A Study of Nuclear Dissipation Effects on Different Features of the Fission Dynamics of \( ^{188}\text{Pt} \) Produced in Heavy-Ion Induced Reactions

H. Eslamizadeh

Physics Department, Persian Gulf University 75169, Bushehr, Iran
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A stochastic approach that treats fission dynamics on the basis of one-dimensional Langevin equations is used to investigate the effect of the nuclear dissipation on the pre-scission neutron multiplicity and fission probability for the compound nucleus \( ^{188}\text{Pt} \) in an intermediate range of excitation energies. A modified wall and window dissipation with a reduction coefficient, \( k_s \), has been applied in the Langevin equations. It is shown that by using values of \( k_s \) in the range \( 0.29 \leq k_s \leq 0.52 \) the results of the calculations are in good agreement with the experimental data.

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I. INTRODUCTION

The mechanisms responsible for the dissipation of collective energy in nuclei are still an open problem in nuclear physics. At present there are several models for dissipation, but they give dependencies which are very different from each other. For example, the linear response theory [1, 2] predicts that dissipation increases with temperature, whereas the model of two body dissipation [3] predicts a decrease of dissipation with temperature as \( T^{-2} \). On the other hand, there are certain indications that the nuclear dissipation is deformation dependent [4]. In paper [4] the authors have assumed that the nuclear dissipation coefficient is constant up to the saddle point, and that it would sharply increase between the saddle and scission points, while many authors for the analysis of different aspects of the nuclear fission assumed a constant nuclear dissipation [5–8]. Furthermore, many authors have used one-body dissipation in order to describe the pre-scission neutron multiplicity data (see, for example, [9]).

The dynamics of nuclear fission can be simulated in terms of the Fokker-Planck equation or the Langevin equations. It should be stressed that the Fokker-Planck equation is a partial differential equation, and it can be solved only by using cumbersome procedures and by invoking various assumptions, but the Langevin equations can be solved on the basis of conventional numerical methods without recourse to additional assumptions.

In this paper we want to reproduce simultaneously the experimental data on the pre-scission neutron multiplicity and fission probability for the compound nucleus \( ^{188}\text{Pt} \) formed in heavy-ion induced fusion reactions \( ^{19}\text{F}+^{169}\text{Tm} \) in an intermediate range of excitation energies.
energies. For this purpose we consider a modified wall and window dissipation with a reduction coefficient [10, 11] in the Langevin equations. It should be stressed that in our calculation we want to consider the magnitude of the reduction coefficient as a free parameter.

The present paper has been arranged as follows. In Sec. II we describe the model and basic equations. The results of the calculations are presented in Sec. III. Finally, concluding remarks are given in Sec. IV.

II. DETAILS OF THE MODEL

In the present investigation, we use the Langevin equations [12] to simulate the dynamics of fission $^{188}$Pt. In order to specify the shape collective coordinates for a dynamical description of nuclear fission, we use the shape parameters $c, h, \alpha$, as suggested by Brack et al. [13]. However, for simplicity we used only the elongation parameter $c$, while the parameters $h$ and $\alpha$ are assumed to be zero. Therefore, in terms of the one-dimensional potential $V(c)$, the coupled Langevin equations can be considered as

$$\frac{dc}{dt} = \frac{p}{m(c)},$$

$$\frac{dp}{dt} = -\frac{p^2}{2} \frac{\partial}{\partial c} \left( \frac{1}{m} \right) - \frac{\partial F}{\partial c} - \eta \dot{c} + R(t).$$

(1)

The shape-dependent collective inertia and the friction coefficient in the above equations are denoted by $m$ and $\eta$, respectively. $R(t)$ is a random force with the properties $\langle R(t) \rangle = 0$ and $\langle R(t)R(t') \rangle = 2\eta T \delta(t-t')$, and $F$ is the free energy of the system,

$$F(c, T) = V(c) - a(c)T^2;$$

(2)

here $T$ is the temperature of the system and $a(c)$ is the level density parameter. The coordinate dependent level density parameter can be considered as

$$a(c) = a_vA + a_sA^{2/3}B_s(c),$$

(3)

where $A$ is the mass number of the compound nucleus and $B_s$ is the dimensionless functional of the surface energy in the liquid drop model. The values of the parameters $a_v = 0.073$ MeV$^{-1}$ and $a_s = 0.095$ MeV$^{-1}$ in Eq. (3) are taken from the work of Ignatyuk et al. [14].

The surface of a nucleus of mass number $A$ with elongation $c$ can be considered as

$$\rho^2(z) = \left( 1 - \frac{z^2}{c_0^2} \right) \left( a_0 c_0^2 + b_0 z^2 \right);$$

(4)

here $c_0 = cR$ and $R = 1.16A^{1/3}$. The parameters $a_0$ and $b_0$ can be expressed as

$$a_0 = \frac{1}{c^3} - \frac{b_0}{5},$$

$$b_0 = \frac{c - 1}{2}.$$  

(5)
For the friction coefficient, we use the modified wall and window dissipation formula, as given in Refs. [10, 11]:

\[
\eta = \frac{1}{2} \rho_m \bar{v} \left\{ \left( \frac{\partial r}{\partial c} \right)^2 \Delta \sigma + k_s \pi \left[ \int_{z_{\text{min}}}^{z_{N}} \left( \frac{\partial \rho^2}{\partial c} + \frac{\partial \rho^2}{\partial z} \frac{\partial D_1}{\partial c} \right)^2 \left( \rho^2 + \left( \frac{1}{2} \frac{\partial \rho^2}{\partial z} \right)^2 \right)^{-1/2} dz \right. \\
+ \left. \int_{z_{N}}^{z_{\text{max}}} \left( \frac{\partial \rho^2}{\partial c} + \frac{\partial \rho^2}{\partial z} \frac{\partial D_2}{\partial c} \right)^2 \left( \rho^2 + \left( \frac{1}{2} \frac{\partial \rho^2}{\partial z} \right)^2 \right)^{-1/2} dz \right] \right\},
\]

(6)

where \( \bar{v} \) is the average nucleon speed inside the nucleus, \( \rho_m \) is the mass density of the nucleus, \( r \) is the distance between the centers of masses of the future fission fragments, \( \rho^2 \) is the surface of the nucleus, \( \Delta \sigma \) is the area of the window between the two parts of the system, \( z_{\text{min}} \) and \( z_{\text{max}} \) are the two extreme ends of the nuclear shape along the \( z \)-axis, \( z_N \) is the position of the neck plane, and \( D_1, D_2 \) are the positions of the centers of mass of the two parts of the fissioning system relative to the center of mass of the whole system.

The potential energy \( V(A, Z, L, c) \) is obtained from the modified liquid drop model (MLDM) [15],

\[
V(A, Z, L, c) = S'(c) E_s^0(Z, A) + 0.7053 C(c) \frac{Z^2}{A^{1/3}} + \frac{L(L + 1) \hbar^2}{2(I(c) \frac{2}{5} M R_0^2 + 4 M a^2)}.
\]

(7)

The MLDM [15] is a fast method for obtaining the corrected Sierk’s finite range model (FRM) potential energy surfaces [16]. In Eq. (7) \( E_s^0 \) is the surface energy of the spherical system and \( C(c) \) is the Coulomb energy of a sharp-surfaced nucleus in units of the Coulomb energy of the sharp-surfaced spherical system. \( I(c) \) is the moment of inertia determined assuming a sharp-surfaced nucleus in units of the spherical value, and \( M \) is the total mass of the nucleus. In our calculation, we take \( S'(c), C(c), I(c) \) according to Ref. [15]. It should be mentioned that the FRM uses \( a = 0.7 \) fm in Eq. (7) but the MLDM uses \( a = 0.6 \) fm, it was found that a significantly better reproduction of the angular momentum dependence of the FRM fission barriers is obtained if one uses \( a = 0.6 \) fm. This change from \( a = 0.7 \) fm to \( a = 0.6 \) fm produces a few hundred keV increase in the high angular momentum fission barriers.

In calculation, we should specify the entrance channel through which a compound nucleus is formed. Assuming complete fusion of the projectile with the target, the angular momentum distribution of the compound nucleus can be described by the formula

\[
\frac{d\sigma(L)}{dL} = \frac{2\pi}{k^2} \frac{2L + 1}{1 + \exp \left( \frac{L - L_c}{\delta L} \right)},
\]

(8)

where \( \delta L \) is the diffuseness and \( L_c \) is the critical angular momentum. The parameters \( \delta L \) and \( L_c \) can be approximated by the relations presented in Ref. [17]. The initial angular momentum of the compound nucleus can be obtained by sampling the above angular momentum distribution function. Figure 1 shows the calculation results for the partial cross sections as a function of angular momentum for \(^{188}\)Pt. It can be seen from Fig. 1 that at
higher center-of-mass energy a compound nucleus formed with a larger value of angular momentum.

The collective inertia, $m$, is obtained by assuming an incompressible irrotational flow and making the Werner-Wheeler approximation [18].

The evaporation of light pre-fission particles is simulated according to the following scheme. We calculate the decay widths for emission $n, p, \alpha, \gamma$ at each Langevin time step $\Delta t$. The emission of a particle is allowed by asking at each time step $\Delta t$ along the trajectory whether the ratio of the Langevin time step $\Delta t$ to the particle decay time $\tau_{\text{part}}$ is larger than a random number $\xi$,

$$\Delta t/\tau_{\text{part}} > \xi \quad (0 \leq \xi \leq 1),$$

where $\tau_{\text{part}} = h/\Gamma_{\text{tot}}$ and $\Gamma_{\text{tot}} = \sum_{\nu} \Gamma_{\nu}$ with ($\nu = n, p, \alpha, \gamma$). If this is the case a particle is emitted, and we ask for the kind of the particle $\nu$ by a Monte Carlo selection with the weights to be equal to $\Gamma_{\nu}/\Gamma_{\text{tot}}$. After the emission act of a particle of kind $\nu$, the kinetic energy $\varepsilon_{\nu}$ of the emitted particle can be obtained by a hit and miss Monte Carlo procedure. Then the angular momentum and intrinsic excitation energy in the Langevin equation are recalculated and the dynamics is continued. The loss of angular momentum is taken into account by assuming that a neutron carries a way $1h$, a proton $1h$, an $\alpha$-particle $2h$, and a $\gamma$ quanta $1h$.

The particle emission width of a particle of kind $\nu$ can be determined as in Ref. [19]:

$$\Gamma_{\nu} = (2s_{\nu} + 1) \frac{m_{\nu}}{\pi^{2}h^{2} \rho_{c}(E_{\text{int.}})} \times \int_{0}^{E_{\text{int}} - B_{\nu}} d\varepsilon_{\nu} \rho R(E_{\text{int}} - B_{\nu} - \varepsilon_{\nu}) \varepsilon_{\nu} \sigma_{\text{inv}}(\varepsilon_{\nu});$$

FIG. 1: The partial cross sections as a function of angular momentum for $^{188}\text{Pt}$. 
here \( s_{\nu} \) is the spin of the emitted particle \( \nu \), \( m_{\nu} \) is its reduced mass with respect to the residual nucleus, \( \varepsilon \) is the energy of the emitted particle, \( E_{\text{int}} \) is the intrinsic excitation energy of the parent nucleus, and \( B_{\nu} \) is liquid drop binding energies of the emitted particle \( \nu \). The intrinsic energy can be given by \( E_{\text{int}} = E^* - V(c) \), where \( E^* \) is the total excitation energy of the system and \( V(c) \) is the potential energy. Also \( \rho_c(E_{\text{int}}) \) and \( \rho_R(E_{\text{int}} - B_{\nu} - \varepsilon_{\nu}) \) are the level densities of the compound and residual nuclei.

The inverse cross sections can be written as
\[
\sigma_{\text{inv}}(\varepsilon_{\nu}) = \begin{cases} \pi R_{\nu}^2 (1 - V_{\nu}/\varepsilon_{\nu}) & \text{for } \varepsilon_{\nu} > V_{\nu}, \\ 0 & \text{for } \varepsilon_{\nu} < V_{\nu}, \end{cases}
\] (11)
with
\[
R_{\nu} = 1.21 \left[ (A - A_{\nu})^{1/3} + A_{\nu}^{1/3} \right] + \left( 3.4/\varepsilon_{\nu}^{1/2} \right) \delta_{\nu,n},
\] (12)
where \( A_{\nu} \) is the mass number of the emitted particle \( v = n, p, \alpha \), and \( \delta_{\nu,n} \) is the Kronecker delta function. The barriers for the charged particles are
\[
V_{\nu} = [(Z - Z_{\nu})Z_{\nu}K_{\nu}]/(R_{\nu} + 1.6),
\] (13)
with \( K_{\nu} = 1.32 \) for an \( \alpha \) particle and 1.15 for a proton.

The \( \gamma \)-ray decay width at each time step is calculated as in Ref. [20],
\[
\Gamma_{\gamma} \approx \frac{3}{\rho_c(E_{\text{int}})} \int_{0}^{E_{\text{int}}} d\varepsilon \rho_c(E_{\text{int}} - \varepsilon) f(\varepsilon); \] (14)
here \( \varepsilon \) is the energy of the emitted \( \gamma \)-ray and \( f(\varepsilon) \) is calculated by
\[
f(\varepsilon) = \frac{4}{3\pi \hbar c} \frac{e^2}{mc^2} \frac{N}{A} \frac{\Gamma_G \varepsilon^4}{(\Gamma_G \varepsilon)^2 + (\varepsilon^2 - E_G^2)^2},
\] (15)
with \( E_G = 80A^{-1/3} \), \( \Gamma_G = 5 \) MeV, and \( k = 0.75 \) [21]; \( E_G \) and \( \Gamma_G \) are the position and width of the giant dipole resonance, respectively.

III. RESULTS AND DISCUSSION

A stochastic approach that treats fission dynamics on the basis of one-dimensional Langevin equations was used to calculate the average pre-scission neutron multiplicities, the fission probabilities for the compound nucleus \(^{188}\text{Pt}\). Figure 2 shows the calculation results for the pre-scission neutron multiplicity with different values of the reduction coefficient for \(^{188}\text{Pt}\) isotopes. In order to obtain further insight into the dynamics of fission, we also calculate the pre-saddle and post-saddle (saddle to scission) contributions of the pre-scission neutron multiplicity for \(^{188}\text{Pt}\). Figure 3 shows the results of the pre-saddle and post-saddle contributions of the pre-scission neutron multiplicity for \(^{188}\text{Pt}\) calculated with \( k_s = 0.29 \) and \( k_s = 0.52 \).
FIG. 2: Experimental data and results of the calculations of pre-scission neutron multiplicity for $^{188}\text{Pt}$ calculated with different values of $k_s$. The experimental data (filled circles) are taken from Refs. [22, 23].

It is clear from Fig. 2 that at lower excitation energies the experimental data and the values of the pre-scission neutron multiplicity calculated with different values of the reduction coefficient are very close together, but at higher excitation energies the experimental data can be reproduced by considering values of $k_s$ in the range $0.29 \leq k_s \leq 0.52$. This can be explained as follows, at lower excitation energy the height of the fission barrier for a compound nucleus is large, because at low excitation energy the compound nucleus is
formed with a lower value of angular momentum (see Fig. 4), and so the fission widths are very smaller than the neutron widths. Therefore, if we use different values of the reduction coefficient the neutrons have enough time to be emitted before fission. On the other hand a compound nucleus at higher excitation energy is formed with a larger value of angular momentum. Thus the fission barrier height will be reduced, and therefore the fission widths are comparable to the neutron widths, so in this case the value of $k_s$ is a very important parameter for reproducing the pre-scission neutron multiplicity.

The experimental data and the calculated results for the fission probability of $^{188}\text{Pt}$ are shown in Fig. 5. It can be seen from Fig. 5 that at lower excitation energies the experimental data can be reproduced by considering $k_s$ in the range $0.29 \leq k_s \leq 0.52$. But, at higher excitation energies the fission probability calculated with the different values of $k_s$ are very close and in agreement with the experimental data. This can be understood as follows, at lower excitation energy a compound nucleus is formed with a lower value of angular momentum, but with increasing excitation energy a compound nucleus is formed with a higher value of angular momentum, and then the fission barrier height is decreased (see Fig. 6). Therefore, the value of $k_s$ is not very important in calculating the fission probability.

It should be mentioned that the MLDM [15] or the FRM [16] uses many degrees of freedom when searching for the saddle-point configurations. In Fig. 6 we compare the results of calculations for the fission barrier height of $^{188}\text{Pt}$ on the basis of the MLDM and the FRM with the results of calculations on the basis of the LDM [25], which does not use many degrees of freedom. From Fig. 6 the importance of considering many degrees of freedom in the calculation of the potential energy surfaces can be seen.

**FIG. 4:** Potential energy surfaces at $L = 0, 35, 45, 55, 65\hbar$. $R_0$ is the radius of the spherical nucleus.
FIG. 5: Experimental data and results of the calculations of fission probability for $^{188}$Pt calculated with different values of $k_s$. The experimental data (filled circles) are taken from Ref. [24].

FIG. 6: Fission barrier height of $^{188}$Pt as a function of angular momentum $L$ on the basis of the MLDM, FRM, and the LDM.

IV. CONCLUSIONS

A stochastic approach that treats fission dynamics on the basis of one-dimensional Langevin equations was used to calculate the average pre-scission neutron multiplicities and
fission probabilities for the compound nucleus $^{188}$Pt in an intermediate range of excitation energies. In the Langevin equations, we have used a modified wall and window dissipation with a reduction coefficient and assumed the magnitude of the reduction coefficient as a free parameter. It was shown that the pre-scission neutron multiplicity and fission probability can be reproduced with $k_s$ in the range $0.29 \leq k_s \leq 0.52$ for the compound nucleus $^{188}$Pt. It should be mentioned that our result for $k_s$ is consistent with the other research [26]. The authors in Ref. [26] have performed a systematic study of many different systems and showed that for reproducing simultaneously the measured neutron multiplicities and the variance of the fission fragment mass-energy distribution, the reduced coefficient of the contribution from a wall formula has to be decreased at least by half of the one body dissipation strength ($0.25 \leq k_s \leq 0.5$).

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References