Two-Dimensional Spatiotemporal Soliton Dynamics in the Inhomogeneous Kerr Media

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The generalized (2+1)-dimensional nonlinear Schrödinger equation with variable coefficients can be used to describe the optical soliton dynamics and interaction in an inhomogeneous nonlinear Kerr media. Via the Hirota method and symbolic computation, analytic bright one- and two-soliton solutions for such an equation are obtained under restrictive conditions. Based on the one-soliton solutions, soliton dynamics with different choices of the group velocity dispersion coefficient $\sigma(z)$ and Kerr effect parameter $K(z)$, with $z$ as the coordinate along the propagation direction of the carrier wave, is discussed. The soliton will propagate periodically when $\sigma(z)$ and $K(z)$ are periodic functions, or stably when $\sigma(z)$ and $K(z)$ are Gaussian functions. Through a graphic and asymptotic analysis on the two-soliton solutions, several cases of the interactions between the two solitons are illustrated with the results that the total energy of the solitons is conserved, and also the two-soliton patterns remain unchanged before and after the interactions.

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I. INTRODUCTION

Optical solitons are localized electromagnetic waves that propagate stably without any decay in nonlinear media [1, 2]. Temporal solitons in single-mode optical fibres are one type of optical solitons theoretically predicted [3] and experimentally observed [4]. Due to the group-velocity dispersion (GVD), light pulses spread in time as they propagate in an optical medium, and each Fourier component of the pulse has a different velocity [5]. In particular, the normal GVD would reinforce the temporal separation between the low- and high-frequency components of the pulse, contributing to its rapid spread [2]. However, anomalous GVD, which also occurs in some materials, may compensate for the nonlinearity-induced spreading [2]. In contrast, nonlinear effects generally accelerate the disintegration of a wavepacket [5]. With the magnitudes of dispersion and intensity matched, the balance between GVD and nonlinear effects may give rise to robust pulses, which are referred to as temporal solitons [3].

Spatial solitons, on the other hand, originate from a balance of self-focusing effect and diffraction [2]. In this case, a laser beam experiences a nonlinear contribution to the index of refraction which follows the spatial (rather than temporal) profile of the intensity.
and thus acts as a lens [6]. In one transverse dimension, the diffraction formally resembles the anomalous dispersion in the temporal domain. Therefore the intensity-dependent lens can exactly compensate for the diffraction, and the resulting beam may propagate without spreading or self-compression [6]. Such a beam has the form of a stripe in a planar waveguide [7].

Spatiotemporal solitons (STSs) result from the simultaneous balance of diffraction and dispersion by the medium’s nonlinearity, and they are localized in all the spatial transverse dimensions, as well as in time [2]. STSs have been said to be the promising objects for both fundamental and applied research in ultrafast all-optical processing devices [8]. In contrast to the extensive developments in the studies of temporal and spatial solitons in one and two dimensions, experimental progress toward the production of STSs in the three-dimensional case has been slow [3]. To date, neither three-dimensional (3D) STSs in a bulk medium nor their two-dimensional (2D) counterparts in a planar waveguide have been experimentally observed [3, 8]. The experimental finding has only been reported in the form of stable quasi-2D solitons in 3D crystals with quadratic nonlinearity, and pulses of 120 femtosecond duration at 800 nm have been converted into a narrow stripe by means of cylindrical optics [9]. Solitons in the Kerr-type self-focusing media are governed by the cubic nonlinear Schrödinger (NLS) equation, and they are known to be unstable in 2D and 3D homogeneous media, because of the possibility of wave collapse [10, 11]. Some schemes to avoid collapse had been proposed, such as the use of weaker saturable or quadratic nonlinearities, and the application of nonlinearity and/or GVD management [12, 13]. It had been predicted that a 2D spatially cylindrical soliton can be stabilized in a bulk layered medium, with the opposite signs of the Kerr coefficient in adjacent layers, corresponding to self-focusing and self-defocusing, respectively [14]. A similar mechanism has been predicted [15] which can support stable 2D solitons in the temporal domain in Bose-Einstein condensates, where the sign in front of the cubic nonlinear term is subject to a periodic sinusoidal modulation in time via the Feshbach resonance. As to quadratic media, stable STSs can be predicted in a medium of “tandem” structures which are composed of periodically-alternating linear-dispersive and nonlinear layers [16].

Within the framework of the approach based on the slowly varying amplitudes of the electromagnetic field and paraxial approximation for the diffraction, evolution of the local-field amplitude \( u \) in a dispersive multidimensional nonlinear medium is governed by the generalized NLS equation as follows [17]:

\[
\begin{align*}
  iu_z + (1/2)(\nabla^2_\perp u + \sigma u_{\tau\tau}) + f(|u|^2)u = 0,
\end{align*}
\]  

(1)

where \( z \) is the coordinate along the propagation direction of the carrier wave, \( \tau = t - z/V_{gr} \) is the “reduced time”, with \( t \) being the time, \( V_{gr} \) being the group velocity of the carrier wave and the reduced time having the same meaning as that in fibre optics [18], \( \sigma \) is the GVD coefficient, which depends on the wavelength of the carrier wave, \( \nabla^2_\perp \) is the Laplacian acting on the transverse coordinates \( (x, y) \), and the function \( f(|u|^2) \) describes a nonlinear correction to the refractive index of the medium [2]. The normal Kerr effect corresponds to \( f(|u|^2) = K|u|^2 \), where \( K \) is a constant [2]. We denote the cases with a self-focusing and self-defocusing cubic nonlinearity, respectively, as \( K > 0 \) and \( K < 0 \) [3]. The cases with
\( \sigma > 0 \) and \( \sigma < 0 \) in Eq. (1) are referred to as anomalous and normal GVD [3].

In the 2D case, \( \nabla^2_\perp \) in Eq. (1) is replaced by \( (\partial^2/\partial x^2) \) [19]. At the same time, with the GVD coefficient and Kerr effect parameter varying along the propagation direction \( z \) in the Kerr media, people consider Eq. (1) as [8, 12, 14, 19]

\[
iu_z + \left( \frac{1}{2} \right) \left[ u_{xx} + \sigma(z) u_{\tau\tau} \right] + K(z) |u|^2 u = 0.
\]

which is composed of alternating self-focusing and self-defocusing layers, corresponding to \( K(z) \) periodically jumping between positive and negative values and supporting stable (2+1)D spatial solitons [14]. The dispersion management (in the multidimensional medium) corresponds to \( \sigma(z) \), periodically varying between positive and negative values [8, 19].

Eq. (2) with \( K(z) = 1 \), which means that the medium has a cubic self-focusing nonlinearity and dispersion management, has been studied [8] via a variational approximation and a stability region for the 2D STS was identified. Eq. (2) has been investigated [12] through a variational approximation and numerical simulation, and it was shown that the stable quasi-stationary (2+1)-dimensional solitons exist in a layered structure where the Kerr nonlinearity alternates between self-focusing and self-defocusing.

In this paper, we will study Eq. (2) analytically and discuss the soliton dynamics and interaction in media with different types of dispersion management and Kerr nonlinearity. In Section II, we will investigate Eq. (2) via the Hirota method [20] and deduce its bilinear forms. With the help of symbolic computation [21, 22], bright one- and two-soliton solutions for Eq. (2) will be obtained under certain restrictive conditions. In Section III, based on those solutions and the different choices of the GVD coefficient and Kerr effect parameter, the dynamics of the one soliton and soliton interaction will be studied graphically. Section IV will be our conclusions.

II. BILINEAR FORMS AND SOLITON SOLUTIONS

II-1. Bilinear forms

Introducing the dependent variable transformation,

\[
u = \frac{g}{f},
\]

where \( g \) is a complex differentiable function with respect to \( x, z \) and \( \tau \), and \( f \) is a real one, we derive the bilinear forms for Eq. (2) as follows:

\[
\left\{ i \frac{D_z}{} + \left( \frac{1}{2} \right) \left[ D_x^2 + \sigma(z) D_{\tau\tau}^2 \right] \right\} g \cdot f = 0,
\]

\[
\left[ D_x^2 + \sigma(z) D_{\tau\tau}^2 \right] f \cdot f - 2K(z)gg^* = 0,
\]

(3)
where \( \ast \) denotes the complex conjugate, while \( D_x, D_z, \) and \( D_\tau \) are the Hirota operators defined by [20]

\[
D_x^j D_z^j D_\tau^j a(x, z, \tau) \cdot b(x, z, \tau) = \frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)^j_1 \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial z'} \right)^j_2 \left( \frac{\partial}{\partial \tau} - \frac{\partial}{\partial \tau'} \right)^j_3 a(x, z, \tau) b(x', z', \tau')|_{x'=x, z'=z, \tau'=-\tau},
\]

where \( a(x, z, \tau) \) and \( b(x, z, \tau) \) are differentiable functions, \( x' \), \( z' \), and \( \tau' \) are the formal variables, \( j_1, j_2 \), and \( j_3 \) are nonnegative integers.

II-2. Soliton solutions

Based on the bilinear forms (3), we can derive the soliton solutions by expanding \( g \) and \( f \) as

\[
g = \varepsilon g_1 + \varepsilon^3 g_3 + \varepsilon^5 g_5 + \cdots, \\
f = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 + \varepsilon^6 f_6 + \cdots,
\]

where \( \varepsilon \) is a formal parameter, \( g_j (j = 1, 3, 5, \cdots) \) are the complex functions to be determined, and \( f_h (h = 2, 4, 6, \cdots) \) are the real ones.

To obtain the one-soliton solutions, we truncate expression (5) as \( g = \varepsilon g_1 \) and \( f = 1 + \varepsilon^2 f_2 \), and substitute them into the bilinear forms (3). Then we get the one-soliton solutions as

\[
u = \frac{g_1}{1 + f_2},
\]

where

\[
g_1 = a_1 \exp \theta_1, \quad f_2 = b_1 \exp (\theta_1 + \theta_1^*) , \quad b_1 = \frac{K(z) a_1 a_1^*}{(k_1 + k_1^*)^2 + \sigma(z)(\omega_1 + \omega_1^*)^2}, \\
\theta_1 = k_1 x + \omega_1 \tau + \frac{i}{2} \int [k_1^2 + \sigma(z) \omega_1^2] \, dz,
\]

with \( a_1, k_1, \) and \( \omega_1 \) as complex constants, \( k_1 \) as the wave number, and \( \omega_1 \) as the frequency. Then, substituting Eq. (6) into Eq. (2), we obtain the following restriction between \( \sigma(z) \) and \( K(z) \):

\[-(k_1 + k_1^*)^2 \frac{\partial K(z)}{\partial z} + (\omega_1 + \omega_1^*)^2 \left[ -\sigma(z) \frac{\partial K(z)}{\partial z} + K(z) \frac{\partial \sigma(z)}{\partial z} \right] = 0,\]

which can be solved out as \( \sigma(z) = c_1 K(z) - \frac{(k_1 + k_1^*)^2}{(\omega_1 + \omega_1^*)^2} \), where \( c_1 \) is a non-zero integer.

Truncating expression (5) as \( g = \varepsilon g_1 + \varepsilon^3 g_3 \) and \( f = 1 + \varepsilon^2 f_2 + \varepsilon^4 f_4 \), we can obtain the two-soliton solutions as

\[
\varphi = \frac{g_1 + g_3}{1 + f_2 + f_4},
\]
where

\[ g_1 = a_1 \exp \theta_1 + a_2 \exp \theta_2, \quad g_3 = \chi_1 e^{\theta_1+\theta_2+\theta_1^*} + \chi_2 e^{\theta_1+\theta_2+\theta_2^*}, \]

\[ f_2 = b_{11} \exp (\theta_1 + \theta_1^*) + b_{12} \exp (\theta_1 + \theta_2^*) + b_{21} \exp (\theta_2 + \theta_1^*) + b_{22} \exp (\theta_2 + \theta_2^*), \]

\[ f_4 = \Omega \exp (\theta_1 + \theta_2 + \theta_1^* + \theta_2^*), \]

\[ \theta_p = k_p x + \omega_p \tau + \frac{i}{2} \int \left[ k_p^2 + \sigma(z) \omega_p^2 \right] dz, \]

\[ b_{pq} = \frac{K(z)^{a_p} a_q^*}{(k_p + k_q^*)^2 + \sigma(z)(\omega_p + \omega_q)^2}, \quad (p, q = 1, 2), \]

\[ \chi_1 = -\frac{a_1 b_{21} \left[ (k_1 - k_1^* - k_2) \left( (k_1 - k_1^* - k_2) \right)^2 + (\omega_1 - \omega_1^* - \omega_2) \left( \sigma(z) - 2i(m_1 - m_2 - m_1^*) \right) \right]}{(k_1 + k_1^* + k_2)^2 + (\omega_1 + \omega_1^* + \omega_2) \left( \sigma(z) - 2i(m_1 + m_2 + m_1^*) \right)} \]

\[ -\frac{a_2 b_{11} \left[ (k_1 + k_1^* - k_2)^2 + (\omega_1 + \omega_1^* - \omega_2) \left( \sigma(z) - 2i(m_1 - m_2 + m_1^*) \right) \right]}{(k_1 + k_1^* + k_2)^2 + (\omega_1 + \omega_1^* + \omega_2) \left( \sigma(z) - 2i(m_1 + m_2 + m_1^*) \right)}, \]

\[ \chi_2 = -\frac{a_2 b_{12} \left[ (k_1 - k_2 + k_2^*) \left( (k_1 - k_2 + k_2^*) \right)^2 + (\omega_1 - \omega_2 + \omega_2^*) \left( \sigma(z) - 2i(m_1 - m_2 + m_2^*) \right) \right]}{(k_1 + k_2 + k_2^*)^2 + (\omega_1 + \omega_2 + \omega_2^*) \left( \sigma(z) - 2i(m_1 + m_2 + m_2^*) \right)} \]

\[ -\frac{a_1 b_{22} \left[ (k_1 - k_2 - k_2^*) \left( (k_1 - k_2 - k_2^*) \right)^2 + (\omega_1 - \omega_2 - \omega_2^*) \left( \sigma(z) + 2i(m_1 - m_2 + m_2^*) \right) \right]}{(k_1 + k_2 + k_2^*)^2 + (\omega_1 + \omega_2 + \omega_2^*) \left( \sigma(z) + 2i(m_1 + m_2 + m_2^*) \right)}, \]

\[ m_1 = \frac{i}{2} [k_1^2 + \omega_1^2 \sigma(z)], \quad m_2 = \frac{i}{2} [k_2^2 + \omega_2^2 \sigma(z)], \]

\[ \Omega = \frac{-b_{11} b_{22} \left[ (k_1 + k_1^* - k_2 - k_2^*) \left( (k_1 + k_1^* - k_2 - k_2^*) \right)^2 + (\omega_1 + \omega_1^* - \omega_2 - \omega_2^*) \left( \sigma(z) \right) \right]}{(k_1 + k_1^* + k_2 + k_2^*)^2 + (\omega_1 + \omega_1^* + \omega_2 + \omega_2^*) \left( \sigma(z) \right)} \]

\[ -\frac{b_{12} b_{21} \left[ (k_1 - k_1^* - k_2 + k_2^*) \left( (k_1 - k_1^* - k_2 + k_2^*) \right)^2 + (\omega_1 - \omega_1^* - \omega_2 + \omega_2^*) \left( \sigma(z) \right) \right]}{(k_1 + k_1^* + k_2 + k_2^*)^2 + (\omega_1 + \omega_1^* + \omega_2 + \omega_2^*) \left( \sigma(z) \right)} \]

\[ K(z)(a_1 \chi_1^* + a_2 \chi_1 + a_1^* \chi_2 + a_2^* \chi_1), \]

with \( a_1, a_2, k_1, k_2, \omega_1, \) and \( \omega_2 \) as complex constants, \( k_j \) and \( \omega_j \) \((j = 1, 2)\) satisfying the conditions

\[ \frac{k_1 + k_1^*}{\omega_1 + \omega_1^*} = \frac{k_2 + k_2^*}{\omega_2 + \omega_2^*} = \frac{(k_1 - k_1^*) - (k_2 - k_2^*)}{(\omega_1 - \omega_1^*) - (\omega_2 - \omega_2^*)}. \]

\[ (8) \]

\( k_1 \) and \( k_2 \) are the wave numbers, with \( \omega_1 \) and \( \omega_2 \) being the frequencies. Then, substituting Eq. (7) into Eq. (2), we obtain the following restriction between \( \sigma(z) \) and \( K(z) \):

\[ \frac{\partial K(z)}{\partial z} \left[ \frac{(k_1 + k_1^*)(k_2 + k_2^*)}{(\omega_1 + \omega_1^*)(\omega_2 + \omega_2^*)} + \sigma(z) \right] - K(z) \frac{\partial \sigma(z)}{\partial z} = 0, \]

which can be solved out as \( \sigma(z) = c_2 K(z) - \frac{(k_1 + k_1^*)(k_2 + k_2^*)}{(\omega_1 + \omega_1^*)(\omega_2 + \omega_2^*)} \), where \( c_2 \) is a non-zero integer.
III. DYNAMICS OF THE SOLITONS AND SOLITON INTERACTIONS

We will investigate the dynamics of the solitons and soliton interactions for Eq. (2). According to solutions (6), the one-soliton solutions can be rewritten as

\[ u = \frac{a_1}{2\sqrt{b_1}} \text{sech}\left(\frac{1}{2} \ln b_1\right) \exp i \theta_1, \]

which

\[ \theta_1 = \theta_1R + i \theta_1I, \quad k_1 = k_1R + i k_1I, \quad \omega_1 = \omega_1R + i \omega_1I, \]

\[ \theta_1R = (k_1Rx + \omega_1R\tau) - \int [k_1Rk_1I + \omega_1R\omega_1I\sigma(z)] \, dz, \]

\[ \theta_1I = (k_1Ix + \omega_1I\tau) + \frac{1}{2} \int [k_1^2R - k_1^2I + (\omega_1^2R - \omega_1^2I)\sigma(z)] \, dz, \]

where \( R \) and \( I \) represent the real part and the imaginary part, respectively. From the solutions (9), the soliton amplitude can be derived as

\[ A = \sqrt{\frac{k_1^2R + \omega_1^2I\sigma(z)}{K(z)}} = \sqrt{c_1\omega_1^2R}. \]

In Fig. 1 and 2, \( \sigma(z) \) and \( K(z) \) are both chosen as cosine functions, which indicate that the GVD and Kerr effects vary periodically along the \( z \) direction and jump between positive and negative values. In Fig. 1, we can see that the solitons in (a) and (b) both follow a periodic evolution pattern along the \( z \) direction. According to a comparison between Fig. 1(b) and 1(a), the soliton propagates along the negative direction of the \( x \) axis as \( \tau \) increases. Meanwhile, the amplitude does not change along the \( z \) direction. In order to investigate the effects of \( k_1I \) and \( \omega_1I \) on the soliton propagation, we choose two different cases, and the corresponding phenomena can be seen in Fig. 2. Fig. 2(a) shows that the extent of the changes in direction increases with an increase of \( \omega_1I \). Fig. 2(b) illustrates that the direction of the soliton propagation has changed. Periods of the soliton propagation in the two cases remain unchanged despite the changes of \( \omega_1I \) and \( k_1I \). Therefore, we can choose different \( \omega_1I \) and \( k_1I \) values to change the features of soliton dynamics. Different from Fig. 1 and 2, the soliton’s propagation patterns in Fig. 3 can be explained with different choices of \( \sigma(z) \) and \( K(z) \), whose profiles are taken as the Gaussian type, and the soliton propagates stably with the amplitude remaining the same.

On the basis of solutions (7), we will investigate the soliton interaction via the asymptotic analysis. More on the soliton interaction can be seen, e.g., in Refs. [23].

(1) Before the interaction \((z \to -\infty)\),

(a) \( \theta_1 + \theta_1^* \sim 0 \), \( \theta_2 + \theta_2^* \to -\infty \):

\[ u^{1-} \to \frac{a_1 \exp \theta_1}{1 + b_{11} \exp (\theta_1 + \theta_1^*)} = A_1 \text{sech} \left( \frac{\theta_1 + \theta_1^* + \ln b_{11}}{2} \right), \]

\[ (11) \]
\( \sigma(z) = 2 \cos z - 1, \; K(z) = \cos z, \; k_{1R} = 1, \; k_{1I} = 1, \; \omega_{1R} = 1, \; \omega_{1I} = 1, \) and \( a_1 = 1. \)

\( \text{FIG. 1: One 2D soliton via solutions (9) with } \sigma(z) = 2 \cos z - 1, \; K(z) = \cos z, \; k_{1R} = 1, \; k_{1I} = 1, \; \omega_{1R} = 1, \; \omega_{1I} = 1, \) and \( a_1 = 1. \)

\( \text{FIG. 2: One 2D soliton via solutions (9) with } \sigma(z) = 2 \cos z - 1, \; K(z) = \cos z, \; k_{1R} = 1, \; \omega_{1R} = 1, \; a_1 = 1, \; \tau = -8, \) (a) \( k_{1I} = 1 \) and \( \omega_{1I} = 1.5, \) (b) \( k_{1I} = 0.5 \) and \( \omega_{1I} = 1. \)

\( \theta_2 + \theta_2^* \sim 0, \; \theta_1 + \theta_1^* \rightarrow +\infty: \]

\[ u^2^- \rightarrow \frac{\chi_1 \exp \theta_2}{b_{11} + \Omega \exp (\theta_2 + \theta_2^*)} = A_2 \text{sech} \left( \frac{\theta_2 + \theta_2^* + \ln \frac{\Omega}{b_{11}}}{2} \right), \]  

where \( u^j^- (j = 1, 2) \) denote the asymptotic expressions for the two solitons before the
interaction and \( A_1 = \frac{a_1}{2 \sqrt{b_{11}}} \exp(\frac{\theta_1 - \theta_1^*}{2}) \), \( A_2 = \frac{\chi_1}{2 \sqrt{b_{11}}} \exp(\frac{\theta_2 - \theta_2^*}{2}) \).

(2) After the interaction \((z \to \infty)\),

(a) \( \theta_1 + \theta_1^* \sim 0 \), \( \theta_2 + \theta_2^* \to +\infty \):

\[
u^1_+ \to \frac{x_2 \exp(\frac{\theta_1}{b_{22} + \Omega \exp(\theta_1 + \theta_1^*)})}{b_{22} + \Omega \exp(\theta_1 + \theta_1^*)} = B_1 \text{sech} \left( \frac{\theta_1 + \theta_1^* + \ln \frac{\Omega}{b_{22}}}{2} \right),
\]

(13)

(b) \( \theta_2 + \theta_2^* \sim 0 \), \( \theta_1 + \theta_1^* \to -\infty \):

\[
u^2_+ \to \frac{a_1 \exp(\frac{\theta_2}{1 + b_{22} \exp(\theta_2 + \theta_2^*)})}{1 + b_{22} \exp(\theta_2 + \theta_2^*)} = B_2 \text{sech} \left( \frac{\theta_2 + \theta_2^* + \ln b_{22}}{2} \right),
\]

(14)

where \( \nu^j_+ (j = 1, 2) \) denote the asymptotic expressions for the two solitons after the interaction, and \( B_1 = \frac{x_2}{2 \sqrt{b_{11}}} \exp(\frac{\theta_1 - \theta_1^*}{2}) \), \( B_2 = \frac{a_1}{2 \sqrt{b_{11}}} \exp(\frac{\theta_2 - \theta_2^*}{2}) \). Amplitudes of the two solitons before and after the interaction all depend on the forms of the group velocity dispersion coefficient \( \sigma(z) \) and Kerr effect parameter \( K(z) \). According to the above expressions, it can be proven that the total energy of the solitons is independent of \( \sigma(z) \) or \( K(z) \) and conserved before and after the interaction under condition (8), that is,

\[ |A_1|^2 + |A_2|^2 = |B_1|^2 + |B_2|^2. \]

(15)

Therefore, the interaction between the solitons is elastic, and there is only a phase shift of the soliton position.

Fig. 4 shows the interactions of the two solitons with different values of \( \sigma(z) \) and \( K(z) \). Fig. 4(a) shows the elastic interaction between the two solitons when \( \sigma(z) \) and \( K(z) \)
are chosen as constants, which indicates the standard NLS equation. When \( \sigma(z) \) and \( K(z) \) are cosine-type functions, the two solitons both propagate periodically along the \( z \) direction, and the periodic variations of the soliton amplitudes do not change after the interaction, as seen in Fig. 4(b). In Fig. 4(c), when \( \sigma(z) \) and \( K(z) \) are linearly-decreasing functions, the interaction between the two solitons occurs in the decreasing-dispersion media, and the two solitons both have a phase shift. Fig. 4(d) displays the interaction with the dispersion profile taken as a Gaussian function \( \sigma(z) = \exp[-\ln(18)z^2] \), where the amplitudes and shapes of the two solitons remain unchanged before and after the interaction. When we change the values of \( k_{1I} \) and \( k_{2I} \), we can find that the two solitons both change their propagation directions and their interactions are elastic, as seen in Fig. 5.

IV. CONCLUSIONS

In this paper, we have investigated the \((2+1)\)-dimensional nonlinear Schrödinger equation with variable dispersion and nonlinearity coefficients, which governs the optical soliton propagation and interaction in the Kerr optical media. Via the Hirota method and symbolic computation, bright one- and two-soliton solutions (6) and (7) for Eq. (2) have been derived, in which the \( k_j \) are wave numbers and \( \omega_j (j = 1,2) \) are the frequencies. On the basis of solutions (7), the asymptotic analysis has been carried out on the soliton interaction, and has indicated that the total energy of the solitons is conserved before and after the interaction.

Based on solutions (9), when the group velocity dispersion coefficient \( \sigma(z) \) and Kerr effect parameter \( K(z) \) are periodic functions, the soliton propagation follows a periodic evolution pattern along the \( z \) direction and the soliton propagates along the negative \( x \) direction as \( \tau \) increases, as seen in Fig. 1. In Fig. 2, due to the changes of \( \omega_{1I} \) and \( k_{1I} \), the direction of the soliton propagation has changed, while the periods of the soliton propagation in the two cases remain unchanged. The soliton propagates stably with the amplitude remaining the same when \( \sigma(z) \) and \( K(z) \) have been changed to Gaussian-type functions, as seen in Fig. 3. Via solutions (7), elastic interactions of two solitons have been illustrated in Fig. 4, when \( \sigma(z) \) and \( K(z) \) are constants, periodic functions, linearly-decreasing functions, or Gaussian functions. Fig. 5 shows the corresponding cases when the values of \( k_{1I} \) and \( k_{2I} \) have been changed.

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FIG. 4: Interactions of the 2D two solitons via solutions (7). Parameters are $k_1 = 1 + i$, $k_2 = 1$, $\omega_1 = 2/3$, $\omega_2 = 2/3(1 - i)$, $a_1 = a_2 = 1$, $\tau = -10$, (a) $\sigma(z) = 1$, $K(z) = 1$, (b) $\sigma(z) = 2.5 \cos z - 9/4$, $K(z) = \cos z$, (c) $\sigma(z) = -5/4 - z/8$, $K(z) = 1 - z/8$, (d) $\sigma(z) = \exp[-\ln(18) z^2]$, $K(z) = \exp[-\ln(18) z^2] + 9/4$.

References


FIG. 5: Interactions of the 2D two solitons via solutions (7). Parameters are $k_1 = 1 + 2i$, $k_2 = 1 + i$, $\omega_1 = 2/3$, $\omega_2 = 2/3(1-i)$, $a_1 = a_2 = 1$, $\tau = -10$, (a) $\sigma(z) = 1$, $K(z) = 1$, (b) $\sigma(z) = 2.5 \cos z - 9/4$, $K(z) = \cos z$, (c) $\sigma(z) = -5/4 - z/8$, $K(z) = 1 - z/8$, (d) $\sigma(z) = \exp[-\ln(18)z^2]$, $K(z) = \exp[-\ln(18)z^2] + 9/4$.