetic field tensor $F_{\mu\nu}$ can be rewritten as

$$F_{\mu\nu}=\partial_\mu(B_\nu\cos\vartheta+\hat{\varphi}\cdot\mathbf{W}_\nu\sin\vartheta)-\partial_\nu(B_\mu\cos\vartheta+\hat{\varphi}\cdot\mathbf{W}_\mu\sin\vartheta)$$
$$+\frac{\sin\vartheta}{ig}\{\varphi^+\partial_\mu\varphi\varphi^+\partial_\nu\varphi-\varphi^+\partial_\mu\tilde{\varphi}\tilde{\varphi}^+\partial_\nu\varphi+\tilde{\varphi}^+\partial_\mu\varphi\varphi^+\partial_\nu\tilde{\varphi}-\tilde{\varphi}^+\partial_\mu\tilde{\varphi}\tilde{\varphi}^+\partial_\nu\tilde{\varphi}-(\mu\leftrightarrow\nu)\}, \quad (4\text{-}7)$$

where (see Appendix C)

$$\hat{\varphi}=Tr\{\mathbf{T}(\varphi\varphi^+-\tilde{\varphi}\tilde{\varphi}^+)\} \quad (4\text{-}8)$$

is the unit local vector in isospin space. This unit isovector is introduced to characterize the Higgs doublet in the unitary gauge through the following eigenvalue equation

$$\varphi\cdot\mathbf{T}\phi=\lambda. \quad (4\text{-}9)$$

The magnetic current, also known as the topological current, is expressed by the divergence of the dual of $F_{\mu\nu}$,

$$k_\mu=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial^\nu F^{\alpha\beta}, \quad (4\text{-}10)$$

which can be computed in terms of the unit isovector $\hat{\varphi}$,

$$k_\mu=\frac{\sin\vartheta}{g}\epsilon_{\mu\nu\alpha\beta}\epsilon^{abc}\partial^\nu\{\hat{\varphi}^a\partial^\alpha\hat{\varphi}^b\partial^\beta\hat{\varphi}^c\}. \quad (4\text{-}11)$$

For the static state solutions in which all fields are indepent of time, the magnetic field in eq. (4-7) reduces to

$$H_i=(\nabla\times\mathbf{A})_i+\frac{\sin\vartheta}{ig}\epsilon_{ijk}\{\varphi^+\partial_j\varphi\varphi^+\partial_k\varphi-\varphi^+\partial_j\tilde{\varphi}\tilde{\varphi}^+\partial_k\varphi+\tilde{\varphi}\partial_j\varphi\varphi^+\partial_k\tilde{\varphi}-\tilde{\varphi}^+\partial_j\tilde{\varphi}\tilde{\varphi}^+\partial_k\tilde{\varphi}\}, \quad (4\text{-}12)$$

where $\mathbf{A}=(A_1, A_2, A_3)$ with

$$A_i=\cos\vartheta B_i+\sin\vartheta\hat{\varphi}\cdot\mathbf{W}_i. \quad (4\text{-}13)$$

The singularity only appears as a sourceless Abelian string, which behaves like the point Dirac monopole situating at one end of a semi-infinitive string (called Dirac string for his first discovery). Since $\hat{\varphi}$ can also be expressed as

$$\hat{\varphi}=\frac{1}{|\phi|^2}(\phi^+\mathbf{T}\phi), \quad (4\text{-}14)$$

one finds immediately that the parallel component of the non-Abelian gauge field is combined with B_μ to form a new Abelian gauge field of $U(1)_{EM}$ that has zero net flux contribution. The strength of the parallel component of the non-Abelian gauge field is uniquely determined by the electro-weak mixing angle ϑ. The second term of the right hand side of eq. (4-12) which contributes to total magnetic charge of system, is free from the string singularity. By taking the spatial divergence of the magnetic field, one finds that

$$\nabla\cdot\mathbf{H}=\frac{\sin\vartheta}{g}\epsilon_{ijk}\partial^i(\hat{\varphi}\cdot\partial^j\hat{\varphi}\times\partial^k\hat{\varphi}). \tag{4-15}$$

The maghetic charge of the $SU(2)\times U(1)$ monopole can also be quantized just as that of O(3) monopole.

5. THE DISCUSSION AND CONCLUSIONS

The result which we have reached in eq. (4-15) at the end of previous section seems against the general proporties of the homotopy theory that the second homotopic group of $SU(N)$ is trivially zero[16~18]. Instead of searching for monopole carrying multiplicative charges in $SU(2)\times U(1)$ gauge theory, it has been shown that the group U(2), isomorphic to $SU(2)\times U(1)/Z_2$, could be taken as another gauge group for analysis[19].

By introducing the dyon charge conjugation in the $SU(2)$ X $U(1)$ gauge theory of pure bosonic fields system, a nontrivial topological current can be contructed if the specific transformation property under DCC is satisfied. In order to see the possibility of such a current exists, we have to investigate the orbits in the group space for $SU(2)$ gauge transformation of both the Higgs field ϕ and its DCC field $\tilde{\phi}$. Let φ be the unit isovector that characterizes the Higgs doublet ϕ as given in the eigenvalue equation of eq. (4-9). The eigenvalue A can be shown to be real because of the hermiticity of T. With the condition eq. (2-14) of D, one has

$$\hat{\varphi}\cdot\mathbf{T}\tilde{\phi}(x)=-\lambda\tilde{\phi}(x). \tag{5-1}$$

If $\hat{\tilde{\varphi}}$ is the local unit isovector that characterizes the DCC field $\tilde{\phi}$, we can immediately verify that

$$\hat{\tilde{\varphi}}=-\hat{\varphi}. \tag{5-2}$$

Therefore the $SU(2)$ gauge transformation of ϕ and $\tilde{\phi}$ correspond respectively to the rotations of $\hat{\varphi}$ and $\hat{\tilde{\varphi}}$ in the group space with opposite direction.

The electromagnetic field tensor given in eq. (4-3) is manfest gauge invariant, as well as odd under DCC, the gauge transformation property of $F_{\mu\nu}$ will reduce to that of SO(3) because the rotations of $\hat{\varphi}$ and $\hat{\tilde{\varphi}}$ are performed simultaneously with the two points on the diametric opposite on the sphere in the group space being identified. This is precisely why the explicit evaluation of the topological current depends upon the four-divergence of SO(3) invariant instead of $SU(2)$'s. The solutions can also be classified according to different values of the topological charges that represent the field configurations of various megnetic charge states.

The eigenvalue equation of eq. (4-9) can also provide us with another justification of the previous conclusion by looking at the orbits of the Higgs fields in field space. If we choose the ansatz $\hat{\varphi}=\hat{r}$ in the unitary gauge, the relative phase between the two componetns of the Higgs field is fixed. Therefore the eigenvalue equation could serve as a constraint on the Higgs doublet so that the region of Higgs vacuum corresponds to the orbit of a S^2 in field space, that is the projection of a S^3 (i.e. sphere of $|\phi|^2=c^2$) on a hyperplane passing through the origin.

The structure of magnetic monopole in the bosonic Weinberg-Salam's theory has not yet been fully explored. The imposition of the DCC discrete symmetry for the system could only provide partial information on the electromagnetic properties. The problems concering the $SU(5)$ monopole transition to monopoles of lower gauge symmetry deserves further investigation, from which one might have a better understanding of the electro-weak mixing angle.

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APPENDIX A

The Lagrangian under D-transformation for both cases can be expressed as

$$\tilde{\mathcal{L}}=\mathcal{L}(\tilde{W}_\mu,\ \tilde{B}_\mu,\ \tilde{\phi};\ -g,\ \pm g')$$
$$=-\frac{1}{2}Tr(\tilde{W}_{\mu\nu}\tilde{W}^{\mu\nu})-\frac{1}{4}\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu}+(\widetilde{D_\mu\phi})^+(\widetilde{D^\mu\phi})-V(\tilde{\phi}^+\tilde{\phi}), \tag{A-1}$$

where

$$\tilde{W}_{\mu\nu}=\partial_\mu\tilde{W}_\nu-\partial_\nu\tilde{W}_\mu+ig[\tilde{W}_\mu,\ \tilde{W}_\nu], \tag{A-2}$$
$$\tilde{B}_{\mu\nu}=\partial_\mu\tilde{B}_\nu-\partial_\nu\tilde{B}_\mu, \tag{A-3}$$
$$\widetilde{D_\mu\phi}=(\partial_\mu+ig\tilde{W}_\mu+ig'\gamma^{-1}\tilde{B}_\mu/2)\phi^*. \tag{A-4}$$

With the eq. (2-6) to eq. (2-10), we obtain

$$Tr\tilde{W}_{\mu\nu}\tilde{W}^{\mu\nu}=Tr(DW^*_{\mu\nu}W^{\mu\nu *}D^{-1})=TrW_{\mu\nu}W^{\mu\nu})^*, \tag{A-5}$$
$$\tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu}=\gamma^2(B_{\mu\nu}B^{\mu\nu})^*, \tag{A-6}$$
$$(\widetilde{D_\mu\phi})^+(\widetilde{D^\mu\phi})=(D_\mu\phi)^TD^+D(D^\mu\phi)^*=(D_\mu\phi)^T(D^\mu\phi)^*, \tag{A-7}$$
$$V(\tilde{\phi}^+\tilde{\phi})=V(\phi^T\phi^*). \tag{A-8}$$

Therefore, for $\gamma=\pm 1$ and reality conditions of fields, $\tilde{\mathcal{L}}$ reduces to

$$\tilde{\mathcal{L}}=\mathcal{L}^*=\mathcal{L} \tag{A-9}$$

APPENDIX B

The Yang-Mills-Higgs Lagrangian of $SU(2)\times U(1)$ symmetry, given as

$$\mathcal{L}=-\frac{1}{2}TrW_{\mu\nu}W^{\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}+(D_\mu\phi)^+(D^\mu\phi)-\lambda(\phi^+\phi-c^2)^2 \tag{B-1}$$

is reduced to that of $U(1)_{EM}$ symmetry by Higgs mechanism with φ defined as

$$\varphi=\phi-\ \langle\phi\rangle_0=\begin{pmatrix}\phi_1\\ \phi_2-c\end{pmatrix}. \tag{B-2}$$

The Lagrangian in eq. (B-l) can be reformulated as

$$\mathcal{L}=-\frac{1}{2}W^+_{\mu\nu}W^{\mu\nu -}+\frac{1}{2}g^2c^2W^+_\mu W^{\mu -}-\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu}+\frac{c^2}{4}(g^2+g'^2)Z_\mu Z^\mu$$
$$-\frac{1}{4}A_{\mu\nu}A^{\mu\nu}+\mathcal{L}_{int}(\varphi,\ Z_\mu,\ A_\mu,\ W^\pm_\mu), \tag{B-3}$$

where

$$W^\pm_\mu=\frac{1}{\sqrt{2}}(W^1_\mu\pm iW^2_\mu), \tag{B-4}$$
$$Z_\mu=\frac{1}{\sqrt{g^2+g'^2}}(gW^3_\mu-g'B_\mu), \tag{B-5}$$
$$A_\mu=\frac{1}{\sqrt{g^2+g'^2}}(g'W^3_\mu+gB_\mu), \tag{B-6}$$

and

$$W^\pm_{\mu\nu}=\partial_\mu W^\pm_\nu-\partial_\nu W^\pm_\mu, \tag{B-7}$$
$$Z_{\mu\nu}=\partial_\mu Z_\nu-\partial_\nu Z_\mu, \tag{B-8}$$
$$A_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu \tag{B-9}$$

The charged vector bosons $W_\mu^\pm$ obtain the mass M_W

$$M_W = cg, \tag{B-10}$$

and the neutral vector boson Z_μ has different mass M_Z

$$M_Z = c\sqrt{g^2+g'^2}, \tag{B-11}$$

but the photon A_μ remains massless.

Since the mixing angle ϑ is related to the coupling constant g, g' by

$$\cos\vartheta = g/\sqrt{g^2+g'^2}, \tag{B-12}$$

one has

$$M_W = M_Z \cos\vartheta. \tag{B-13}$$

APPENDIX C

Intorduce $\tilde{\varphi}$ and φ as

$$\varphi = \phi/|\phi|, \tag{C-1}$$

$$\tilde{\varphi} = \tilde{\phi}/|\tilde{\phi}|, \tag{C-2}$$

the eigenvalue equations of eq. (4-9) and eq. (5-2) can be re-expressed in the following forms

$$\hat{\varphi}\cdot\mathbf{T}\varphi_\lambda = \lambda\varphi_\lambda, \tag{C-3}$$

$$\hat{\varphi}\cdot\mathbf{T}\tilde{\varphi}_\lambda = -\lambda\tilde{\varphi}_\lambda. \tag{C-4}$$

Since $\tilde{\varphi}_\lambda = \varphi_{-\lambda}$, the closure relation

$$\varphi_\lambda\varphi_\lambda^+ + \varphi_{-\lambda}\varphi_{-\lambda}^+ = I, \tag{C-5}$$

become

$$\varphi_\lambda\varphi_\lambda^+ + \tilde{\varphi}_\lambda\tilde{\varphi}_\lambda^+ = I. \tag{C-6}$$

Therefore one obtians from eq. (C-3) and eq. (C-4) that

$$\hat{\varphi}\cdot\mathbf{T} = \frac{1}{2}(\varphi_\lambda\varphi_\lambda^+ - \tilde{\varphi}_\lambda\tilde{\varphi}_\lambda^+). \tag{C-7}$$

With the normalization

$$Tr(T^aT^b) = \frac{1}{2}\delta^{ab}, \tag{C-8}$$

one has

$$\hat{\varphi} = Tr\{\mathbf{T}(\varphi\varphi^+ - \tilde{\varphi}\tilde{\varphi}^+)\} \tag{C-9}$$