

## Extended Symmetries in the Special Theory of Relativity

S. ARIF KAMAL

*Department of Physics, Indiana University,  
Bloomington, Indiana 47405, U. S. A.*

(Received 18, February 1981)

The behavior of particles with velocities greater than the velocity of light is re-examined and a theory is presented which is based on the symmetry principles of nature.

MANY authors have considered the possibility of describing within special theory of relativity, particles which travel with velocities greater than the velocity of light. Such particles are named as tachyons<sup>(1)</sup>. In the Feinberg's theory<sup>(2)</sup> proper mass  $m_0$  is replaced by  $i\mu_0$  where  $\mu_0$  is real. As the velocity approaches infinity, energy approaches zero and momentum takes a value  $\mu_0 c$  so that infinite speed particle carries momentum but no energy<sup>(2)</sup>. Recami and Mignani<sup>(3)</sup> gave the generalized Lorentz transformations as

$$x' = \pm (x - Vt)K^{-1/2} \quad (1a)$$

$$y' = \pm y(1 - V^2/c^2)^{1/2}K^{-1/2} \quad (1b)$$

$$z' = \pm z(1 - V^2/c^2)^{1/2}K^{-1/2} \quad (1c)$$

$$t' = \pm (t - Vx/c^2)K^{-1/2} \quad (1d)$$

where  $K = |1 - V^2/c^2|$ . The range of  $V$  (the frame velocity of  $S'$  with respect to  $S$ ) is now from  $-\infty$  to  $\infty$ . The double sign is required by the invertibility Lorentz transformations. In the classical theory of tachyons we note that

- (i) At least two of the coordinates in (1) are imaginary as  $V$  exceeds  $c$ .
- (ii) At least two components of the velocity are imaginary as  $V$  exceeds  $c$ .
- (iii) At infinite velocity energy is zero but momentum is  $\mu_0 c$ . This shows an asymmetry in energy and momentum values for the ranges 0 to  $c$  and  $c$  to infinity.
- (iv) In Feinberg's theory<sup>(2)</sup> the concept of imaginary proper mass for tachyons is not very attractive.
- (v) Feinberg's<sup>(2)</sup> and Tanaka's<sup>(4)</sup> theories for tachyons are not Lorentz covariant.
- (vi) The correct time ordering of the events cannot be retained for superluminal frames.

Also the fact that no experimental evidence for tachyons is available upto now<sup>(5)</sup>, suggests that there might be a modification needed in the existing superluminal theories so that a better physical picture is obtained.

In this paper an attempt is made to present a theory which is based on the symmetry principles of nature. The invariant quantity  $c$  in the Lorentz transformations comes out to be equal to the speed of light in vacuum as a consequence of Maxwell's equations. This does not exclude the

- (1) G. Feinberg, Phys. Rev. 159 (1967) 1089.
- (2) E. van der Spuy, Nuovo Cimento 3A (1971) 822.
- (3) E. Recami and R. Mignani, Riv. Nuovo Cimento 4 (1974) 209; 398.
- (4) S. Tanaka, Prog. Theor. Phys. 24 (1960) 171.
- (5) Particle Data Group, Revs. Mod. Phys. 52 (1980) S1.

possibility of other types of radiations travelling at other speeds which are also invariant in all frames of reference. If we look at the properties of particles which travel with speeds less than the speed of light (bradyons) and the particles which travel with speeds greater than the speed of light (tachyons), we think why nature has preferred the speed  $u=c$  to be invariant in all frames of reference and why this becomes, a demarkation line on each side of which there exist particles with remarkably different properties. For massive particles the range 0 to  $c$  is allowed as well as the range  $c$  to infinity but  $u=c$  is not permitted. Extending this concept to negative values we note that  $u=+c$  and  $u=-c$  are invariant in all frames (by  $u=-c$ , we mean the particles corresponding to the lightlike worldline described by  $x+ct=0$ ). The interval  $(-c, c)$  is very small as compared to the interval  $(-\infty, -c)$  or  $(c, \infty)$ . Therefore we note that the allowed range of speeds for bradyons is very much less than the allowed range of speeds for tachyons<sup>(6)</sup>.

This fact presents another question. Are there only two values  $u=+c$  and  $u=-c$  which are invariant under all transformations? If so, then why preference is given only to these values. There must be analogues to these speeds in the space-time continuum which fit symmetrically in such a way that there is no preference to any such speed. These speeds must be selected in such a way that if we choose any point on the speed axis which is equidistant from the two invariant speeds, we must obtain a symmetrical distribution of invariant speeds. Let  $c_1, c_2, c_3, \dots$  be the set of invariant speeds with which the particles can travel in the space-time continuum. The difference between two consecutive invariant speeds must be  $2c$  because this is the difference between  $c$  and  $-c$ <sup>(7)</sup>. In general for any two invariant speeds

$$c_p - c_q = 2nc; \quad p, q, n = \text{integers} \tag{2}$$

Denoting  $c$  by  $c_1$ ,  $-c$  by  $c_0$  and taking  $q=0, p=n$  we get

$$c_n = (2n-1)c; \quad n = 0, \pm 1, \pm 2, \dots \tag{3}$$

The invariant speeds are now

$$\dots, -5c, -3c, -c, c, 3c, 5c, \dots$$

This idea is further supported if we define

$$m(u) = m_0(1-u^2/c^2)^{-1/2}, \quad -c < u < c \tag{4a}$$

$$= m(u-2c), \quad u > c \tag{4b}$$

$$= m(u+2c), \quad u < -c \tag{4c}$$

This can be expressed as a Fourier series

$$m(u) = \frac{1}{2} a_0 + \sum_{j=1}^{\infty} [a_j \cos(j\pi u/c) + b_j \sin(j\pi u/c)] \tag{5}$$

where

$$a_j = (1/c) \int_{-c}^c m(u) \cos(j\pi u/c) du, \quad b_j = 0,$$

because  $m(u)$  is an even function of  $u$ . Mathematical calculation gives

$$m(u) = \pi m_0 \left[ \frac{1}{2} + \sum_{j=1}^{\infty} J_0(j\pi) \cos(j\pi u/c) \right] \tag{6}$$

where  $J_0(j\pi)$  is the Bessel function of order zero. The momentum and energy are still given by

$$p(u) = um(u); \quad E(u) = c^2 m(u) \tag{7}$$

Figures 1 and 2 show  $E/m_0c^2$  and  $p/m_0c$  as function of  $u/c$  for values of  $u/c$  between  $-3$  and  $+3$ .

If  $c < u < 3c$ ,  $u$  can be written as  $v+2c$ , where  $-c < v < c$  and so  $m(u) = m(v)$ . If  $3c$  is also an

(6) S. A. Kamal, Luxon-Bradyon Transformation, Thesis, Univ. Karachi, 1978, pp. 49-54 (unpublished).

(7) S. A. Kamal, Beyond the speed of light, Bull. Amer. Phys. Soc. 27 (1982) 33.

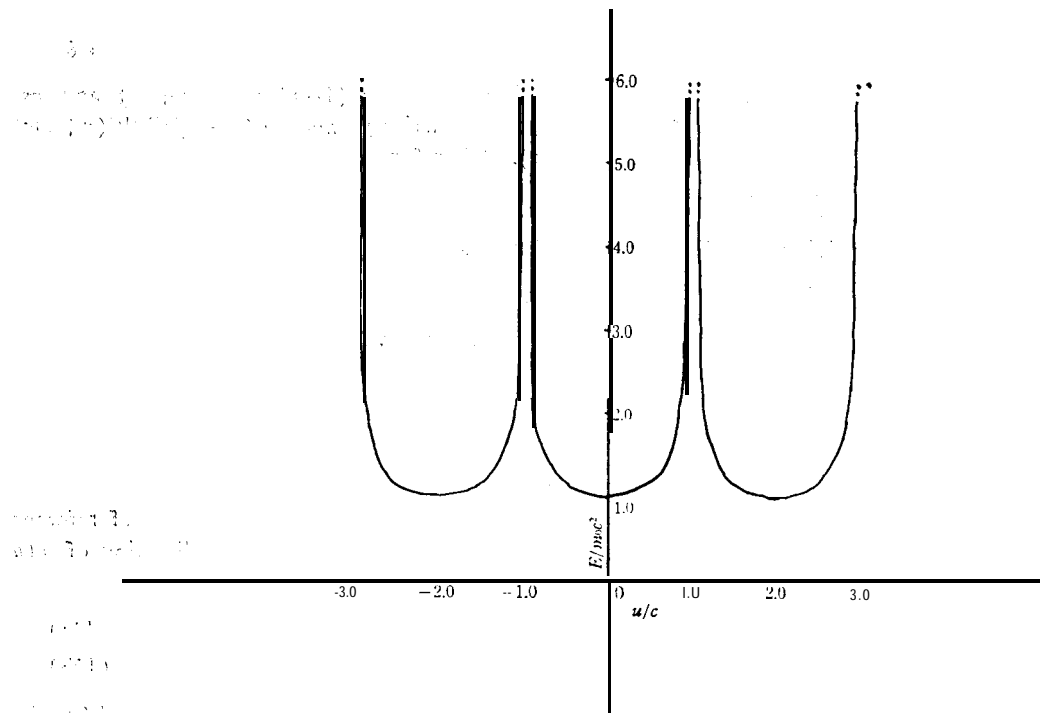


Fig. 1.  $\dot{E}/m_0c^2$  as a function of  $u/c$ . Note that energy is an even function of velocity.

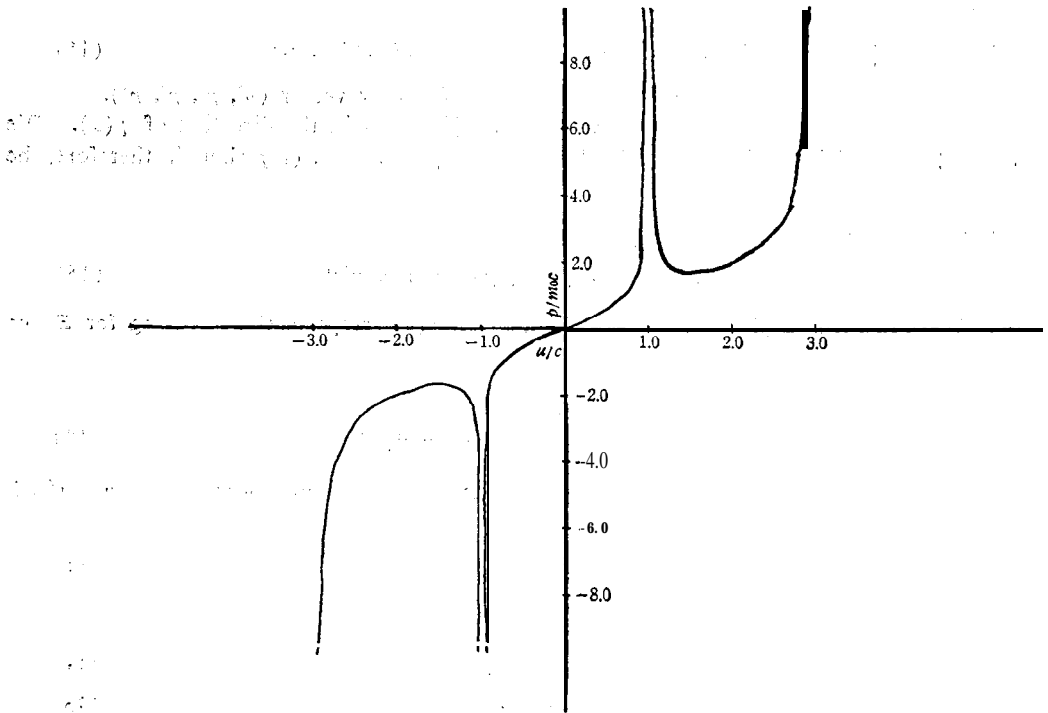


Fig. 2.  $p/m_0c$  as a function of  $u/c$ . Note that momentum is an odd function of velocity.

invariant speed we must have

$$(p-3mc)(p-mc)=-m_0^2c^2 \quad (8)$$

The above relation can be verified by substituting  $p=mu=m_0(1-v^2/c^2)^{-1/2}(v+2c)$  and  $m=m_0(1-v^2/c^2)^{-1/2}$ . In general if  $(2n-1)c < u < (2n+1)c$ , we can write  $p=mn=m_0(1-v^2/c^2)^{-1/2}(v+2nc)$ ;  $m=m_0(1-v^2/c^2)^{-1/2}$  where  $u=v+2nc$  ( $n$  is an integer). Therefore

$$[p-(2n+1)mc][p-(2n-1)mc]=-m_0^2c^2 \quad (9)$$

Using  $m=E/c^2$  and solving the above equation for  $E$  we get

$$E=(4n^2-1)^{-1}[2ncp \pm \{c^2p^2 - m_0^2c^4(1-4n^2)\}^{1/2}] \quad (10)$$

Correct sign can be chosen by the requirement that  $E=m_0c^2$  when  $u=2nc$ . We therefore, have

$$E=(4n^2-1)^{-1}[2ncp - \{c^2p^2 + m_0^2c^4(1-4n^2)\}^{1/2}] \quad (11)$$

For  $n=0$ , eqs. (9) and (11) reduce to

$$(p+mc)(p-mc)=-m_0^2c^2; \quad E=(c^2p^2+m_0^2c^4)^{1/2} \quad (12)$$

If  $E(u)$  and  $p(u)$  are the energy and momentum of a particle in laboratory frame of reference and  $E(u')$  and  $p(u')$  are the values in a frame moving with velocity  $\beta c$  in the direction of  $p(u)$  ( $|\beta| < 1$ ), the energy and momentum are related by

$$cp(u')-2n'E(u')=(1-\beta^2)^{-1/2}[\{cp(u)-2nE(u)\}-\beta E(u)] \quad (13a)$$

$$E(u')=(1-\beta^2)^{-1/2}[E(u)-\beta\{cp(u)-2nE(u)\}] \quad (13b)$$

where  $u'=2n'c+v'$  ( $n'$  is an integer),  $u=2nc+v$ ,  $-c < v' < c$ ,  $-c < v < c$ ,  $v'=(1-\beta v/c)^{-1}$  (V-PC). The integers  $n, n'$  are defined by the conditions

$$|[cp(u)/E(u)]-2n| < 1; \quad |[cp(u')/E(u')]-2n'| < 1 \quad (14a, b)$$

Using eq. (11) it can easily verified that

$$[E(u')]^2 - [cp(u')-2n'E(u')]^2 = [E(u)]^2 - [cp(u)-2nE(u)]^2 = m_0^2c^4 \quad (15)$$

Therefore  $(E(u), 0, 0, cp(u)-2nE(u))$  are the components of a four vector  $(p^0, p^1, p^2, p^3)$ .

Let us now consider a frame moving with velocity  $\alpha c$  ( $|\alpha| > 1$ ) in the direction of  $p(u)$ . We can write  $\alpha = \beta_0 + 2n''$  ( $n''$  is an integer such that  $|\alpha - 2n''| < 1$ ). Eq. (15) should, therefore, be modified as

$$\begin{aligned} & [(1-2n'')E' + (cp' - 2n'E')] [(1+2n'')E' - (cp' - 2n'E')] \\ & = [E' + (cp' - 2n'E')] [E' - (cp' - 2n'E')] = m_0^2c^4 \end{aligned} \quad (16)$$

where  $E', p', E, p$  are used in place of  $E(u'), p(u'), E(u), p(u)$  respectively. Solving for  $E'$  we get

$$\begin{aligned} E' & = (4n'^2 + 4n''^2 + 8n'n'' - 1)^{-1} [2(n' + n'')cp' \\ & \quad - \{c^2p'^2 + (1 - 4n'^2 - 4n''^2 - 8n'n'')m_0^2c^4\}^{1/2}] \end{aligned} \quad (17)$$

Correct sign for the square root is chosen by the condition  $E=m_0c^2$  when  $p=0, n'=n''=0$ . Eq. (16) can be rewritten as

$$E'^2 - [cp' - 2(n' + n'')E']^2 = E^2 - (cp - 2nE)^2 = m_0^2c^4 \quad (18)$$

Therefore eq. (13) is modified as

$$cp' - 2(n' + n'')E' = (1 - \beta_0^2)^{-1/2} [(cp - 2nE) - \beta_0 E] \quad (19a)$$

$$E' = (1 - \beta_0^2)^{-1/2} [E - \beta_0 (cp - 2nE)] \quad (19b)$$

Eqs. (15) and (18) are written using the matrix

$$\eta = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (20)$$

An experiment is suggested to verify this theory. If this theory is correct we should be able to observe particles (antiparticles) with energy  $m_0c^2$  and momentum  $2m_0c$  (corresponding to  $u=2c$ ). Consider a photon incident along positive x-axis having energy 1.02 MeV. Its momentum is 1.02 MeV/c. If this theory is valid it is possible that this photon may produce an electron-positron pair (Fig. 3). The electron and positron should equally divide the energy and momentum and travel with speeds  $2c$  making an angle  $60^\circ$  on opposite sides of z-axis. We now have

$$\begin{aligned} E_e &= 1.02 \text{ MeV}, p_e = 1.02 \text{ MeV}/c, E_{e^-} = 0.51 \text{ MeV}, p_{e^-} = 1.02 \text{ MeV}/c, \\ (p_x)_{e^-} &= 0.51 \text{ MeV}/c, (p_z)_{e^-} = 0.51 \sqrt{3} \text{ MeV}/c, E_{e^+} = 0.51 \text{ MeV}, \\ p_{e^+} &= 1.02 \text{ MeV}/c, (p_x)_{e^+} = 0.51 \text{ MeV}/c, (p_z)_{e^+} = -0.51 \sqrt{3} \text{ MeV}/c. \end{aligned}$$

Therefore

$$E = E_{e^-} + E_{e^+}; p = (p_x)_{e^-} + (p_x)_{e^+}; 0 = (p_z)_{e^-} + (p_z)_{e^+}$$

A determination of velocity, energy and momentum of these particles would provide a test of this theory. Another test is proposed elsewhere".

The theory presented here is Lorentz covariant, does not involve imaginary quantities, satisfies the symmetry principles of nature, explains the kinematics of the particles and preserves the definition of momentum and energy.

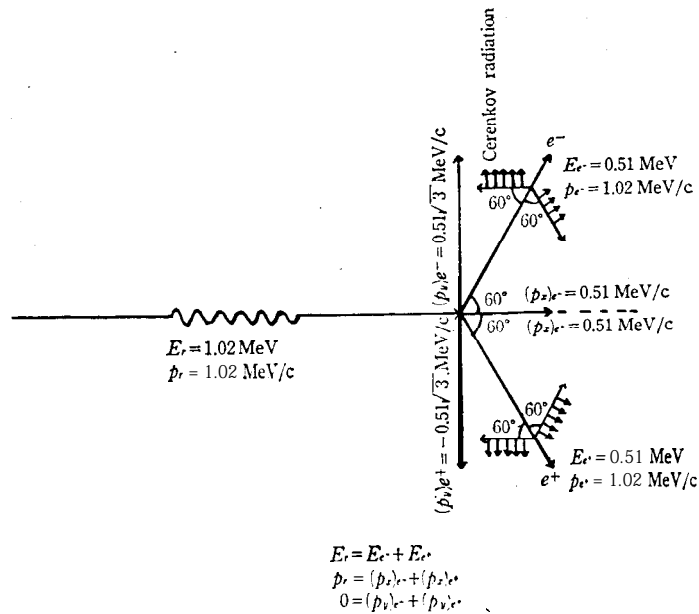


Fig. 3. A photon decaying into electron-positron pair. Both electron and positron should travel with a speed twice the speed of light.

(8) S. A. Kamal, Experimental test for the behavior of massive particles near and beyond the speed of light, Amer. Phys. Soc. Meeting, Washington, D. C., April 1982 (submitted).