The THz Instability in a Two Dimensional Quantum Gated Electron Gas with Scattering of Carriers

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Plasma wave generation is a result of the wave amplification due to the reflection from the device boundaries, and typical plasma frequencies lie in the terahertz (THz) range in nanometer field effect transistors (FETs). In this paper, the influence of the scattering of carriers on the instability of plasma waves in nanometer FETs is reported with quantum effects. The quantum effects enhance this instability considerably, as the imaginary part of the wave increases with quantum effects, but the external friction associated with electron scattering reduces the instability. Accordingly, the quantum effects can be used to compensate or overcome the stabilizing role of the friction due to the scattering of carriers on plasma wave generation. These properties could make the nanometer FETs advantageous for the design of terahertz plasmonic sources.

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I. INTRODUCTION

During the past decade, the study of THz devices and their applications has attracted extensive attention of researchers in various areas, such as homeland security, biomedical imaging, radio astronomy, industrial controls, and short range covert and space communications [1–3]. Recently, Dyakonov and Shur proposed a new THz device utilizing the plasma resonance effect of highly dense two-dimensional conduction electrons in the FET channel [4–8]. The asymmetric boundary conditions with a fixed voltage at the source and fixed current at the drain lead to an instability of the steady state with a dc current. This instability, providing a realizable new mechanism for plasma wave excitation, was interpreted to result from a plasma wave amplification due to the wave re-flection from the device boundaries. The nonlinear properties of the FET channel may be used for detection and frequency mixing in the THz domain [9]. Continuous shrinking of the channel length of a FET has led to the development of important quantum effects, such as quantum statistical pressures for electrons, electron tunneling, and many electron exchange correlation [10, 11]. Quantum effects may alter device performance in several ways, for example, quantum confinement keeps charge away from the boundaries of the channel. Quantum-mechanical effects must be accounted, due to the relevance of quantum confinement on the distribution of the channel charge. Thus, understanding of the collective dynamics of a

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quantum electron gas is of great importance for the design of a new generation of micro-electronic devices, including metallic nanometer structures, thin metal films, semiconductor quantum wells, and quantum dots. For nanometer transistors, the radiation frequencies of the lowest fundamental plasmas mode is in the THz range, and the emission peak can be tuned by the gate parameters, such as the gate bias and gate length.

In the presence of external friction associated with electron scattering by impurities and/or phonons, carriers in the FET channel will be affected, which influences the motion of carriers. The electron scattering will be important for the instability of current in the FET channel. A theoretical investigation was made of the plasma-wave instability mechanism in a two-dimensional electron fluid in a ballistic FET in the presence of electron scattering [12]. However, the influence of the electron scattering on the plasma wave excitation and THz radiation in a nanometer FET with quantum effects are still unclear.

Therefore, in this article, based on a quantum hydrodynamic model (QHM), the influence of the quantum effects and the electron scattering on the current instability in a nanometer FET with asymmetrical boundary conditions are studied both analytically and numerically. The numerical results show that the quantum effects enhance the radiation power, but the external friction associated with electron scattering reduces the radiation power. Furthermore, the quantum effects enhance the radiation frequencies, but the external friction has an insignificant effect on the radiation frequencies. To our best knowledge, no attempts to observe this instability have been reported. The obtained results allow us to determine device parameters required for the experimental observation of the instability.

II. MODEL

Electron-electron scattering rates can be large compared to other frequency scales of interest for sufficiently high electron concentrations, while collisions with phonons and/or impurities during the transit time may be weak and can be neglected for sufficiently short channels. In this limit, the electrons behave as a fluid moving in the channel without external friction, and the system can be described by a frictionless hydrodynamic model. Moving away from this limit, some quantitative accuracy may be lost, but these approximations should still support qualitative analysis of the quantum effects of interest here. We use a self-consistent QHM that was originally derived [13]–[14] starting from the Wigner–Poisson system and has been extensively used in the study of quantum plasmas and metallic nanostructures:

\[
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = -\frac{e}{m} \frac{\partial U}{\partial x} + \frac{\hbar^2}{2m^2} \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{U}} \frac{\partial^2 \sqrt{U}}{\partial x^2} \right) - \frac{v}{\tau},
\]  

(1)

\[
\frac{\partial U}{\partial t} + \frac{\partial (Uv)}{\partial x} = 0,
\]  

(2)

where \(v\) is the average electron flux velocity, \(U(x,t) = U_{gc}(x,t) - U_T\), \(U_{gc}\) is the local gate-to-channel voltage, \(U_T\) is the threshold voltage, and \(e\) and \(m\) are the electron charge and the
effective mass, respectively. Eq. (1) is the Euler equation. The second term on the right-hand of the Eq. (1) is the so-called Bohm potential, which contains all the quantum effects that are present in the model. The external friction associated with electron scattering by impurities and/or phonons give an additional term \( v/\tau \) in the right-hand side of Eq. (1), where \( \tau \) is the momentum relaxation time. Eq. (2) is, in fact, the continuity equation since the electron density in the channel, \( n \), is related to the voltage swing, \( U \), by the relation [15]

\[
en = CU,
\]

where \( C \) is the gate to channel capacitance per unit area. This equation holds if the scale of the variation of the potential in the channel is large compared to the gate-to-channel separation (the graduate channel approximation).

At small currents the steady electron flow with a constant \( U_0 \) and \( v_0 \) is shown to be unstable against small perturbations when the following boundary conditions are fulfilled [4]:

\[
\begin{align*}
  u_1(x = 0) &= 0, \\
  U(L, t)v(L, t) &= U_0v_0,
\end{align*}
\]

where \( L \) is the distance between the source and drain contacts. These boundary conditions can be realized by grounding the source either directly or via a very large capacitance presenting a short at plasma wave frequencies, and by attaching the drain to the power supply via an inductance that presents an open circuit at plasma wave frequencies.

### III. DISPERSION RELATION

To explore the stability of this steady state, we put \( U = U_0 + u_1 \), \( v = v_0 + v_1 \), with \( u_1, v_1 \sim \exp(-i\omega t + ikx) \) and linearize Eqs. (1)–(2) with respect to \( u_1 \) and \( v_1 \). Then we find

\[
(\omega - kv_0 + iv)v_1 = \left( \frac{\hbar^2}{4m^2U_0}k^3 + \frac{e}{m}k \right) u_1,
\]

\[
(\omega - kv_0)u_1 - kU_0v_1 = 0.
\]

Now, using Eqs. (5) and (6) and eliminating \( u_1 \) and \( v_1 \), the dispersion equation is obtained:

\[
\frac{\hbar^2}{4m^2}k^4 + \left( \frac{eU_0}{m} - v_0^2 \right) k^2 + (2\omega + iv)v_0k - \omega^2 - iv\omega = 0.
\]

It can been seen from Eq. (7) that the wave number \( k \) has four roots. However, numerical results indicate that \( k \) has only two real roots corresponding to waves propagating downstream and upstream. We set them as \( k_1 \) and \( k_2 \), respectively. The general solution for \( u_1 \) and \( v_1 \) can be found using Eq. (5):

\[
u_1 = A \exp(ik_1x) + B \exp(ik_2x),\]
\[ v_1 = \frac{k_1 A}{\omega - k_1 v_0} \exp(ik_1 x) + \frac{k_2 B}{\omega - k_2 v_0} \exp(ik_2 x), \]  

where \( A \) and \( B \) are constants to be determined from the boundary conditions. From Eqs. (4), (8) and (9), we obtain

\[ \exp[i(k_1 - k_2)L] = \frac{k_1}{k_2}. \]  

Eq. (10) allows us to find both the real and imaginary parts of the complex frequency \( \omega = \omega' + i\omega'' \), and the sign of the imaginary part will determine the stability of the steady state. A positive imaginary part \( \omega'' > 0 \) corresponds to instability.

We normalize the electron flux velocity \( v \) to the velocity of the plasma waves \( s = \sqrt{eU_0/m} \), the length to \( L \), the time and the momentum relaxation time to \( L/s \), the frequency to \( s/L \). We then obtain the dimensionless dispersion relation:

\[ HK^4 + (1 - \beta^2)K^2 + (2\Omega + i\gamma)\beta K - \Omega^2 - i\gamma\Omega = 0, \]  

and Eq. (10) becomes

\[ \exp[i(K_1 - K_2)] = \frac{K_1}{K_2}, \]  

where \( \Omega \) and \( K \) are the dimensionless frequency and wave number, \( H = \hbar^2/4m^2s^2L^2 \) represents quantum effects depending on the dimension of the device. \( \beta = v_0/s \) is the dimensionless electron flux velocity, \( \gamma = L/s\tau \) represents the external friction associated with electron scattering by impurities and/or phonons.

When the size of the device is a little larger, the quantum effects due to size may be neglected. That is, in this case, we put \( H \sim 0 \) in Eq. (11). Eq. (11) reduces to \( (1 - \beta^2)K^2 + (2\Omega + i\gamma)\beta K - \Omega^2 - i\gamma\Omega = 0 \). For given \( \Omega \), we find two values corresponding to oblique waves propagating downstream and upstream: \( K_{1,2} = [\pm q \pm \sqrt{(q^2 - 4pr)}]/2p \), with \( q = (2\Omega + i\gamma)\beta \), \( p = (1 - \beta^2) \), and \( r = -\Omega^2 - i\gamma\Omega \). In the absence of external friction, the results of Ref. [4] for \( \omega_r \) and \( \omega_i \) are \( \omega_r = (s^2 - v_0^2)\pi n/2Ls \) and \( \omega_i = (s^2 - v_0^2)\ln |(s + v_0)/(s - v_0)|/2Ls \). Note that for \( \beta \ll 1 \), the mode-independent instability increment is \( \omega_i = v_0/L \), as reported in Ref. [7].

IV. NUMERICAL EXAMPLES

For the general case, both the instability increment corresponding to the radiation power and the radiation frequencies should depend on the quantum effects and the external friction associated with electron scattering by impurities and/or phonons. Typical parameters are used in this paper: the effective electron mass \( m = 0.067m_e \), the effective dielectric permeability \( \epsilon = 13\epsilon_0 \) (where \( \epsilon_0 \) is the vacuum dielectric constant), the equilibrium density \( U_0 \sim 0.5 \text{ V} \), the length of the channel \( L \sim 10-100 \text{ nm} \). Using these values, we obtain the radiation frequencies \( \sim 10 \text{ THz} \), which indicates that radiation with a THz frequency
is generated. In order to analyze the essential features of the plasma wave instability, we numerically solve Eq. (11) and (12). Fig. 1 shows the dependence of the instability increment on the dimensionless electron flux velocity $\beta$ for different $H$ and $\gamma$. The steady flow is called unstable if $\Omega'' > 0$, i.e., the wave grows. From Fig. 1, one can see that the external friction associated with electron scattering shortens the instability range of $\beta$ and reduces the instability increment, but the quantum effects broaden the instability range of $\beta$ and enhance the instability increment. Furthermore, it can be seen from Fig. 1 that for low Mach number $\beta$ the quantum effects have insignificant influence on the instability increment, but for high Mach number $\beta$ the quantum effects have significant influence on the instability increment.

In Fig. 2, we show the instability increment and the radiation frequencies as a function of the external friction associated with electron scattering by impurities and/or phonons ($\gamma$) for different $H$. As shown in Fig. 2, the radiation frequencies increase with the quantum effects, but monotonically decrease with the external friction. It is clear from Fig. 2 that the external friction dramatically influences the instability increment, but influences the instability increment insignificantly.
Numerical results demonstrate that for a nanometer FET with asymmetrical source and drain boundary conditions, the current-carrying state becomes unstable in the transistor channel, and a relatively low drain current should induce plasmas oscillations in the THz frequency range. Compared with the traditional electron gas, quantum effects broaden the instability range of $\beta$ and enhanced frequencies that might correspond to the plasma oscillations excited in the transistor channel of a nanometer FET. Much higher frequencies result in a much higher-quality factor, $\omega_\tau$. The quantum effects are associated with the size of the device, i.e., when the size of the device becomes smaller and smaller the quantum effects become more and more important. So when the size of device decreases the radiation frequencies and the radiation power of the THz plasmas oscillations increase. We have demonstrated that while the role of external friction is subtractive, the quantum effects plays an additive role. By adjusting the quantum effects associated with the size of the device, the effect of external friction can be compensated or even overcome.

V. CONCLUSION

In summary, based on the quantum hydrodynamic model, the current instability and resulting plasma waves generation owing to the wave reflection from the device boundaries in a two dimensional quantum electron gas realizable in a short FET channel with external friction are studied. we found that the quantum effects enhanced the radiation power and radiation frequencies of the plasmas oscillations in the THz frequency range, but the external friction reduced the radiation power. These properties could make the nanometer FET advantageous for the realization of practical THz oscillations. The obtained results allow us to determine the device parameters required for the experimental observation of the instability.
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