THz Radiation of Vortex Flux Flow in Superconductor Media at the Approximately Non-Laminar Regime

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The electromagnetic radiation due to flux flow from type II superconductors has been investigated. The effect of small non-laminar flux motion is simulated, and the radiation power is calculated and analyzed. The calculations show that for a little non-laminar effect, the harmonic frequencies do not change, but the power will be changed proportional to this effect. Therefore, the system will radiate at the harmonics of the famous washboard frequency with a widening of its frequency. Considering this unwanted effect is necessary for optimization and modeling of flux flow based devices, especially THz devices. The behavior of the obtained power is similar to the experimental results.

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I. INTRODUCTION

Superconducting radiation based on flux flow dynamics was predicted first by Fiory [1] for laminar dynamics and then observed experimentally [2]. However, the exact analysis of the radiation spectrum was proposed by Bulaevskii and Chudnovsky [3]. They investigated a system in a magnetic field that is biased with a constant current. Other groups also investigated similar phenomena based on the flux flow dynamics [4–6]. In such systems, the vortices are supposed to move constantly due to a very large viscosity of the environment. The constant velocity of the flux lines will cause radiation at special frequencies that are harmonics of the washboard frequency, \( \omega_0 = \frac{2\pi v}{a} \), in which \( a \) is the vortex lattice parameter and \( v \) is the constant velocity of the flux lines. The surface Meissner current and the back effect of the radiation on the lattice motion are usually ignored [3, 6]. It should be noted that the largest harmonics of the radiation could not exceed the gap frequency of the superconductor that is in the range of THz. Because the power of the radiation increases with frequency [3], it is obvious that the most important parts of the radiation are situated at the THz range. Manipulating the washboard frequency by changing the applied magnetic field (which will change the parameter \( a \)) and the applied current (which will change the parameter \( v \)) helps us to reach the desired radiation frequencies.

In the case of a prefect crystal lattice that is necessary for large radiation power, the damping force is never large enough to satisfy the laminar condition, so this approximation is not exact [7, 8]. Here it is assumed that the damping force is not large enough for the
laminar condition, but it is large enough to use perturbation theory. Therefore I supposed that flux lines are moving with non-constant velocity but with small nonzero acceleration. This condition is usually called non-laminar media. The same approach as [3] for radiation power calculations has been used.

II. THEORETICAL MODEL

For the calculation of the electromagnetic radiation from the superconductor slab, a geometry similar to [3, 6] is selected. The specification of the selected geometry is illustrated elsewhere [3, 6], so I want to go directly to the calculations. For small velocity rather than near the speed of light the magnetic field due to the flux lines are obtained from following equations [9]:

\[ a) \nabla \times \nabla \times B + B/\lambda^2 = \left( \Phi_0 e_z/\lambda^2 \right) \sum_n \int dz \delta \left[ r - r_n(t, z) \right] \Theta \left[ -x_n(t) \right] \Theta \left[ L_x + x_n(t) \right] , \]

\[ b) \ c \nabla \times E = -\partial B/\partial t, \]

in which \( L_x \) is the length of superconductor segment in the \( x \) direction, \( r_n \) is the position and \( x_n \) is the \( x \) component of the position of \( n \)th flux line, \( e_z \) is the \( z \) direction unit vector, \( \Theta \) is the unit step function, \( \lambda \) is the London penetration depth, \( E \) and \( B \) are the electric and magnetic fields, respectively, \( c \) is the light speed, and \( \Phi_0 \) is the magnetic flux quantum.

To derive the equation of motion of the flux lines we should solve the Newton equation, \( F = ma \), where \( F \) is the sum of the Lorentz force and all the interactions inside the superconductor and \( a \) is the acceleration. Also the parameter \( m \) is due to the mass of the vortex current that produced a flux line, so it is equal to \( N m_e \), in which \( N \) in the number of electrons inside the vortex current and \( m_e \) is the electron mass. In laminar regimes, the inside interactions act as a large damping force, and lead to a constant velocity \( F/\alpha \) in which \( \alpha \) is the damping force. For the large Lorentz forces, the forces between the flux lines could be ignored, so the position and velocity of the vortices will be:

\[ x_n(t) = an + \frac{F}{\alpha} t - \frac{m}{\alpha} \left( v_{t=0} - \frac{F}{\alpha} \right) \exp \left( -\frac{\alpha}{m} t \right) , \quad v(t) = \left( v_{t=0} - \frac{F}{\alpha} \right) \exp \left( -\frac{\alpha}{m} t \right) + \frac{F}{\alpha} . \]

\[ (2) \]

in which \( v_{t=0} \) is the initial velocity. By inserting \( v_{t=0} = 0 \) and \( F/\alpha = v \), and the damping coefficient is large, so

\[ a) x_n = an + vt - v\tau \exp \left( -\frac{t}{\tau} \right) , \quad \tau \equiv \frac{m}{\alpha} \ll \frac{a}{v} . \quad \tau \exp \left( -\frac{t}{\tau} \right) \equiv \mu(t) \ll \frac{a}{v} , \]

\[ b) x_n = an + vt + v\mu(t) , \quad y_p(z) = ap + \delta_p(z) . \]

\[ (3) \]

in which \( \tau \) is mass divided by the damping coefficient and shows the amount of deviation from laminar motion. For \( \tau = 0 \) or \( \alpha \to \infty \) the motion is totally laminar. Using the Taylor series expansion and small \( \mu \), the step function expands at first order of approximation as below:

\[ \Theta (-x_n) = \Theta (-an/v - t) - \mu (an/v) \delta (-an/v - t) . \]

\[ (4) \]
The product of the Dirac delta and step function is written down (right hand side of Equation (1)) in the wave number domain as

\[
\delta(r - r_n) \left[ \Theta \left( -\frac{an}{v} - t \right) + \mu \left( \frac{an}{v} \right) \delta \left( -\frac{an}{v} - t \right) \right] = \int \int \int \mathcal{D}k e^{-jk_y \eta_p} e^{jk_z z} \\
\times \left[ e^{-jk_r (an + vt)} \Theta \left( -\frac{an}{v} - t \right) - e^{-jk_r (an + vt)} - jk_x m \frac{\mu}{a} e^{-t/\tau} \Theta \left( -\frac{an}{v} - t \right) + \frac{\mu}{a} \delta \left( -\frac{an}{v} - t \right) \right]
\]

where \( j \) is the imaginary unit. By the substitution of this equation into Equation (1a) the magnetic field at the boundary in the frequency domain is obtained as

\[
B_{zv}(\omega, k) = \sum_n \int dz e^{jk_z z - jk_y \eta_p} \left\{ \frac{\delta_n}{1 + \lambda^2 k^2} \phi_0 \left( \frac{e^{-j \omega \eta_p / v}}{\omega - j k_x v - \epsilon} - \tau e^{-\frac{\omega \eta_p}{v} - \frac{j \omega \eta}{v}} + e^{-\frac{\omega \eta_p}{v} - \frac{j \omega \eta}{v}} \frac{k_x}{j \omega - j k_x v + 1/\tau} \right) \right\}
\]

(5)

In which \( \omega \) is the frequency of the radiated electromagnetic waves. Also it is fruitful to apply the following mathematical identities:

\[
a) \sum_{n=0}^{N} e^{-(j \omega \eta_p / v) n} = 2\pi e^{-j A} \sum_m \delta_n \left( \omega a / v - 2\pi m \right), \quad A \equiv \omega a (N - 1) / 2v,
\]

\[
b) 2\pi \sum_m \delta_N \left( \omega a / v - 2\pi m \right) \equiv \frac{\sin \left( j N \omega a / 2v \right)}{\sin \left( j \omega a / 2v \right)}, \quad \sum_{n=0}^{N} e^{-\frac{\omega \eta_p}{v} - \frac{j \omega \eta}{v}} \cong \frac{1}{1 - e^{-\frac{a \eta}{2\pi v}} e^{-\frac{j \omega \eta}{v}}}
\]

(7)

In the last relation large \( N \) and small \( \tau \) is supposed. By substituting (7) (a and b) into (6) the magnetic field is obtained as

\[
B_{zv}(\omega, k) = \frac{2\pi v}{a} \sum_p \int dz e^{jk_z z - jk_y \eta_p} \left\{ \sum_m \delta_n \left( \omega - m \omega_0 \right) \frac{k_x}{j \omega - j k_x v - \epsilon} - \frac{-\tau a}{2\pi v} e^{-\frac{\omega \eta_p}{v} - \frac{j \omega \eta}{v}} \frac{k_x}{j \omega - j k_x v - 1/\tau} \right\}
\]

(8)

\[
\omega_0 = 2\pi v / a.
\]

By a Fourier transform for the \( k_x \) component of the wave number and putting \( x = 0 \), the magnetic field at the right boundary is obtained as

\[
B_{zv}(\omega, x = 0, k_y, k_z) = \sum_p \int dz \frac{\phi_0 e^{jk_z z - jk_y \eta_p}}{a} \left[ \frac{e^{-j A} \sum_m \delta_n \left( \omega - m \omega_0 \right)}{2i \lambda \omega} + \frac{-\tau a}{2\pi v} \frac{1}{1 - e^{-\frac{\omega \eta_p}{v} - \frac{j \omega \eta}{v}}} \right].
\]

(9)
By using the magnetic field at the boundary, it is possible to calculate both the magnetic and electric field outside of the superconductor media [3]. Continuing a similar approach used in [3], and define the parameter $\zeta$ as the ratio of the electrical to the magnetic field:

$$\zeta(\omega, k_\perp) = \frac{|k_\omega| \Theta (k_\omega^2 - k_\perp^2)}{\sqrt{k_\omega^2 - k_\perp^2}} - \frac{j k_\omega \Theta (k_\perp^2 - k_\omega^2)}{\sqrt{k_\perp^2 - k_\omega^2}},$$

(10)

where the $k_\omega$ is $\omega/c$ and $k_\perp$ is $(k_x, k_y)$. The radiated power spectrum is calculated as

$$P_{rad}^r(\omega) = \frac{c}{8\pi} \int \frac{dk_\perp}{(2\pi)^2} Re \left[ \zeta^{-1}(\omega, k_\perp) \right] |B_z(\omega, 0, k_\perp)|^2 = c \frac{k_\omega}{8\pi} \int \frac{dk_\perp}{(2\pi)^2} \frac{1}{\sqrt{k_\perp^2 - k_\omega^2}} \lambda^2 k_\omega^2 |B_{zv}(\omega, 0, k_\perp)|^2.$$

(11)

The upper index $r$ denotes the right-hand radiated power. It should be noted that there is a small difference between the right-propagating and left-propagating radiation powers [3]. By neglecting this small effect, the radiation power can be written as

$$P_{rad}(\omega) = \frac{c k_\omega}{32\pi} \int \frac{dk_\perp}{\pi^2} \frac{\lambda^2 k_\omega^2 N_y L_z}{\sqrt{k_\perp^2 - k_\omega^2}} \sum_p \int \frac{dz e^{ik_y y_p(z)}}{a^2} \phi_0^2 v_0^2 \times \sum_m \delta_N(\omega/\omega_0 - m)$$

$$\left[ \sum_m \delta_N(\omega/\omega_0 - m)/(4\omega^2\lambda^2) + \frac{1}{2\lambda} \left| \frac{\tau a}{4\pi v} \lambda \right|/\left| 1 - e^{-\frac{\tau a}{\sqrt{4\pi v}}} e^{-ij\omega v} \right| \right].$$

(12)

By defining the structure function, the radiation power can be rewritten as

$$P_{rad}(\omega) = \frac{L_y L_z k_\omega^2 \phi_0^2 v_0^2}{32\pi c a^2} \int \frac{dk_\perp}{\pi^2} \frac{1}{\sqrt{k_\perp^2 - k_\omega^2}} S(k_\perp) \times \sum_m \delta_N(\omega/\omega_0 - m)$$

$$\left[ \frac{1}{4} \sum_m \delta_N(\omega/\omega_0 - m) + \frac{\tau \omega^2 a}{8\pi v} \times \left( 1 + e^{-\frac{\tau a}{\sqrt{4\pi v}}} - 2 e^{-\frac{\tau a}{\sqrt{4\pi v}}} \cos \left( \frac{a\omega}{v} \right) \right)^{-1/2} \right].$$

(13)

where the structural factor $S(k_\perp)$ is given by

$$S(k_\perp) = \sum_p \int (dz/a) \exp \left[ ik_z z - ik_y y_p(z) \right].$$

III. RESULTS AND DISCUSSION

By neglecting the variation of the denominator with respect to $k_\perp$, the integral part of the radiation power is calculated easily as

$$\int dk_\perp/(2\pi)^2 S(k_\perp) = 1/a^2.$$
The radiation spectrum of the superconductor is obtained for different limits as introduced in [3, 6]. Different cases are classified in three sections, according to the magnitude of the dimensionless transversal ($l_y$) and longitudinal ($l_z$) correlation lengths for moving vortex lattice, which are expressed in units of the intervortex spacing $a$ [3, 6, 11]. These three approximation regions are considered as weak disorder and large crystal (WDL), weak disorder and small crystal (WDS), and strong disorder (SD).

For the case of WDL, $1 \ll k_\omega l_y \ll k_\omega a l_y$, so the radiation power is obtained as

$$P_{WDL}(\omega) = \frac{L_y L_z v^2 B^2}{32\pi c} \sum_m \delta_N \left( \frac{\omega}{\omega_0} - m \right) \times \left\{ \sum_m \delta_N \left( \frac{\omega}{\omega_0} - m \right) + \frac{\tau_\omega^2}{\omega_0} \left( 1 + e^{\frac{-2a}{v\tau}} - 2e^{\frac{-a}{v\tau}} \cos \left( \frac{a\omega}{v} \right) \right)^{-1/2} \right\}.$$ (14)

For the case of WDS, we have $1 \gg k_\omega l_y \ll k_\omega a l_y$, and for SD $1 \gg k_\omega l_y \gg k_\omega a l_y$, so the radiation powers are, respectively:

$$P_{WDS}(\omega) = \frac{1}{32\pi c} \left( \frac{v B l_y L_z \omega}{c} \right)^2 \sum_m \delta_N \left( \frac{\omega}{\omega_0} - m \right) \times \left\{ \sum_m \delta_N \left( \frac{\omega}{\omega_0} - m \right) + \frac{\tau_\omega^2}{\omega_0} \left( 1 + e^{\frac{-2a}{v\tau}} - 2e^{\frac{-a}{v\tau}} \cos \left( \frac{a\omega}{v} \right) \right)^{-1/2} \right\},$$ (15)

$$P_{SD}(\omega) = \frac{L_y L_z l_y l_z}{32\pi c} \left( \frac{v B a \omega}{c} \right)^2 \sum_m \delta_N \left( \frac{\omega}{\omega_0} - m \right) \times \left\{ \sum_m \delta_N \left( \frac{\omega}{\omega_0} - m \right) + \frac{\tau_\omega^2}{\omega_0} \left( 1 + e^{\frac{-2a}{v\tau}} - 2e^{\frac{-a}{v\tau}} \cos \left( \frac{a\omega}{v} \right) \right)^{-1/2} \right\}.$$ (16)

In general the radiation power could be written in the form

$$a) P(\omega) = C(\omega) \sum_m \delta_N \left( \frac{\omega}{\omega_0} - m \right) \left[ \sum_m \delta_N \left( \frac{\omega}{\omega_0} - m \right) + f(\omega) \right],$$

$$b) g(\omega) \equiv \sum_m \delta_N \left( \frac{\omega}{\omega_0} - m \right) + f(\omega),$$

$$c) f(\omega) \equiv \frac{\tau_\omega^2}{\omega_0} \left( 1 + e^{\frac{-2a}{v\tau}} - 2e^{\frac{-a}{v\tau}} \cos \left( \frac{a\omega}{v} \right) \right)^{-1/2},$$ (17)

where $C(\omega)$ is the function of frequency that appeared in Equations (14)–(16) for different approximations. If $\tau = 0$ the $f(\omega)$ is zero, that corresponds to the unperturbed problem. For nonzero $\tau$, $f(\omega)$ has an infinite relative maximum exactly at the frequencies $n\omega_0$. This result shows that the resonance frequencies of the radiation do not change in the perturbation process.
In numerical study, for simplification the dimensionless parameter $\tau$ is introduced by multiplying it by $\omega_0$. For other parameters, the typical values are as $B \sim 1\,\text{T}$ so $a \sim 45.5\,\text{nm}$ and $v \sim 455\,\text{m/s}$ is chosen, therefore $\omega_0$ is approximately $0.063\,\text{THz}$. Here, the first 10 harmonics of radiation from a superconductor has been considered. The superconductor length is 100 times more than $a$, $L_x \sim 4.55\,\mu\text{m}$, that is in range of mesa structures that are fabricated and tested experimentally [10, 12–14]. The other dimensions ($y$ and $z$) are very much larger than the $x$ dimension. By choosing these values, the perturbation is valid for a very much smaller amount of $\tau$ rather than 1.

It is necessary to mention that there is an important constraint on the upper limit of the frequency due to the nature of the superconductivity. Because the Cooper pairs inside the superconductor cannot withstand the energies larger than its gap energy, so this analysis is valid only for frequencies below the gap frequency of the superconductor [3, 9]. The largest amount of the energy gap for the bismuth strontium calcium copper oxide (BSSCO) superconductor for optimum doping at ($T = 0\,\text{K}$) is about $50\,\text{meV}$ [10] equivalent to $12\,\text{THz}$. For the higher temperatures the gap decreases quickly. Therefore for frequencies below $1\,\text{THz}$, there is no disturbance due to the gap energy and the relations are valid.

Fig. 1 shows the behavior of $g(\omega)$ for different values of normalized $\tau$ as a function of frequency ($\omega$). For small magnitude $\tau$ the relative maximums (broad peaks) of $f(\omega)$ are short and by increasing $\tau$ they increase quickly, as is obvious in Fig. 1. The sharp peaks on top of the $f(\omega)$ maximums are due to the Dirac delta functions. This figure shows perfectly that the two peaks are exactly on top of each other. It should be noted that the results for $\tau = 0.6$ have larger error, because it is near to the upper range of perturbation.

![FIG. 1: $g(\omega)$ versus angular frequency.](image)

The total power for $\tau = 0.2$ is depicted in Fig. 2. Fig. 2a shows the total radiated power versus angular frequency for WDL and Fig. 2b shows the total radiated power for both the WDS and SD approximations. These diagrams reveal that the power will be
increased by increasing the frequency, especially for WDS and SD, so the most part of the radiation belongs to larger frequencies below the gap frequency.

Fig. 2: Total power for $\tau = 0.2$: a) WDL, and b) for both WDS and SD.

Fig. 3 shows the 10th harmonic of the radiated power for the unperturbed state in comparison with 2 perturbed states $\tau = 0.1$ and $\tau = 0.2$. Although the perturbation coefficient ($\tau$) is small, it has an important effect on the total radiated power, so that for $\tau = 0.1$ ($\tau = 0.2$) the peak of the power is 1.61 (2.24) times greater than for the unperturbed state. Furthermore, the tails of the curve for the non-laminar regime are different from the laminar regime and are more similar to the experimental data [10].

Fig. 3: Comparison of the total power at the 10th harmonic for the perturbed states and the unperturbed state
IV. SUMMARY AND CONCLUSION

The non-laminar effects of a superconductor media on flux flow radiation have been investigated and calculated. The computations show that the fields and thereupon the radiation power inside the superconductor for non-laminar media are different from laminar media. The results reveal that the small non-laminar effect did not change the frequency of the harmonics of the radiation but added a new term with wider peaks to it, exactly at the same frequency as the laminar harmonics.

The results for the total power declare that the most important parts of power are the highest frequencies below the gap frequency that are unchanged due to the gap, and by increasing the parameter of the non-laminar effect ($\tau$), the total power increases quickly, but the radiation peaks spread. These results are in comparison with the experimental data [10].

References