Elastic Electron Scattering from Some Light Nuclei

F. I. Sharrad,1,2,* A. K. Hamoudi,3 R. A. Radhi,3 Hewa Y. Abdullah,4,5 A. A. Okhunov,1 and H. Abu Kassim1

1Department of Physics, Faculty of Science, University of Malaya, 50603 Kuala Lumpur, Malaysia
2Department of Physics, College of Science, University of Kerbala, Karbala, Iraq
3Department of Physics, College of Science, University of Baghdad, Baghdad, Iraq
4Department of Physics, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia
5Department of Physics, College of Science Education, Salahaddin University, Erbil, Iraq

(Received May 8, 2012; Revised July 9, 2012)

The effects of the two-body short range correlation (SRC) and the occupation probability (η) of higher states on the elastic electron scattering longitudinal form factors F(q) are investigated. Considering the effect of higher occupation probabilities and the effect of SRCs is important in getting a good agreement between the calculated elastic longitudinal electron scattering F(q)s and those of the experimental data for 4He, 12C, 16O, 28Si, 32S, and 40Ca nuclei.

DOI: 10.6122/CJP.51.452 PACS numbers: 21.10.Ft, 21.60.Cs, 25.30.Bf

I. INTRODUCTION

The scattering of electrons from nuclei gives the most precise information about the nuclear size and charge distribution, providing important information about the electromagnetic currents inside the nuclei. Electron scattering can provide a good test for such calculations, since it is sensitive to the spatial dependence of the charge and current densities [1–3]. The electron-nucleus interaction is considered [4] in the first Born approximation as an exchange of virtual photons carrying a momentum transfer q. In this case the initial and the final particles are considered to be free and can be represented by a plane wave.

In the Born approximation the interaction of the electron with the charge distribution of a nucleus is considered as an exchange of a virtual photon with zero angular momentum along the direction of q, this is known as longitudinal or Coulomb scattering. The form factor measurements were reported by [5] for the electroexcitation of the 0+(6.052 MeV), 3− (6.131 MeV), and 2+(6.916 MeV) states of 16O, in the momentum transfer region of 0.5 to 1.0 fm−1. The elastic electron scattering cross section for the nucleus 12C was measured in the momentum transfer range of 0.25 to 2.75 fm−1 [6]. Analytical expressions of the one and two body terms in the cluster expansion of the charge form factors and density distributions of sp- and sd-shell nuclei with Z = N were derived by [7, 8]. Electron scattering Coulomb form factors for the single-particle quadrupole transition in 1p and

*Electronic address: fadhil altaie@gmail.com

sd-shell nuclei have been studied by [9], taking into account the core-polarization effects derived from first-order perturbation theory.

In this paper, an effective two-body density operator for a point nucleon system folded with the two-body short range correlations (SRCs), which take account of the strong short range repulsion in the nucleon-nucleon forces, is produced and used to calculate the longitudinal elastic form factors, applicable for various closed and open shell nuclei with \( N = Z \).

II. ELASTIC ELECTRON SCATTERING

The elastic electron scattering form factor from spin zero nuclei (\( J = 0 \)) can be determined by the ground-state charge density distributions (CDD). In the plane wave Born approximation (PWBA), the incident and scattered electron waves are considered as plane waves, and the CDD is real and spherically symmetric, therefore the form factor is simply the Fourier transform of the CDD. Thus [10–12]

\[
F(q) = \frac{4\pi}{Z} \int_0^\infty \rho_o(r) j_0(qr)r^2 dr,
\]

(1)

where, it is not in the following.

\[
\rho_o(r) = \langle \hat{\rho}^{(2)}_{\text{eff}}(\vec{r}) \rangle = \sum_{i<j} \langle ij | \hat{\rho}^{(2)}_{\text{eff}}(\vec{r}) | ij \rangle - | ji \rangle,
\]

(2)

and

\[
f(r_{ij}) = \begin{cases} 1, & i = j \text{ and } r_c = 0 \\ 1 - \exp\left[-\beta(r_{ij} - r_c)^2\right], & i \neq j \end{cases}
\]

(4)

where \( r_c \) is the radius of a suitable hard core, and \( \beta = 25 \text{ fm}^{-2} \) [12] is a correlation parameter. The two-body charge density distribution (2BCDD) of closed shell nuclei is given by the expectation values of the effective two-body charge density operator of Eq. (2) and is expressed as

\[
\langle \Psi | \hat{\rho}^{(2)}_{\text{eff}}(\vec{r}) | \Psi \rangle = \sum_{i<j} \langle ij | \hat{\rho}^{(2)}_{\text{eff}}(\vec{r}) | ij \rangle - | ji \rangle = \sum_{i<j} (2i_j + 1)(2j_j + 1)C(nle, NLM_L)\hat{\rho}^{(2)}_{\text{eff}}(\vec{r})|n'le'm'_e, N'L'M'_L|.
\]

(5)
where $C$ is a constant.

\[ j_0(qr) = \frac{\sin(qr)}{qr} \] is the zeroth order of the spherical Bessel function and $q$ is the momentum transfer from the incident electron to the target nucleus. Eq. (1) can be expressed as

\[ F(q) = \frac{4\pi}{qZ} \int_0^\infty \rho_o(r) \sin(qr) r dr. \] \hspace{1cm} (6)

Inclusion of the finite nucleon size correction $F_{fs}(q)$ and the center of mass correction $F_{cm}(q)$ in our calculations requires multiplying the form factor of Eq. (3) by these corrections. $F_{fs}(q)$ is considered as a free nucleon form factor and is assumed to be the same for protons and neutrons. This correction takes the form \[ F_{fs}(q) = e^{-0.43q^2/A}, \] \hspace{1cm} (7)

and the correction $F_{cm}(q)$ removes the spurious state arising from the motion of the center of mass when the shell model wave function is used and is given by

\[ F_{cm}(q) = e^{q^2b^2/4A}, \] \hspace{1cm} (8)

where $A$ is the nuclear mass number. Introducing these corrections into Eq. (3), we obtain

\[ F(q) = \frac{4\pi}{qZ} \int_0^\infty \rho_o(r) \sin(qr) r dr F_{fs}(q) F_{cm}(q). \] \hspace{1cm} (9)

In the limit of $q \to 0$ the target will be considered as a point particle, and from Eq. (1) with the help of the following equation:

\[ Z = 4\pi \int_0^\infty \rho_o(r) r^2 dr, \] \hspace{1cm} (10)

the form factor of this target nucleus is equal to unity, i.e., $F(q \to 0) = 1$. The elastic longitudinal electron scattering form factor with the inclusion of the effect of the short-range correlation in light nuclei can now be obtained by introducing the ground state two-body charge density distribution together with those of Eqs. (7) and (8) into Eq. (9).

We also wish to mention that we have written all computer programs needed in this study by using the language capabilities of a Fortran 90 power station.

### III. RESULTS AND DISCUSSION

The elastic electron scattering longitudinal form factors $F(q)$ from the considered spin-zero nuclei are determined in terms of the calculated two-body charge density distributions of the ground state and momentum-transfer $(q)$ using the plane wave Born approximation (PWBA), where the charge form factor is a Fourier transform of the ground state two-body charge density distributions and vice versa.
In Figures 1–6, the calculated $F(q)$s are compared with those of the experimental data for $^4$He, $^{12}$C, $^{16}$O, $^{28}$Si, $^{32}$S, and $^{40}$Ca nuclei, respectively. Parts a and b of these figures are the calculated results based on case 1 (which are based on the prediction of the simple shell model) and case 2 (which included the higher occupation probabilities) of Tables I and II, respectively. In these tables the parameters can be defined as: $\hbar \omega$ is the oscillator parameter used in the radial part of the harmonic oscillator wave function and is given by $[12]: \hbar \omega = \frac{413742}{b^2}$, where $b$ is the harmonic oscillator size parameter. The $\hbar \omega$ values are determined by fitting the experimental values of the root mean square charge radii $\langle r^2 \rangle^{1/2}$, and $\eta_{n_i\ell_i j_i}$ and $\eta_{n_j\ell_j j_j}$ are the occupation probabilities of the states $n_i\ell_i j_i$ and $n_j\ell_j j_j$, respectively.

**TABLE I: Adopted values for the parameters used in the calculation of case 1.**

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$^4$He</th>
<th>$^{12}$C</th>
<th>$^{16}$O</th>
<th>$^{28}$Si</th>
<th>$^{32}$S</th>
<th>$^{40}$Ca</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hbar \omega$ (MeV)</td>
<td>23</td>
<td>15</td>
<td>12.6</td>
<td>12</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>$\eta_{S_{1/2}}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_{P_{3/2}}$</td>
<td>—</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_{P_{1/2}}$</td>
<td>—</td>
<td>—</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_{D_{5/2}}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_{2S_{1/2}}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_{2D_{1/2}}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>1</td>
</tr>
<tr>
<td>$\langle r^2 \rangle^{1/2}_{r_c=0}$ fm</td>
<td>1.644</td>
<td>2.437</td>
<td>2.715</td>
<td>3.071</td>
<td>3.267</td>
<td>3.504</td>
</tr>
<tr>
<td>$\langle r^2 \rangle^{1/2}_{r_c=0.5}$ fm</td>
<td>1.678</td>
<td>2.454</td>
<td>2.728</td>
<td>3.082</td>
<td>3.278</td>
<td>3.513</td>
</tr>
<tr>
<td>$\langle r^2 \rangle^{1/2}_{SRC}$ fm</td>
<td>0.336</td>
<td>0.288</td>
<td>0.266</td>
<td>0.260</td>
<td>0.259</td>
<td>0.251</td>
</tr>
<tr>
<td>$\langle r^2 \rangle^{1/2}_{\text{exp}[7]}$ fm</td>
<td>1.676 (5)</td>
<td>2.471 (3)</td>
<td>2.730 (25)</td>
<td>3.086 (18)</td>
<td>3.248 (11)</td>
<td>3.479 (3)</td>
</tr>
</tbody>
</table>

These parameters are equal to zero or one for closed shell nuclei with $Z = N$, while for open shell nuclei with $Z = N$ they are larger than zero and less than one (i.e., $0 < \eta_{n_i\ell_i j_i} < 1$ and $0 < \eta_{n_j\ell_j j_j} < 1$), where the total number of nucleons in the level is conserved. In case 1, the occupation probability $\eta$ of the states in the considered nuclei are assumed according to the prediction of the simple shell model. While in case 2, the occupation probability of the higher states in the considered nuclei have been taken into account as free parameters in addition to those of case 1. For example in Table I the $^4$He nucleus has 4 nucleons (2
FIG. 1: (Color online) Elastic form factors for the $^4\text{He}$ nucleus. The dotted symbols are the experimental data [14, 15].

protons and 2 neutrons in the $S_{1/2}$ state). In Table II, the occupation probability in the $S_{1/2}$ state equals 0.6, i.e., $0.6 \times 2 = 1.2$ protons or neutrons, and in the $P_{3/2}$ state this is equal to 0.2, $0.2 \times 4 = 0.8$ protons or neutrons. So the total numbers for protons or neutrons are $1.2 + 0.8 = 2$. Also the $\eta$ values are determined by fitting the experimental values of the root mean square charge radii $\langle r^2 \rangle_{\exp}^{1/2}$. The root mean square charge radii
FIG. 2: (Color online) Same title as Figure 1, but this is for the $^{12}$C nucleus. The dotted symbols are the experimental data [16].

The root mean square (rms) of the considered nuclei are obtained by [12]

$$
\langle r^2 \rangle^{1/2} = \left[ \frac{4\pi}{Z} \int_0^\infty \rho(r) r^4 dr \right]^{1/2}.
$$

(12)

Furthermore, the calculated root mean square charge radii with short range correlations, $\langle r^2 \rangle_{r_e=0.5}$, and without them, $\langle r^2 \rangle_{r_e=0}$, together with those of the experimental $\langle r^2 \rangle_{\text{exp}}^{1/2}$ are compared. The comparison between $\langle r^2 \rangle_{r_e=0}$ and $\langle r^2 \rangle_{r_e=0.5}$ demonstrates an increase
FIG. 3: (Color online) Same title as Figure 1 but this is for the \(^{16}\)O nucleus. The dotted symbols are the experimental data [16].

in the values of \(\langle r^2 \rangle_{r_c=0.5}^{1/2}\) due to the introduction of two-body SRCs in the calculations. The contribution of the two-body SRCs to the calculated \(\langle r^2 \rangle^{1/2}\) charge radii, \(\langle r^2 \rangle_{SRC}^{1/2} = \sqrt{\langle r^2 \rangle_{r_c=0.5fm} - \langle r^2 \rangle_{r_c=0}}\), are displayed as well in these tables.

Our calculations of \(\langle r^2 \rangle^{1/2}\) are in good agreement with the experimental data from previous works [7, 8]. We used the Shell model in our calculation, but in [7, 8] they used the cluster expansion.
FIG. 4: (Color online) Same title as Figure 1 but this is for the $^{28}$Si nucleus. The dotted symbols are the experimental data [17].

In the above figures the calculated $F(q)$s are plotted as a function of $q$, while those of the experimental data are plotted as a function of $q_{\text{eff}}$, where $q_{\text{eff}} = q \left[1 + \frac{3}{2} \frac{\langle r^2 \rangle}{E_i R_c} \right]$ and $R_c = \sqrt{\frac{2}{3}} \left\langle r^2 \right\rangle^{1/2}$ [13], where $E_i$ is the initial energy and $\left\langle r^2 \right\rangle^{1/2}$ is the root mean square. The solid and dashed curves are the calculated $F(q)$s with $(r_c = 0.5)$ and without $(r_c = 0 \text{ fm})$ inclusion of the effect of the two-body short range correlation functions (SRCs), respectively, whereas the dotted symbols are those of the experimental data.
III-1. $^4$He nucleus

The elastic longitudinal $F(q)$s of the $^4$He nucleus are displayed in Figure 1. As is obvious from parts (a) and (b) of this figure, the solid curves are better in describing the experimental data [14, 15] than those of the dashed curves. The solid curves in (a) and (b) agree quite well with the data up to $q = 3.2$ fm$^{-1}$ and underestimate these data at $q > 3.2$ fm$^{-1}$. Besides, these solid curves indicate that the inclusion of two-body SRCs tends to move the locations of the diffraction minima into regions having less momentum transfer.
FIG. 6: (Color online) Same title as Figure 1 but this is for the $^{40}$Ca nucleus. The dotted symbols are the experimental data [18].

than those of the dashed curves.

The quality of agreement between the calculated $F(q)$s and those of the experimental data becomes even better in part (b) than part (a), since the solid curve in (b) becomes closer to the data at a higher momentum transfer of $q > 3.2 \, \text{fm}^{-1}$ than that of part (a). Besides, in part (b) the location of the diffraction minimum is reproduced in the correct place, while in part (a) there is a shift of about $\Delta q = 0.15 \, \text{fm}^{-1}$ between the locations of the calculated and experimental diffraction minima, as seen in the solid curves of this figure. It
TABLE II: Adopted values for the parameters used in the calculation of case 2.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$^4\text{He}$</th>
<th>$^{12}\text{C}$</th>
<th>$^{16}\text{O}$</th>
<th>$^{28}\text{Si}$</th>
<th>$^{32}\text{S}$</th>
<th>$^{40}\text{Ca}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hbar \omega$ (MeV)</td>
<td>26</td>
<td>14.665</td>
<td>12.5</td>
<td>11.63</td>
<td>10.9</td>
<td>10</td>
</tr>
<tr>
<td>$\eta_{1S_{1/2}}$</td>
<td>0.6</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_{1P_{3/2}}$</td>
<td>0.2</td>
<td>0.95</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_{1P_{1/2}}$</td>
<td>—</td>
<td>—</td>
<td>0.97</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_{1d_{5/2}}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.8</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_{2S_{1/2}}$</td>
<td>—</td>
<td>0.1</td>
<td>0.03</td>
<td>0.6</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>$\eta_{1d_{3/2}}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.15</td>
<td>1</td>
</tr>
<tr>
<td>$\eta_{1p_{3/2}}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.15</td>
</tr>
<tr>
<td>$\langle r^2 \rangle_{r_c=0}^{1/2}$ fm</td>
<td>1.633</td>
<td>2.454</td>
<td>2.723</td>
<td>3.069</td>
<td>3.255</td>
<td>3.490</td>
</tr>
<tr>
<td>$\langle r^2 \rangle_{r_c=0.5}^{1/2}$ fm</td>
<td>1.669</td>
<td>2.472</td>
<td>2.737</td>
<td>3.081</td>
<td>3.266</td>
<td>3.499</td>
</tr>
<tr>
<td>$\langle r^2 \rangle_{\text{SRC}}^{1/2}$ fm</td>
<td>0.344</td>
<td>0.297</td>
<td>0.276</td>
<td>0.271</td>
<td>0.267</td>
<td>0.250</td>
</tr>
<tr>
<td>$\langle r^2 \rangle_{\text{exp}}^{1/2}$ [7]</td>
<td>1.676 (8)</td>
<td>2.471 (6)</td>
<td>2.730 (25)</td>
<td>3.086 (18)</td>
<td>3.248 (11)</td>
<td>3.479 (3)</td>
</tr>
</tbody>
</table>

is concluded from this figure that considering the higher state $1p_{3/2}$ with occupation probability $\eta_{1p_{3/2}} = 0.2$ together with the introduction of the two-body SRCs in the calculation (the solid curve of part b) leads to reproducing better the results for the calculated $F(q)$s than that of part a (case1).

III-2. $^{12}\text{C}$ nucleus

The elastic longitudinal form factors of the $^{12}\text{C}$ nucleus are presented in Figure 2. It is noticed from this figure that both the magnitude and the behavior of the calculated form factors in case 1 and case 2 are in a very good agreement with those of the experimental data [16] throughout the whole range of momentum transfer $q$. Besides, the locations of the calculated diffraction minima in both cases are reproduced in the correct place. It is also noticed that the introduction of the two-body SRCs in the calculations of both cases (the solid curves) leads to enhancing slightly the values of the calculated form factors in the region of momentum transfer $q \geq 1.7$ fm$^{-1}$, and consequently tends to improve the calculated results of the form factors as seen by the solid curves, since they become closer to the experimental data than those of the dashed curves. An inspection of parts (a) and (b) of this figure gives an indication that case 1 better represents the experimental data of the form factors in the $^{12}\text{C}$ nucleus than that of case 2.

III-3. $^{16}\text{O}$ nucleus

In Figure 3 we explore the calculated results for the form factors of the $^{16}\text{O}$ nucleus. It is evident from this figure that the calculated results obtained in both cases are nearly
the same. This is attributed to the chosen values of the occupation probabilities in case 2, which are nearly the same as those of case 1. It seems that the calculated results of case 2 are not affected by the small change that we have made in the values of the occupation probabilities of case 2. However, the dashed and solid curves of both cases are in reasonable agreement with those of the experimental data [16] up to momentum transfer \( q = 2.8 \text{ fm}^{-1} \).

The first diffraction minimum which is known from the experimental data is very well reproduced by the calculations of case 1 and case 2. It is seen from this figure that there is a disagreement between the experimental and calculated form factors of this nucleus at momentum transfer \( q \approx 3 \text{ fm}^{-1} \), where it seems that there is a second diffraction minimum in the experimental data which cannot be reproduced in the correct place for both cases. This figure also demonstrates that the effect of the two-body SRCs starts at the region of momentum transfer \( q \geq 2.8 \text{ fm}^{-1} \), where the solid curve, in both cases, deviates from the dashed curve in this region of \( q \).

### III-4. \( ^{28}\text{Si} \) nucleus

The form factors of the \( ^{28}\text{Si} \) are displayed in Figure 4. It shows that the calculations of case 2 are better in reproducing the experimental data than that of case 1. In the calculations of both case 1 and case 2, the experimental data [17] of the \( ^{28}\text{Si} \) nucleus are very well reproduced up to \( q = 2.3 \text{ fm}^{-1} \). In the region of \( q > 2.3 \text{ fm}^{-1} \) of case 1, both the behavior and the magnitude of the calculated form factors do not predict the data. It is obvious that the calculated second diffraction minimum (located at \( q \approx 2.7 \text{ fm}^{-1} \)) is not in agreement with that of the experimental data (located at \( q \approx 2.35 \text{ fm}^{-1} \)). Besides, the third diffraction minimum which is presented in the experimental data (located at \( q \approx 3.4 \text{ fm}^{-1} \)) is not reproduced by the calculation of case 1. It is seen that the effect of the two-body SRCs begins at the region of momentum transfer \( q \geq 2.8 \text{ fm}^{-1} \) (see the solid curve in case 1) and leads to reducing the enhancement of the calculated result of the form factors. Whereas in case 2, this effect starts at the region of \( q = 1.25 \text{ fm}^{-1} \) (see the solid curve in case 2) and leads to increasing the enhancement of the calculated form factors, which consequently tends to improve the calculated result of case 2. It is noted, in case 2, that the behavior of the calculated results (the solid curve) is in very good agreement with that of the experimental data throughout the whole range of \( q \). Besides, the first, second, and third diffraction minima which are presented in the experimental data are quite well described by the solid curve. It is also noted that the solid curve of case 2 underestimates the data at the region of momentum transfer \( q \geq 2.3 \text{ fm}^{-1} \).

### III-5. \( ^{32}\text{S} \) nucleus

The form factors of \( ^{32}\text{S} \) nucleus are presented in Figure 5. It is very clear that there is a disagreement between the calculated result of case 1 and those of the experimental data [17], where the behavior, the magnitude of the calculated form factors at \( q \geq 1.2 \text{ fm}^{-1} \), and the locations of the calculated diffraction minima are not in accordance with the experimental data. In case 2, the calculated result of the form factors are in good agreement with those of the experimental data up to the region of momentum transfer \( q \approx 3 \text{ fm}^{-1} \).
The behavior of the calculated results accord very well with the data. In addition, all the experimental diffraction minima of this nucleus are reproduced in the correct places, as seen in the solid curve of case 2. This figure shows that the effect of the two-body SRCs is small up to $q \approx 2.7$ fm$^{-1}$, while for higher $q$ it becomes progressively larger, since it reduces the calculated form factor significantly at this region of $q$, as seen in the solid curves of case 1 and case 2.

III-6. $^{40}$Ca nucleus

Figure 6 displays the form factors of the $^{40}$Ca nucleus. It is noted from case 1 that the calculated form factors are in a good agreement with those of the experimental data [18] for all ranges of $q$, with the exception of the region $1$ fm$^{-1} < q < 2$ fm$^{-1}$. In this region, the calculated form factors slightly underestimate the experimental data, and the calculated second diffraction minimum is not reproduced in the correct place. In fact, an improvement for the calculated form factors is obtained by considering the higher state $2p_{3/2}$ in the calculations of case 2, since the occupation probability of this state is taken as $\eta_{2p_{3/2}} = 0.15$. As is evident from case 2 of this figure, there is a very good agreement between the calculated and experimental form factors throughout the whole range of momentum transfer $q$. Here, both of the behavior and magnitude of the calculated curves are in a very good agreement with those of the experimental data. Besides, the calculated first and second diffraction minima are in coincidence with those of the experimental data. The same argument can be applied here for the effect of the two-body SRCs, as shown in Figure 5, i.e., the effect is small up to $q \approx 2.8$ fm$^{-1}$, whereas for higher $q$ it becomes larger and it decreases the calculated form factors, as seen in the solid curves of case 1 and case 2.

IV. CONCLUSIONS

The effect of higher occupation probabilities and SRC’s are well established for the analysis of elastic scattering, and we see a good agreement between the calculated elastic longitudinal $F(q)$s and those of the experimental data, i.e., we have included the higher occupation probabilities of $\eta_{n,\ell,i}$ and $\eta_{n,\ell,j,j'}$ and the SRC (case 2).

Acknowledgements

We thank the Islamic Development Bank (IDB) for supporting this work under grant No. 36/11201905/35/IRQ/D31. Furthermore, we thank the University of Malaya - Faculty of Science as well as the Department of Physics, University of Kerbala - College of Science - Department of Physics and the University of Baghdad - College of Science - Department of Physics for supporting this work.
References