Multichannel Faddeev Equations and its Application to The ^{9}Be Bound States*

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The binding energies of the ground state $3/2^-$ and an excited state $1/2^+$ of ^{9}Be are calculated on a multichannel three-body model in which the internal structure of the α particle is approximately represented by a two-state system. In this calculation the $n\alpha$ interaction is represented by a 2x2 matrix and the $\alpha\alpha$ interaction by a 3x3 matrix. Both potentials are Z-dependence and determined from the two-body scattering data at low energies.

I. INTRODUCTION

THE studies") of ^{9}Be are usually treated by the Hartree-Fock calculations. The results are not very good. The energy difference between the lowest odd parity state and the lowest even parity state obtained by this method is 4.3 MeV, which does not agree with the experimental value 1.75 MeV. Grubman and Witten[2] use Faddeev equations to calculate the ground state $3/2^-$ energy of ^{9}Be. They find an energy of 1.22 MeV compared to the experimental result 1.571 MeV. They attribute this discrepancy to the neglect of the composite nature of the α particle.

Recently, calculations of the bound states of ^{6}Li and ^{12}C have been made on the multichannel three-body model[3,4]. In such a model, the α particle involved in the a-nucleon three-body system is assumed to be a two-level particle to take care of its internal structure approximately. The $n\alpha$ and $\alpha\alpha$ two-body potentials are obtained phenomenologically multichannel analyses of the low energy scattering data. Thus the picture has effectively taken into account all the inelastic channel contributions except for breakup of the a particle. With this picture of internal structure, the $\alpha\alpha$ interaction becomes a 3x3 matrix and the a-nucleon interaction a 2×2 matrix. They are both I-dependent with each matrix element represented by a square well. The Faddeev equations are then generalized to allow spin and internal structure quantum numbers of the particles and are solved in the separable t-matrix approximation for bound state energies. The results calculated for both the ^{6}Li and ^{12}C nuclei[3,4] agree very well with the

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experimental values. It seems therefore interesting to carry out the same model calculation for ^{9}Be. In this paper we shall consider the ^{9}Be as a multichannel three-particle $(n\alpha\alpha)$ problem. The binding energies for the $3/2^-$ and $1/2^+$ states have been calculated and are found in good agreement with experiments. In Sec. II, we modify the Faddeev equations for three non-elementary particles and make a complete angular momentum reduction of the modified Faddeev equations. In Sec. III, we describe how the multichannel $n\alpha$ potential is obtained. The results of the calculation are discussed in Sec. IV.

II. MULTICHANNEL FADDEEV EQUATIONS

The formulation is essentially the same as that of ^{6}Li, which has been described in detail in Ref. 3. We shall only give a brief outline of the procedures.

We shall consider a general three-body system of particles that may have different spin or internal structure states. The three-body states in general have three parts :
the internal structure part

$$|\gamma_i\rangle_i \equiv |r_i m_{r_i};\ (r_j r_k) R_i m_{R_i}\rangle_i, \tag{1}$$

the spin part

$$|\beta_i\rangle_i \equiv |[s_i(s_j s_k) S_i] S m_S\rangle_i, \tag{2}$$

and the spatial part

$$|\mathbf{p}_i \mathbf{q}_i\rangle,$$

where the internal structure states is specified by the set of quantum numbers (γ, m_r) and we are working in a representation in which particles j and k form a subsystem with particle i left free. Thus the internal state of the subsystem is characterized by $R_i = r_j + r_k$ and $R = R_i + r_i$ is the corresponding quantum number for the three-body system. The transformation between i and j representations can be obtained by means of 3-j and 6-j symbols. Since we are going to calculate the energies of bound states with definite spin-parity J, we shall for convenience use the three-body state in the following form with obvious notation

$$|p_i q_j;\ \alpha_i;\ \gamma_i\rangle_i = |p_i[(s_j s_k) S_i l_i] J_i;\ q_i (S_i L_i) I_i;\ JM;\ \gamma_i\rangle_i, \tag{3}$$

where, as in (1) and (2), we have denoted the set of angular momentum quantum numbers collectively by the symbol α_i.

With the three-body states defined as in (3). the generalized multichannel Faddeev equations can be written in the form

$$\Psi_\alpha^{(i)}(pq\gamma, z) = \Phi_\alpha^{(i)}(pq\gamma, z) - \frac{1}{4}\sum_{j\neq i}\sum_{\alpha_j\gamma_j}\int_0^\infty dp_j^2 \int_0^\infty dq_j^2 \frac{p_j q_j}{p_j^2 + q_j^2 + \epsilon_{r_j m_{r_j}} + \epsilon_{R_j m_{R_j}} - z}$$
$$\times K^{(i,j)}(pq\alpha\gamma|p_j q_j \alpha_j \gamma_j)\Psi(p_j q_j \alpha_j \gamma_j, z), \tag{4}$$

where

$$\Psi_\alpha^{(i)}(pq\gamma, z) \equiv {}_i\langle pq\alpha\gamma|T^{(i)}(z)|n\rangle, \tag{5a}$$

$$\Phi_\alpha^{(i)}(pq\gamma, z) \equiv {}_i\langle pq\alpha\gamma|T_i(z)|n\rangle, \tag{5b}$$

$$K^{(i,j)}(pq\alpha\gamma|p_jq_j\alpha_j\gamma_j)=_i\langle pq\alpha\gamma|T_i(z)|p_jq_j\alpha_j\gamma_j\rangle_j$$
$$=\sum_{\alpha_i}{}_i\langle pq\alpha\gamma|T_i(z)|p_iq_i\alpha_i\gamma_i\rangle_i\,{}_i\langle p_iq_i\alpha_i\gamma_i|p_jq_j\alpha_j\gamma_j\rangle_j, \qquad (5c)$$

where $\varepsilon_{r_jm_{r_j}}$ and $\varepsilon_{R_jm_{R_j}}$ are the internal-state energies of the corresponding sub-systems. The angular momentum states involved in $K^{(i,j)}(pq\alpha\gamma|p_jq_j\alpha_j\gamma_j)$ can be decomposed by expressing the two-body t-matrix in three-body Hilbert space in terms of the ordinary two-body t-matrix and carrying out the angular integrations by rotating the coordinate axes from space-fixed system to body-fixed system. Then the form of the kernel reduces to:

$$K^{(i,j)}(pq\alpha\gamma|p_jq_j\alpha_j\gamma_j)=\delta_{JJ_j}\delta_{MM_j}\sum_{R_im_{R_i}}\sum_{Rm_R}\delta_{rr_i}(-1)^{3r_i+2r_j+r_k-R_j+2m_K}$$
$$\times\sqrt{(2R_i+1)(2R_j+1)}(2R+1)\begin{Bmatrix}r_i & r_k & R_j\\ r_j & R & R_i\end{Bmatrix}\begin{pmatrix}r_i & R_i & R\\ m_{r_i} & m_{R_i} & -m_R\end{pmatrix}$$
$$\times\begin{pmatrix}r_j & R_j & R\\ m_{r_j} & m_{R_j} & -m_R\end{pmatrix}\frac{4\pi^{3/2}}{q}\frac{U(L_{ij},U_{ij},p_j^2)}{\alpha_{ij}\beta_{ij}p_iq_j}\sum_{S_il_i}\sum_L(2L+1)$$
$$\times(-1)^{l_i+l_j-L_i-L_j+S_j+S_k-S_i}[(2l_j+1)(2S_j+1)(2I_j+1)(2J_j+1)(2S_i+1)$$
$$\times(2J_i+1)]^{1/2}\sum_S(-1)^{2S}(2S+1)\begin{Bmatrix}s_i & s_k & S_j\\ s_j & S & S_i\end{Bmatrix}\begin{Bmatrix}s_i & L_i & I_i\\ S_i & l_i & J_i\\ S & L & J\end{Bmatrix}$$
$$\times\begin{Bmatrix}s_j & L_j & I_j\\ S_j & l_j & J_j\\ S & L & J\end{Bmatrix}\sum_{n_{l_i}n_{L_i}}\sum_{n_{L_j}}\begin{pmatrix}l_j & L_j & L\\ 0 & n_{L_j} & -n_{L_j}\end{pmatrix}\begin{pmatrix}l_i & L_i & L\\ n_{l_i} & n_{L_i} & -n_{L_j}\end{pmatrix}$$
$$\times Y^*_{l_in_{l_i}}\left(\theta_{p_ip_j},\frac{\pi}{2}+(-1)^P\frac{\pi}{2}\right)Y^*_{L_in_{L_i}}(\theta_{q_ip_j},\pi)$$
$$\times Y_{L_jn_{L_j}}(\theta_{p_jq_j},0)\,t_i^{ll_i,SS_i,RR_i}(pp_i,z-q^2-\epsilon_{r_im_{r_i}}), \qquad (6)$$

where

$$\alpha_{ij}=\left[\frac{m_im_j}{(m_i+m_k)(m_j+m_k)}\right]^{1/2}, \qquad (7)$$
$$\beta_{ij}=(1-\alpha_{ij}^2)^{1/2}, \qquad (8)$$
$$U_{ij}=(\alpha_{ij}q_j+q)^2/\beta_{ij}^2, \qquad (9a)$$
$$L_{ij}=(\alpha_{ij}q_j-q)^2/\beta_{ij}^2, \qquad (9b)$$

and P is the cyclic permutation of the particle indices i and j. It follows from the relations

$$\mathbf{p}_i=-\alpha_{ij}\mathbf{p}_j-(-1)^P\beta_{ij}\mathbf{q}_j \qquad (10a)$$
$$\mathbf{q}_i=(-1)^P\beta_{ij}\mathbf{p}_j-\alpha_{ij}\mathbf{q}_j \qquad (10b)$$

that the azimuthal angles $\phi_{p_jq_i}=\pi$ and $\phi_{p_jp_i}=\pi/2+(-1)^P\pi/2$. The step function U in (6) is defined as

$$U(L_{ij},U_{ij},p_j^2)=\begin{cases}1, & \text{if } L_{ij}\leqslant p_j^2\leqslant V_{ij}\\ 0, & \text{otherwise}\end{cases} \tag{11}$$

and n_L and n_l are the components of the angular momentum operators along the body-fixed axis.

The Faddeev equations with kernal given by (6) represent a set of coupled integral equations with two continuous variables p_j^2 and q_j^2. It can be further reduced to a set of coupled. integral equations in one variable by means of the separable l-matrix approximation[5].

$$\chi_{n\alpha}^{(i)}(qr_im_{r_i}z)=\eta_{n\alpha}^{(i)}(qr_im_{r_i}z)+\sum_{i\neq j}\sum_{\alpha_j}\sum_{r_jm_{r_j}}\sum_{n_j}\int_0^\infty dq_j^2$$
$$\times K_{n\alpha n_j\alpha_j}^{(i,j)}(qr_im_{r_i},q_jr_jm_{r_j};z)\chi_{n_j\alpha_j}^{(i)}(q_jr_jm_{r_j}z), \tag{12}$$

where the function $\chi_{n\alpha}^{(i)}$ satisfies

$$\Psi_\alpha^{(i)}(pqrz)=\Phi_\alpha^{(i)}(pqrz)+\sum_{n_j}\frac{\lambda_{n_l}^{(i)}(z-q^2-\varepsilon_{r_im_{r_i}})\phi_{n_jl}^{(i)}(pRm_R,z-q^2-\varepsilon_{r_im_{r_i}})}{1-\lambda_{n_j}^{(i)}(z-q^2-\varepsilon_{r_im_{r_i}})}\chi_{n_i\alpha}^{(i)}(qr_im_{r_i}z), \tag{13}$$

the kernel is given by

$$K_{n\alpha,n_j\alpha_j}^{(i,j)}=\sum_{R_jm_{R_j}}\int_{L_{ij}}^{U_{ij}}dp_j^2\frac{\lambda_{n_jl_j}^{(j)}(z-q_j^2-\varepsilon_{r_jm_{r_j}})}{1-\lambda_{n_jl_i}^{(j)}(z-q_j^2-\varepsilon_{r_jm_{r_j}})}W(\alpha\alpha_jr_im_{r_i}r_jp_jq_jz)$$
$$\times\phi_{n_jl_j}^{(j)}(p_jR_jm_{R_j};z-q_j^2-\varepsilon_{r_jm_{r_j}}), \tag{14}$$

the inhomogeneous part

$$\eta_{n\alpha}^{(i)}=\sum_{j\neq i}\sum_{\alpha_jr_j}\int_0^\infty dq_j^2\int_{L_{ij}}^{U_{ij}}dp_j^2W(\alpha\alpha_jr_im_{r_i}r_jp_jq_jz)\Phi_{\alpha_j}^{(j)}(p_jq_jr_jz), \tag{15}$$

and $\phi_{nl}^{(i)}$ satifies

$$\lambda_{nl}^{(i)}(z)\phi_{nl}^{(i)}(pSRm_Rz)=\frac{1}{2}\sum_{R_i}\sum_{S_il_i}\int dp_i^2\frac{p_iU_{ll_i}^i(pSRm_R,p_iS_iR_im_{R_i})}{p_i^2-z}$$
$$\times\phi_{nl_i}^{(i)}(p_iS_iR_im_{R_i}z). \tag{16}$$

with the orthogonality condition

$$\frac{1}{2}\sum_R\sum_{lS}\int_0^\infty dp^2\frac{p\phi_{nl}^{(i)}(p,SRm_Rz)\phi_{ml}^{(i)}(pSRm_Rz)}{p^2-z}=\delta_{nm}. \tag{17}$$

The function W appearing in (14) and (15) is given by

$$W=\frac{-\pi^{3/2}}{\alpha_{ij}\beta_{ij}q}\frac{1}{p_j^2+q_j^2+\varepsilon_{r_jm_{r_j}}+\varepsilon_{R_jm_{R_j}}-z}\sum_{R_im_{R_i}}\sum_{Rm_R}(-1)^{3r_i+2r_j+r_k-R_j+2m_R}[(2R_i+1)$$
$$\times(2R_j+1)]^{1/2}(2R+1)\begin{Bmatrix}r_i & r_k & R_j\\ r_j & R & R_i\end{Bmatrix}\begin{pmatrix}r_i & R_i & R\\ m_{r_i} & m_{R_i} & m_R\end{pmatrix}\begin{pmatrix}r_i & R_j & R\\ m_r & m_{R_j} & -m_R\end{pmatrix}\sum_{S_il_iL}[(2l_j+1)$$
$$\times(2S_j+1)(2I_j+1)(2J_j+1)(2S_i+1)(2I_i+1)(2J_i+1)]^{1/2}$$
$$(2L+1)(-1)^{l_i+l_j-L_i-L_j+s_j+s_k-S_i}$$

(5) J. S. Ball and D. Y. Wong, Phys. Rev. 169, 1362(1968).

$$\times \sum_{S}(-1)^{2S}(2S+1)\begin{Bmatrix} s_i & s_k & S_j \\ s_j & S & S_i \end{Bmatrix}\begin{Bmatrix} s_i & L_i & I_i \\ S_i & l_i & J_i \\ S & L & J \end{Bmatrix}\begin{Bmatrix} s_j & L_j & I_j \\ S_j & l_j & J_j \\ S & L & J \end{Bmatrix}\sum_{n_{l_i} n_{L_i}}\sum_{n_{L_j}} \delta_{l l_i}\delta_{S S_i}$$

$$\times \begin{pmatrix} l_j & L_j & L \\ 0 & n_{L_j} & -n_{L_j} \end{pmatrix}\begin{pmatrix} l_i & L_i & L \\ n_{l_i} & n_{L_i} & -n_{L_j} \end{pmatrix} Y^{*}_{l_i n_{l_i}}\left(\theta_{p_i p_j}, \frac{\pi}{2} \cdot (-1)^{p}\frac{\pi}{2}\right) Y^{*}_{L_i n_{L_i}}(\theta_{q_i p_j}, \pi)$$

$$\times Y_{L_j n_{L_j}}(\theta_{p_j q_j}, 0)\phi^{(i)}_{n_{l_i}}(p_i R_i m_{R_i}, z-q^2-\epsilon_{r_i m_{r_i}}). \tag{18}$$

To apply these multichannel Faddeev equations to the ^{9}Be system, we have two spinless particles with internal states for each, and one structureless particle with spin 1/2. For definiteness, we label the neutron as particle 1 and the two α particles as particle 2 and 3 respectively. We shall denote the internal structure quantum numbers of particles 2 and 3 by r_j so that $r_2=r_3=r$ and $r_1=0$. The total internal structure quantum number of the subsystem$^{(2,3)}$ is denoted by R. Similarly, for the spin quantum numbers we have $S_1=S$ and $S_2=S_3=0$. Then if we make use of the relations of 3--j, 6-j and 9-j symbols, equation (6) becomes :

$$K_j^{(j)}=\frac{4\pi^{3/2}}{4}\delta_{JJ'}\delta_{MM'}\frac{U(L_{ij}, U_{ij}, p_j^2)}{\alpha_{ij}\beta_{ij}p_j q_j}\sum_{Rm_R}[i,\{\hat{t}\}]$$

$$\times[(2j+1)(2j_j+1)(2l_j+1)]^{1/2}\sum_{\mathscr{L}}(2\mathscr{L}+1)[i,(s,l,j,L,\mathscr{L},J)]$$

$$\times[j,(s,l_j,j_j,L_j,\mathscr{L},J)]\sum_{n_{l_i}}\sum_{n_{L_i} n_{L_i}}\begin{pmatrix} l_j & L_j & \mathscr{L} \\ 0 & n_{L_i} & -n_{L_j} \end{pmatrix}\begin{pmatrix} l_i & L_j & \mathscr{L} \\ n_{l_i} & n_{L_j} & -n_{L_j} \end{pmatrix}$$

$$\times Y^{l_i *}_{n_{l_i}}\left(\theta_{p_i p_j}\frac{\pi}{2}+(-1)^{p}\frac{\pi}{2}\right)Y^{L_i *}_{n_{L_i}}(\theta_{p_i p_j}, \pi)\, Y^{L_j}_{n_{L_j}}(\theta_{p_j q_j}, 0), \tag{19}$$

where we have defined

$$[i,\{\hat{t}\}]=\begin{cases} \hat{t}_i^{j, l l_i}(p, p_i, z-q^2) & \text{if } i=1 \\ (-1)^{m_R}(2R+1)\hat{t}_i^{j, l l_i}(p, p_i, z-q^2-\epsilon_{r m_r}) & \text{if } i=2,3 \end{cases} \tag{20}$$

and

$$[i,(s,l,j,L,\mathscr{L},J)]=\begin{cases} \begin{Bmatrix} s & l_1 & j \\ L_2 & J & \mathscr{L} \end{Bmatrix} & \text{if } i=1 \\ (-1)^{2s+2l_2+L_1+j+\mathscr{L}+J}\begin{Bmatrix} s & l_2 & j \\ L_1 & J & \mathscr{L} \end{Bmatrix} & \text{if } i=2,3. \end{cases} \tag{21}$$

The angular momentum states involved in this problem are much complicated than the case of ^{6}Li and ^{12}C. The different angular momentum states can be obtained as follows. The total angular momentum and parity for the ground state of ^{9}Be is $J^{\pi}=3/2^-$, and for the excited states, we consider $J=1/2^+$ only. For the two-body interactions, we include S and P waves for n-6 and S and D waves for α-α interaction. Under this consideration, the angular momentum states involved for $3/2^-$ and $1/2^+$ states of ^{9}Be are listed in Table 1 and 2.

Table 1. Angular momentum states involved in the calculation of the $3/2^-$ state of Be^9

	subsystem	l	L	J	j	J^π
1	n-α	0	1	1	1/2	$3/2^-$
		1	0	0	3/2	$3/2^-$
		1	2	2	1/2	$3/2^-$
		1	2	2	3/2	$3/2^-$
2	α-α	0	1	3/2	0	$3/2^-$
		2	1	1/2	2	$3/2^-$
		2	1	3/2	2	$3/2^-$
		2	3	5/2	2	$3/2^-$
		2	3	3/2	2	$3/2^-$

Table 2. Angular momentum states involved in the calculation of the $1/2^+$ state of Be^9

	subsystem	l	L	J	j	J^π
1	n-α	0	0	0	1/2	$1/2^+$
		1	1	1	1/2	$1/2^+$
		1	1	1	3/2	$1/2^+$
2	α-α	0	0	1/2	0	$1/2^+$
		2	2	3/2	2	$1/2^+$
		2	2	5/2	2	$1/2^+$

III. TWO-BODY POTENTIALS

To obtain the n-α potential for our purpose, we have to make a multichannel analysis of the two-body scattering problem. Since the α particle is assumed to be a two-state particle, the Schrödinger equation for the n-α system can be written as

$$\left[-\frac{\hbar^2}{2\mu}\nabla_\rho^2 + H(r,m_r) + V(\rho,r,m_r)\right]\Psi(\rho,r,m_r) = E\Psi(\rho,r,m_r), \tag{22}$$

where the internal energy operator H has the form

$$H(r,m_r) = \begin{pmatrix} \epsilon_\downarrow & 0 \\ 0 & \epsilon_\uparrow \end{pmatrix}, \tag{23}$$

and the potential energy operator V is given by

$$V(\rho,r,m_r) = \begin{pmatrix} V_1(\rho) & V_3(\rho) \\ V_3(\rho) & V_2(\rho) \end{pmatrix} = \frac{\hbar^2}{2\mu}\begin{pmatrix} v_1(\rho) & v_3(\rho) \\ v_3(\rho) & v_2(\rho) \end{pmatrix} \tag{24}$$

If we now write the total wave function as

$$\Psi(\rho,r,m_r) = \sum_l \frac{2l+1}{k_0\rho} i^l P_l(\cos\theta)\begin{pmatrix} u_{1l}(\rho) \\ u_{2l}(\rho) \end{pmatrix}, \tag{25}$$

the for incident energies below the α excitation energy, the Schrödinger equation (22) reduces to two coupled equations

$$\frac{d^2u_{1l}}{d\rho^2}+\left[k_0^2-\frac{l(l+1)}{\rho^2}-v_1\right]u_{1l}-v_3u_{2l}=0, \tag{26a}$$

$$\frac{d^2u_{2l}}{d\rho^2}-\left[k^2+\frac{l(l+1)}{\rho^2}-v_2\right]u_{2l}-v_3u_{1l}=0, \tag{26b}$$

where we have taken $\epsilon_\downarrow=0$ for convenience and defined k by

$$-k^2=k_c^2-\frac{2\mu}{\hbar^2}\epsilon_\uparrow. \tag{27}$$

The wave functions are required to satisfy the boundary conditions

$$u_{1l}(0)=u_{2l}(0)=0$$

and behave asymptotically like

$$u_{1l}\longrightarrow i^l\sin\left(k_0\rho-\frac{l\pi}{2}\right)+i^l\alpha_l\,e^{i(k_0\rho-l\pi/2)}, \tag{28a}$$

$$u_{2l}\longrightarrow\beta_l\,e^{-k\rho}. \tag{28b}$$

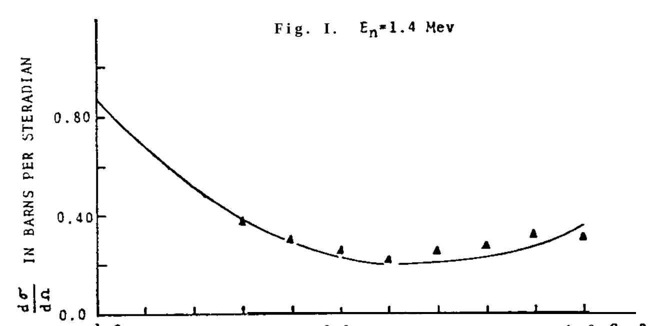
The equations (26) for square well can be solved analytically in the resonance approximation. The differential cross section for elastic scattering is given by

$$\frac{d\sigma}{d\Omega}=\left|\sum_l\frac{2l+1}{k_0}\alpha_lP_l(\cos\theta)\right|^2, \tag{29}$$

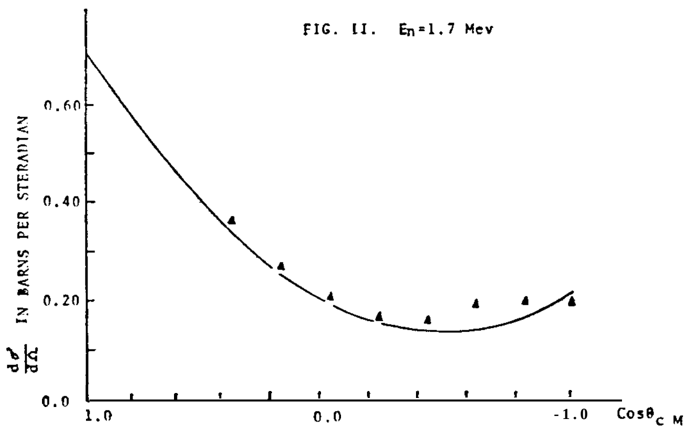
where

$$\alpha_l=\frac{1}{2i}(e^{2i\eta_l}-1)-e^{2i\eta_l}\frac{1/2\Gamma_l}{E-E_c+1/2i\Gamma_l}. \tag{30}$$

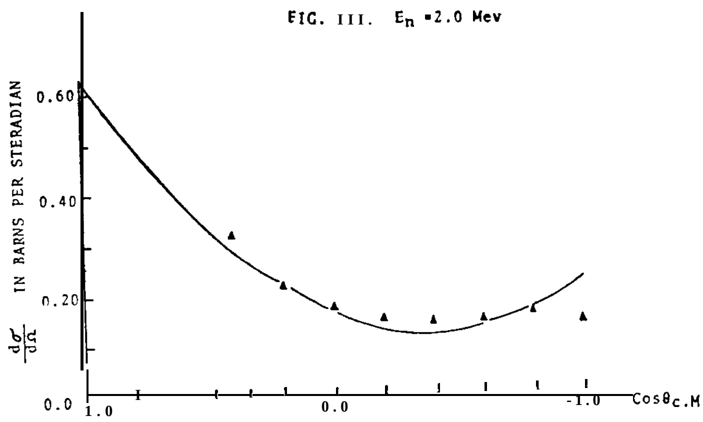
The first term in eq. (30) is due to the direct channel and the second term represents the coupled-channel contributions which is responsible for the resonance at E_c with width Γ_l. The n-α scattering differential cross sections are fitted by an l-dependent potential as shown in the figures 1-5. Two sets of parameters that fit the data equally well are obtained and are listed in Table 3. We have also used the same method to obtain the $\alpha\alpha$ potential and two sets of parameters are thus obtained as listed in the Table 4.



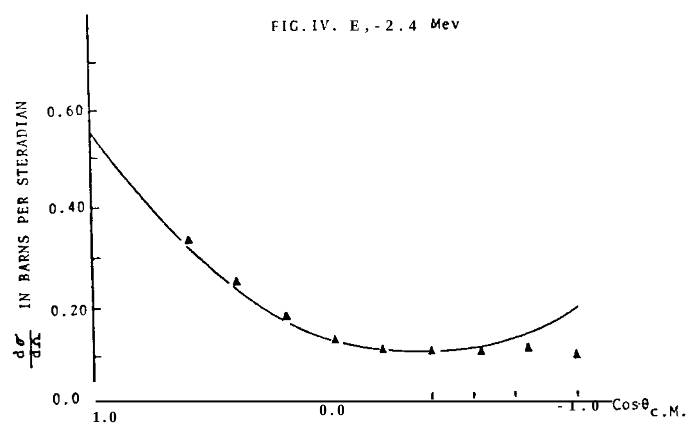
Angular distribution in the C. M. system of neutrons scattered by helium at neutron bombardment energy of 1.4 MeV. The solid curve shows the distribution calculated from Eq.(29). The solid triangle represents the experimental data.



Angular distribution in the C. M. system of neutrons scattered by helium at neutron bombardment energy of 1.7 MeV. The solid curve shows the distribution calculated from Eq. (29). The solid triangle represents the experimental data.

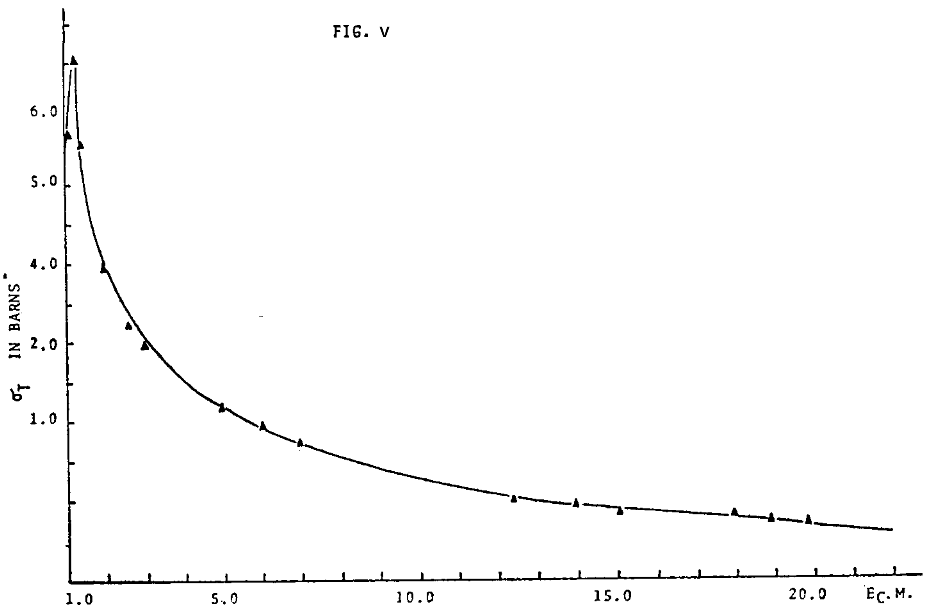


Angular distribution in the C. M. system of neutrons scattered by helium at neutron bombardment energy of 2.0 MeV. The solid curve shows the distribution calculated from Eq. (29). The solid triangle represents the experimental data,



Angular distribution in the C. hl. system of neutrons scattered by helium at neutron bombardment energy of 2.4 MeV. The solid curve shows the distribution calculated from Eq. (29). The solid triangle represents the experimental data.

FIG. V



n-α total cross section as a function of neutron bombardment energy (MeV). The solid curve shows the distribution calculated, The solid triangle represents the experimental data.

Table 3. The n-α interaction potential matrix elements

$$V_{n-\alpha}=\begin{bmatrix} V_1 & V_3 \\ V_3 & V_2 \end{bmatrix}$$

	l	V_1 (MeV)	V_2 (MeV)	V_3 (MeV)	r (fm)
Set I	0	46.815	31.14	0.205	3.5305
	1	21.507	46.364	0.325	3.4936
Set II	0	58.092	34.0	4.358	3.19
	1	21.332	19.082	0.95	3.025

Table 4. The α-α interaction potential matrix elements

$$V_{\alpha-\alpha}=\begin{bmatrix} V_1 & V_2 & V_6 \\ V_5 & V_2 & V_7 \\ V_6 & V_7 & V_3 \end{bmatrix}$$

l	V_1 (MeV)	V_2 (MeV)	V_3 (MeV)	V_5 (MeV)	V_6 (MeV)	V_7 (MeV)	r (fm)
0	23.3	23.32	24.5	8.18	0.5	2.52	4.809
2	24.9	29.5	29.41	4.98	10.2	11.5	4.64

IV. RESULTS AND DISCUSSION

As has been mentioned above, we have obtained two sets of parameters for the n-a potential and the α-α potentical which fit the two-body scattering data equally well. For the ^{9}Be nucleus, we limited the calculations to the energies of the lowest $3/2^-$ and $1/2^+$ states with the set I n-a potential and set I a-a! potential.

We find -3.60 MeV and -1.16 MeV for the $3/2^-$ state and $1/2^+$ state of ^{9}Be repsectively. The coulomb correction for $(n\alpha\alpha)$ system is 1.84 MeV, after this correction we obtain -1.76 MeV for the ground state energy of ^{9}Be and 0.68 MeV for the excited $1/2^+$ state. The corresponding experimental values are -1.57 MeV and 0.18 MeV. The only existing work that we are awere of has been made by Grubman and Witten who obtained -1.22 MeV for the ground state from a single-channel calculation. The comparation is given in Table 4.

Table 5. Binding energies of $3/2^-$ and $1/2^+$ states of Be^9

Authors	n-a interaction	α-α interaction	ground state* $(3/2)^-$ (MeV)	Excited state* $(1/2)^+$ (MeV)
Grubman-Witten	Yukawa form for S-wave Yukau-atexponential for P-wave	Darriulat potential	-1.22	
Present work Solution I	S+P-nave square well	S+D-wave square well	-1.76	0.68
Experimental			-1.57	0.18

* Actual binding energies (has been corrected by the Coulomb repulsion energy 1.84 MeV)

It should be of interest to investigate the importance of the ***effect*** due to the α internal states on the three-body binding energy. We have used a perturbation method for this purpose. Since the alpha particle excitation energy is large compared with the three-body binding energy, it is certainly much larger than the internal structure correction to the three-body binding energy. Therefore, we can treat the contribution due to the internal structure of α particle as a perturbation. It is shown that a simple calculation yields roughly 15% for ^{9}Be. We have also performed a numerical calculation *for* the ground state energy of ^{9}Be by setting all the V_s except V_1 to zero in the n-a and as-a: potentials. We find that the binding energy of ^{9}Be is reduced by about 18%. However, it should be emphasized that this does not mean that the single channel square well n-a! and α-α potentials will produce the same result since V_1 **also** does not fit the scattering data any more.

Therefore we conclude that as far as the bound state energies are concerned. the three-body model seems to be adequate to describe the nuclei ^{6}Li. ^{9}Be and ^{12}C provided that the internal states of the α particle are properly taken into account.