Radiation Squeezing for M Two-Level Atoms Interacting with a Single Mode Coherent Radiation

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The squeezing of the electromagnetic field for M two level atoms existing in the Dicke state $|r, m\rangle$ and interacting with a single mode coherent radiation has been investigated under an approximation based on a fully quantum mechanical approach. The time dependent variance has been calculated and the influence of the number of photons and some other parameters are noted.

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I. INTRODUCTION

The squeezing of a radiation field has attracted a great deal of interest in view of the possibility of the reduction in the noise of an optical signal below the vacuum limit. Earlier, squeezing was studied as an academic interest, but the low-noise property of squeezed states are of interest in connection with practical applications in optical communication and information theory \cite{1, 2}, quantum teleportation \cite{3, 4}, gravitational wave detection \cite{5}, quantum cryptography \cite{6}, etc.

A lot of work has been done on the generation of squeezed states of an electromagnetic field and their experimental detection in various processes, such as multi-wave mixing \cite{7–10}, higher order harmonic generation \cite{11–13}, parametric amplification \cite{14}, multi-photon absorption \cite{15}, degenerate hyper-Raman scattering \cite{16}, etc. The interaction of radiation with matter shows a large number of interesting effects, and different models have been given to explain this \cite{17–19}. The possibility of squeezing in the Jaynes-Cummings model (JCM) was studied in several papers \cite{20–23}. Radiation squeezing in a two atom JCM with one and multiphoton transitions has been studied by a number of authors for different initial field inputs \cite{24–27}. A number of non-classical phenomenon have been described by this model, such as vacuum field Rabi oscillation, subpoissonian photon statistics, squeezing of the radiation field, and collapse and revival phenomenon \cite{28–33}. Earlier we have studied the collapse and revival phenomenon for M two level atoms interacting with single mode coherent radiation \cite{33}.

The aim of present paper is to study radiation squeezing for M two level atoms interacting with single mode coherent radiation and also to investigate the dependence of the squeezing on the photon number and other parameters. We consider the approximation

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that the number of photons is large compared to that of the atoms.

II. DEFINITION OF RADIATION SQUEEZING OF THE ELECTROMAGNETIC FIELD

A quantum state is said to be squeezed if one quadrature of the field has reduced quantum fluctuations at the expense of increased quantum fluctuations in the other quadrature of the field. To define squeezing the most general operator $q_\theta$ can be taken as

$$q_\theta = \frac{1}{\sqrt{2}} \left[ ce^{-i\theta} + c^\dagger e^{i\theta} \right].$$

For this operator, if $\Delta q_\theta \equiv q_\theta - \langle q_\theta \rangle$, the minimum variances (minimum against variation of $\theta$) are seen to be

$$\langle (\Delta q_\theta)^2 \rangle_{\text{min}} = \frac{1}{2} + \frac{1}{2} \left( \langle c^\dagger c \rangle - |\langle c \rangle|^2 \right) - \frac{1}{2} \left| \langle c^2 \rangle - \langle c \rangle^2 \right|.$$  \hspace{1cm} (2)

where $c$ and $c^\dagger$ and $N (= c^\dagger c)$ are the annihilation, creation, and number operators for radiation, respectively.

If $\langle (\Delta q_\theta)^2 \rangle_{\text{min}} < 1/2$, $q_\theta$ is said to be ordinary squeezed (radiation). The condition for this to occur is

$$\langle c^\dagger c \rangle - |\langle c \rangle|^2 < \left| \langle c^2 \rangle - \langle c \rangle^2 \right|.$$ \hspace{1cm} (3)

III. TRILINEAR HAMILTONIAN AND THE EQUATION OF MOTIONS

For a single two-level atom or for $M$ two level atoms interacting with single mode coherent radiation, the Hamiltonian [34] is

$$H = H_0 + H_I, \quad H_0 = H_F + H_A,$$ \hspace{1cm} (4)

where

$$H_F = \omega N, \quad H_A = \omega R_3, \quad H_I = g(cR_+ + c^\dagger R_-), \quad N = c^\dagger c.$$ \hspace{1cm} (5)

Here the subscripts $F, A, I$, refer to the field, atom, and interaction, respectively. Here $g$ is a coupling constant, $\omega$ is the common atomic and radiation frequency, and $R_+$ and $R_3$ are the Dicke collective operators for the atomic assembly [34]. Operators $R_\pm$ and $R_3$ satisfy the commutation relations $[R_+, R_-] = 2R_3$ and $[R_3, R_\pm] = \pm R_\pm$.

For $M$ two level atoms, $R_\pm$ and $R_3$ can be expressed in a two–boson representation [35–37] in the form, $R_+ = a^\dagger b, R_- = ab^\dagger$, and $R_3 = \frac{1}{2} (a^\dagger a - b^\dagger b)$, with $a$ and $b$ as the annihilation operators for the boson modes corresponding to the upper and lower atomic
levels, and we can represent the Dicke state $|r, m\rangle$, defined by $R^2|r, m\rangle = r(r+1)|r, m\rangle$ and $R_3|r, m\rangle = m|r, m\rangle$, by the boson state $|n_a, n_b\rangle$ with $n_a = r + m$, $n_b = r - m$, where $R^2 = R_x^2 + R_y^2 + R_z^2$ and $R_x$, $R_y$, $R_z$ satisfy the commutation relation $[R_x, R_y] = iR_3$. Eq. (5) then takes the form,

$$H_I = g(a^\dagger b c + ab^\dagger c^\dagger).$$

If we define $S_+ \equiv a^\dagger c$, $S_- \equiv ac^\dagger$, we can write $H_I$ as

$$H_I = g(S_+ b + S_- b^\dagger)$$

We can use the Holstein-Primakoff transformation [37, 38] for the operators $S_\pm$ and write

$$S_+ \equiv a^\dagger c = d^\dagger \sqrt{N_a + N_c} = d^\dagger d = d^\dagger \sqrt{N_0 - d^\dagger d},$$

$$S_- \equiv ac^\dagger = \sqrt{N_a + N_c} = d^\dagger dd = \sqrt{N_0 - d^\dagger dd},$$

where $N_0$ is the value of $a^\dagger a + c^\dagger c$, which is constant. For a description in terms of the operators $d$, $d^\dagger$ and $c$, $c^\dagger$, the state $|n_a, n_b, n_c\rangle$ can be written as $|n_d, n_b\rangle$, because $n_d = n_a$ and $n_a + n_c = n_d + n_c$ is constant and specification of $n_d$ and $n_b$ is sufficient to tell $n_a$, $n_b$ and $n_c$, where $n_a, n_b$ are the number of atoms in the upper and lower atomic levels, $n_d$ is the number of excited atoms, and $n_c$ is the number of photons.

We now consider the approximation $N_0 \gg N_A = n_a + n_b$, i.e., $\langle n_a + n_c \rangle \gg \langle n_a + n_b \rangle$. Obviously this holds very well when the mean number of photons $\bar{n}$ is much greater than the number of atoms. Under this approximation

$$\sqrt{N_0 - d^\dagger d} \approx \sqrt{N_0}[1 - (1/2N_0)d^\dagger d]$$

and Equation (7) reduces to the lowest contributing order in perturbation:

$$H_I = G[(d^\dagger b + b^\dagger d)], \quad G = g\sqrt{N_0}.$$  

This gives the time evaluation operator in the interaction picture as $V_0 = e^{-iH_{1t}}$, in the lowest order. Since $n_a + n_c = constant$, $n_c$ need not be specified and $n_a$ (or $n_d = n_a$) will tell the value of $n_c$. In the lowest order of perturbation, where the time evaluation operator is expressed in terms of $b$, $d$, $b^\dagger$, $d^\dagger$ only, we can consider states in the $d$ and $b$ modes only and write the state at time $t$ as $|\psi(t)\rangle$, where $n$ refers to value of $n_c$ at $t = 0$. We then have $|\psi_n(0)\rangle = |r + m, r - m\rangle$ and

$$|\psi_n(t)\rangle = e^{-iH_{1t}} |\psi(0)\rangle = e^{-iGt(d^\dagger b + b^\dagger d)} |r + m, r - m\rangle$$

$$= \frac{1}{\sqrt{(r+m)!(r-m)!}}(d^\dagger \cos Gt - ib^\dagger \sin Gt)^{r+m}(b^\dagger \cos Gt - id^\dagger \sin Gt)^{r-m} |vac\rangle.$$  

(12)
The terms within the brackets can be written as

\[(d^\dagger \cos Gt - i b^\dagger \sin Gt)^{r+m} = \sum_p r+m C_p (-i b^\dagger \sin Gt)^{r+m-p}(d^\dagger \cos Gt)^p,\]

\[(b^\dagger \cos Gt - i d^\dagger \sin Gt)^{r-m} = \sum_q r-m C_q (b^\dagger \cos Gt)^{r-m-q}(-i d^\dagger \sin Gt)^q.\]

We define \(s = p + q\) and write \(q = s - p\). Eq. (12) will then be in the form

\[|\psi_n(t)\rangle = \sum_s (-i)^{r+m+s} D_{s,n} |s, 2r - s, r + m + n - s\rangle,\]  

where \(D_{s,n}\) is given by

\[D_{s,n} = \sum_p \sqrt{(r+m)!(r-m)!} \sqrt{(2r-s)!} \sqrt{(r+m+n-s+1)!} (-1)^p (\sin Gt)^{r+m+s-2p} (\cos Gt)^{r-m-s+2p}.\]  

IV. ORDINARY RADIATION SQUEEZING FOR THE INTERACTION OF M TWO-LEVEL ATOMS WITH SINGLE MODE COHERENT RADIATION

If the field is in a coherent state \(|\alpha\rangle\),

\[|\alpha\rangle = \sum_n e^{-\frac{1}{2} |\alpha|^2} \frac{\alpha^n}{\sqrt{n!}},\]

where \(\alpha = |\alpha| e^{i\psi}\) and \(\bar{n} = |\alpha|^2\) is the initial mean photon number or dimensionless intensity of the field.

For coherent radiation \(|\alpha\rangle\), the field state will be

\[|\psi(t)\rangle = \sum_n e^{-\frac{1}{2} |\alpha|^2} \frac{\alpha^n}{\sqrt{n!}} \sum_s D_{s,n} |s, 2r - s, r + m + n - s\rangle.\]  

From Equation (2) radiation squeezing is given by

\[\langle (\Delta q)^2\rangle_{\text{min}} - \frac{1}{2} = C_{11} - C_{01}^2 - |C_{02} - C_{01}|^2,\]

where \(C_{01}, C_{02}, C_{11}\) are given by

\[C_{01} = \sum_n e^{-|\alpha|^2} \frac{|\alpha|^{2n+1}}{n! \sqrt{n+1}} \sum_s \sqrt{r + m + n - s + 1} D_{s,n} D_{s,n+1},\]

\[C_{02} = \sum_n e^{-|\alpha|^2} \frac{|\alpha|^{2n+2}}{n! (n+1)!} \sum_s \sqrt{(r + m + n - s + 1)(r + m + n - s + 2)} D_{s,n} D_{s,n+2},\]

\[C_{11} = \sum_n e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!} \sum_s (r + m + n - s) D_{s,n}^2.\]
V. RESULT AND DISCUSSIONS

FIG. 1: A graph showing the variation of the variance $\langle (\Delta q)^2 \rangle_{\text{min}}$ with $gt$ for $r = 5$, $m = 5$, and $\bar{n} = 400$.

FIG. 2: A graph showing the variation of the variance $\langle (\Delta q)^2 \rangle_{\text{min}}$ with $gt$ for $r = 5$, $m = -5$, and $\bar{n} = 400$.

As an illustration of our result we present the short time behavior of squeezing (variance) for different field intensities $\bar{n}$ and for fixed $M$. Figs. 1 to 6 show the variation of
the variance with coupling time $gt$ for fixed $M$ and $\bar{n} = 400$, 225, and 64 with the atoms prepared in the different atomic states $|r, m\rangle$. In Fig. 1, we clearly see that there are oscillations with oscillation frequency at interval $gt_o = \pi/\sqrt{\bar{n} + \frac{1}{2} + m} = 0.15$ and the minimum of the variance is 0.37 or the max squeezing is 26%, which is also clear from the expression. On comparison of Fig. 1, Fig. 2, and Fig. 3, it is clear that squeezing and $gt_o$ decreases
FIG. 5: A graph showing the variation of the variance $\langle (\Delta q)^2 \rangle_{\text{min}}$ with $gt$ for $r = 5$, $m = -5$, and $\bar{n} = 225$.

FIG. 6: A graph showing the variation of the variance $\langle (\Delta q)^2 \rangle_{\text{min}}$ with $gt$ for $r = 5$, $m = 5$, and $\bar{n} = 64$.

with a decrease of $|m|$ and $-m$, and on a comparison of Figs. 1, 4, 5, and 6 it is clear that squeezing and $gt_o$ increases with a decrease of the number of photons. It is interesting to note that the number of atoms $M$ does not matter in squeezing, only $r$, $m$, and $n$ the number of photons matter. Earlier we also investigated collapses and revivals for $M$ two level atoms interacting with single mode coherent radiation. A comparison of collapses and
revivals [33] and radiation squeezing show that these two oscillate at the same phase and at the same frequency. The average value of squeezing is always positive, while the average value of $\langle R_3 \rangle$ may be positive, negative, or zero.

VI. CONCLUSIONS

In this paper we have studied the interaction of a system of two level atoms initially in a Dicke state $|r, m\rangle$ with single mode radiation, initially in an intense coherent state with $\bar{n}$ ($\bar{n} \gg r + m$) photons. Earlier ordinary squeezing was studied for two two-level atoms [39], we generalize the problem and analyze the possibility of ordinary squeezing (radiation squeezing) for M-two level atoms. We demonstrated ordinary squeezing using the Holstein-Primakoff transformation and found that if the mean photon number is large enough significant squeezing can be achieved.

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References