Holographic Q-Picture of Black Holes in Five Dimensional Minimal Supergravity

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In this article, we explore the holographic Q-picture description for charged rotating black holes in five-dimensional minimal supergravity. The central charge in the Q-picture depends only on the black hole charge, therefore it can be computed from the near horizon geometry of the extremal and non-rotating counterpart. Moreover, the CFT temperatures can be identified by studying the hidden conformal symmetry, and the related gravity-CFT dictionary can be translated via thermodynamic analysis. The entropy and absorption cross section computed from both the gravity and CFT sides properly agree with each other.

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I. INTRODUCTION

The holographic principle [1–3] is an outstanding concept which provides dual descriptions connecting gravity and field theory. In the past years, numerous substantial successes have been achieved on the holographic dual description for black holes, in particular for the Kerr solutions [4–13] as well as for other generalizations [14–25]. The original paper on the Kerr/CFT correspondence [4] explored the holographic correspondence for the extremal Kerr black holes. More precisely, it was shown that the central charge of the dual CFT can be derived from the asymptotic symmetry group of the near horizon geometry, and the temperatures can be identified via the Boltzmann factor. The major evidence is the fact that the CFT entropy computed by using the Cardy formula exactly reproduces the black hole Bekenstein-Hawking entropy. Soon after this stimulating progress, the investigation on the Kerr/CFT correspondence has been extended to the near extremal cases [7, 9] with a new support that the scattering absorption cross section of a probe field, in suitable limits, agrees with the two-point function of the dual operator. Recently, the Kerr/CFT correspondence was remarkably generalized to the generic non-extremal Kerr black holes [13]. For the non-extremal black holes, the near horizon geometry does not contain an explicit AdS3 structure, nevertheless, there is a local conformal invariance in the solution space of a specified probe field which ensures a dual CFT description. This observation indicates
that even though the near-horizon geometry of a generic Kerr black hole could be distinct from that of an AdS or warped AdS spacetime, the local conformal symmetry on the solution space may still allow us to explore its CFT description. Both the microscopic entropy counting and the low frequency scattering amplitude in the near region support such an intuitive judgement. The study of hidden conformal symmetry has been generalized to various types of black holes [26–43].

For the charged black holes the holographic duality changes to have multiple faces [44–47]. It has been shown in [48], see also [49], that there are two different individual 2D CFTs holographically dual to the Kerr-Newman black holes referring to the two possible limits: neutral Kerr and non-rotating Reissner-Nordstrom (RN) solutions. The twofold holographic descriptions, called the J-picture and Q-picture, distinctly are direct extensions respectively of the Kerr/CFT correspondence and the RN/CFT correspondence [47, 50–53]. Just like the Kerr-Newman solutions, the charged rotating black holes in five-dimensional minimal supergravity [54, 55] can provide another interesting background for verifying the validity of the holographic principle. Several profound investigations have been done, including the duality for the extremal limit, see for example [56, 57], and the associated hidden conformal symmetry [58]. However, all of the studies were only focusing on the angular momentum description, namely the J-picture. It is a natural expectation that there should be a holographic Q-picture description for the black holes in five dimensional minimal supergravity. In this article, we will explore this picture in more detail.

The paper is organized as follows. In Section II, we review some basic properties of black holes in five dimensional minimal supergravity. In Section III, the dynamics of a probe massless charged scalar field propagating in the considered black hole background is studied. We investigate the Q-picture hidden conformal symmetry by analyzing the wave equation of the probe scalar field. As expected, we confirm that the microscopic entropy evaluated by the Cardy formula exactly reproduces the black hole Bekenstein-Hawking entropy. In Section IV, a further support of agreement between the absorption cross section and two point function is checked. Finally, the last section is devoted to the conclusion.

II. BLACK HOLES IN 5D MINIMAL SUPERGRAVITY

In this section we review and examine the black hole solutions in five dimensional minimal supergravity,

\[
S_5 = \frac{1}{16\pi} \left[ \int d^5x \sqrt{-g} \left( R - \frac{1}{4} F^2 \right) - \frac{1}{3\sqrt{3}} \int F \wedge F \wedge A \right].
\]
The electric charged rotating black holes [54, 55], in Boyer-Lindquist coordinates $x^\mu = (t, r, \theta, \varphi_1, \varphi_2)$, have the following non-vanishing metric components:

\[
\begin{align*}
  g_{00} &= -\left(1 - \frac{2m}{\rho^2} + \frac{q^2}{\rho^4}\right), \\
  g_{03} &= -\frac{a(2m\rho^2 - q^2) + bq\rho^2}{\rho^4} \sin^2 \theta, \\
  g_{04} &= -\frac{b(2m\rho^2 - q^2) + aq\rho^2}{\rho^4} \cos^2 \theta, \\
  g_{33} &= (r^2 + a^2) \sin^2 \theta + \frac{a^2(2m\rho^2 - q^2) + 2abq\rho^2}{\rho^4} \sin^4 \theta, \\
  g_{44} &= (r^2 + b^2) \cos^2 \theta + \frac{b^2(2m\rho^2 - q^2) + 2abq^2}{\rho^4} \cos^4 \theta, \\
  g_{34} &= \frac{ab(2m\rho^2 - q^2) + (a^2 + b^2)q\rho^2}{\rho^4} \sin^2 \theta \cos^2 \theta, \\
  g_{11} &= \frac{\rho^2}{\Delta}, \quad g_{22} = \rho^2, \\
\end{align*}
\]

and the gauge potential is

\[
A = -\frac{\sqrt{3}q}{\rho^2} (dt - a \sin^2 \theta d\varphi_1 - b \cos^2 \theta d\varphi_2),
\]

where

\[
\Delta = \frac{(r^2 + a^2)(r^2 + b^2) + q^2 + 2abq - 2m}{r^2}, \quad \rho^2 = r^2 + a^2 \cos^2 \theta + b^2 \sin^2 \theta.
\]

For the above spacetime geometry, the determinant of the metric is $\sqrt{-\det(g_{\mu\nu})} = \sqrt{-g} = r\rho^2 \sin \theta \cos \theta$, and the locations of the event horizons are given by the zeros of the metric function which are the real roots of $r^2 \Delta = 0$. These black hole solutions are characterized by four parameters $m, q, a, b$ representing respectively the mass, charge and two independent angular momenta:

\[
M = \frac{3\pi}{4} m, \quad Q = \frac{\sqrt{3}\pi}{4} q, \quad J_1 = \frac{\pi}{4} (2ma + qb), \quad J_2 = \frac{\pi}{4} (2mb + qa).
\]
The corresponding Hawking temperature, entropy, and the angular velocities and chemical potential on the horizon are given by

\[
T_H = \frac{r^+_H - (ab + q)^2}{2\pi r_+[(r^+_H + a^2)(r^+_H + b^2) + ab]},
\]

\[
S_{BH} = \frac{\pi^2[(r^+_H + a^2)(r^+_H + b^2) + ab]}{2r_+},
\]

\[
\Omega_1 = \frac{(ar^2_+ + ab^2 + bq)}{(r^2_+ + a^2)(r^2_+ + b^2) + abq},
\]

\[
\Omega_2 = \frac{(br^2_+ + a^2b + aq)}{(r^2_+ + a^2)(r^2_+ + b^2) + abq},
\]

\[
\mu_q = \sqrt{3}qr^2_+ \frac{\sqrt{3}qr^2_+}{(r^2_+ + a^2)(r^2_+ + b^2) + abq}.
\]

The central charges associated with the two angular momenta, i.e., the holographic J-picture description, has been discussed in [56, 57], and the related hidden conformal symmetries were analyzed in [58]. However, as pointed out in [47, 48], the charged rotating black holes can have another proper dual CFT description, called the Q-picture, essentially based only on their charge parameters [64], see also [49, 53]. In this paper, we will mainly focus on the Q-picture CFT description for the black holes in five dimensional minimal supergravity. By virtue of the property that the Q-picture is orthogonal of the J-picture, i.e., the associated central charge is independent on the angular momenta, therefore it can be obtained simply from the non-rotating counterparts,

\[
ds_5^2 = -\left(1 - \frac{2m}{r^2} + \frac{q^2}{r^4}\right)dt^2 + \left(1 - \frac{2m}{r^2} + \frac{q^2}{r^4}\right)^{-1}dr^2 + r^2d\Omega_3^2,
\]

\[
A = \sqrt{3}qdr/r^2dt.
\]

where \(d\Omega_3^2 = d\theta^2 + \sin^2\theta d\varphi_1^2 + \cos^2\theta d\varphi_2^2\). In the extremal limit, \(m = q\), the radius of the degenerate black hole horizons is \(r_0 = \sqrt{m} = \sqrt{q}\) and the related near horizon geometry can be achieved by taking the limit \((\epsilon \to 0)\)

\[
r \to r_0 + \epsilon r, \quad t \to \frac{m}{4\epsilon}t.
\]

As expected, the near horizon geometry has an AdS\(_2 \times S^3\) structure and the gauge potential is linear in the radius coordinate:

\[
ds_5^2 = \frac{r_0^2}{4} \left(-r^2dt^2 + \frac{dr^2}{r^2}\right) + r_0^2d\Omega_3^2,
\]

\[
A = \frac{\sqrt{3}}{2}r_0rdt.
\]
Unlike the J-picture, the central charge of the Q-picture is encoded both in the metric and the gauge potential. In order to recover the AdS$_3$ structure, one should embed the near horizon solution (9) into a six-dimensional spacetime [65] [59],

$$ds_6^2 = ds_5^2 + (dy + A)^2,$$

which leads to the following form

$$ds_6^2 = \Gamma \left( -r^2 dt^2 + \frac{dr^2}{r^2} + \alpha d\Omega_3^2 \right) + \gamma (dy + k r dt)^2,$$

with

$$\Gamma = \frac{r_0^2}{4}, \quad \alpha = 4, \quad \gamma = 1, \quad k = \frac{\sqrt{3}}{r_0}.$$

The metric (11) can be expressed in the particular form of

$$ds_6^2 = \Gamma \left( -r^2 dt^2 + \frac{dr^2}{r^2} + \alpha d\theta^2 \right) + \gamma_{ij} (dy^i + k^i r dt)(dy^j + k^j r dt),$$

with

$$y^i = (y, \varphi_1, \varphi_2), \quad \gamma_{ij} = \text{diag}(\gamma, \Gamma \alpha \sin^2 \theta, \Gamma \alpha \cos^2 \theta), \quad k^i = (k, 0, 0).$$

Following the derivation in [60], the left-sector central charge and temperature are given by (here the range of $\theta$ is $0 \leq \theta \leq \pi/2$)

$$c_L = 6 \pi k \int_0^{\pi/2} d\theta \sqrt{\Gamma \alpha \text{det}(\gamma_{ij})} = 6 \pi k \int_0^{\pi/2} d\theta \sqrt{(\Gamma \alpha)^3} \sin \theta \cos \theta = 3 \pi r_0^3 k,$$

$$T_L = \frac{1}{2 \pi k},$$

assuming that the periodicity of the coordinate $y$ is $2\pi$. The Cardy formula for the CFT entropy exactly reproduces the black hole entropy,

$$S_{\text{CFT}} = \frac{\pi^2}{3} c_L T_L = \frac{1}{2} \pi^2 r_0^3 = S_{\text{BH}}.$$

Nevertheless, the Q-picture central charge is ambiguous up to the radius, $\ell$, of the extra circle, namely $y \sim y + 2\pi \ell$ periodicity [51, 52]. Moreover, both the left-sector and right-sector central charges are expected to be identical, since there is no gravitational anomaly. Therefore, the general formula of the central charges is

$$c_L = c_R = \frac{3 \sqrt{3} \pi q^2}{2 \ell}.$$

There are two natural choices for the value of $\ell$: one is $\ell = 1$ (Planck length) and the other is $\ell = r_0$ (about the size of the AdS$_3$). From the brane construction point of view, the later choice corresponds to the configuration of long strings winding on the large extra circle [61].
III. Q-PICTURE HIDDEN CONFORMAL SYMMETRY

In this section, we consider the Klein-Gordon (KG) equation for a probe complex scalar field propagating in the considered black hole background. For a massless complex scalar field carrying the charge $e$, the KG equation,

$$(\nabla_\mu - ieA_\mu)(\nabla^\mu - ieA^\mu)\Phi = 0,$$  

(18)

can be simplified by assuming the following form of the scalar field [62]:

$$\Phi = \exp(-i\omega t + im_1\varphi_1 + im_2\varphi_2)S(\theta)R(r).$$  

(19)

The neutral scalar field ($e = 0$) is able to reveal, from the radial equation, the hidden conformal symmetry (in the J-picture) as shown in [58]. For the Q-picture description we should consider the radial equation with conditions $m_1 = m_2 = 0$ [48, 53], so the radial equation can be expressed as

$$\frac{1}{r} \partial_r (r^2 \partial_r R) + \left[ \left( \frac{[(r^2 + a^2)(r^2 + b^2) + abq\omega - \sqrt{3}eqr^2]^2}{r^4 \Delta} - \frac{a^2b^2\omega^2}{r^2} - \lambda \right) R = 0. \right.$$  

(20)

Defining a new variable $u = r^2$ and $u_+ = r_+^2; \ u_- = r_-^2$, then

$$\tilde{\Delta} \equiv r^2 \Delta = (u - u_+)(u - u_-),$$  

(21)

and the above differential equation becomes

$$4\partial_u (\tilde{\Delta} \partial_u R) + \left[ \frac{\left( [(u + a^2)(u + b^2) + abq\omega - \sqrt{3}equ]^2}{u(u - u_+)(u - u_-)} - \frac{a^2b^2\omega^2}{u} - \lambda \right) R = 0. \right.$$  

(22)

In the limits for the scalar field with low frequency $\omega r_+ \ll 1$ (consequently $\omega m \ll 1, \ \omega a \ll 1, \ \omega b \ll 1$) and small charge $eq \ll 1$, the radial equation in the near region $r\omega \ll 1$ could be simplified as [66]

$$\left[ \partial_u (\tilde{\Delta} \partial_u) + \frac{\left( \beta_+ \omega - \sqrt{3}eqr_+ \right)^2}{4(u - u_+)(u_+ - u_-)} - \frac{\left( \beta_- \omega - \sqrt{3}eqr_- \right)^2}{4(u - u_-)(u_+ - u_-)} \right] R = \frac{l(l + 2)}{4} R, $$  

(23)

where

$$\beta_\pm = \frac{(r_\pm^2 + a^2)(r_\pm^2 + b^2) + abq}{r_\pm}.$$  

(24)

Following the idea proposed in [13], we are going to show that Equation (23) actually can be reproduced by the Casimir operator of the AdS$_3$ space,

$$ds^2_3 = \frac{L^2}{y^2} (dy^2 + dw^+ dw^-).$$  

(25)
Here, the AdS$_3$ radius $L$ is not essential in our discussion. There are two sets of symmetry 
generators

\[ H_1 = i \partial_+, \quad H_0 = i(w^+ \partial_+ + \frac{1}{2} y \partial_y), \quad H_{-1} = i((w^+)^2 \partial_+ + w^+ y \partial_y - y^2 \partial_-), \]  

(26)

and

\[ \bar{H}_1 = i \partial_-, \quad \bar{H}_0 = i(w^- \partial_- + \frac{1}{2} y \partial_y), \quad \bar{H}_{-1} = i((w^-)^2 \partial_- + w^- y \partial_y - y^2 \partial_+), \]  

(27)

each of them satisfies the $SL(2, R)$ algebra:

\[ [H_0, H_1] = \mp i H_1, \quad [H_1, H_1] = -2i H_0, \]  

(28)

and

\[ [\bar{H}_0, \bar{H}_1] = \mp i \bar{H}_1, \quad [\bar{H}_1, \bar{H}_1] = -2i \bar{H}_0. \]  

(29)

Coordinately, the associated quadratic Casimir operator is

\[ H_2 = \bar{H}_2 = -\frac{1}{2}(H_1 H_{-1} + H_{-1} H_1) = \frac{1}{4}(y^2 \partial_y^2 - y \partial_y) + y^2 \partial_+ \partial_- . \]  

(30)

By introducing the following transformations from the conformal space to black hole coordinates:

\[ w^+ = \sqrt{\frac{u-u_+}{u-u_-}} \exp(2\pi T_R \chi + 2n_R t), \]

\[ w^- = \sqrt{\frac{u-u_+}{u-u_-}} \exp(2\pi T_L \chi + 2n_L t), \]

\[ y = \sqrt{\frac{u_+ - u_-}{u_+ - u_-}} \exp(\pi(T_R + T_L) \chi + (n_R + n_L)t), \]  

(31)

the Casimir operator is transformed in terms of $(u, t, \chi)$ coordinates as

\[ \mathcal{H}^2 = \partial_u (\tilde{\Delta} \partial_u) - \frac{u_+ - u_-}{u - u_+} \left( \frac{T_L + T_R}{4A} \partial_t - \frac{n_L + n_R}{4\pi A} \partial_\chi \right)^2 \]

\[ + \frac{u_+ - u_-}{u - u_-} \left( \frac{T_L - T_R}{4A} \partial_t - \frac{n_L - n_R}{4\pi A} \partial_\chi \right)^2, \]  

(32)

where $A = T_R n_L - T_L n_R$. Furthermore, from the black hole side, the radial equation (23) can be reexpressed as

\[ \partial_u (\tilde{\Delta} \partial_u) = \left( \frac{\beta_+ \partial_t - (\sqrt{3} q r_+ / \ell) \partial_\chi}{4(u - u_+)(u_+ - u_-)} + \frac{\beta_- \partial_t - (\sqrt{3} q r_- / \ell) \partial_\chi}{4(u - u_-)(u_+ - u_-)} \right)^2 R = \frac{l(l+2)}{4} R, \]  

(33)

after introducing an operator $\partial_\chi$ acting on the $U(1)$ symmetry internal space of the complex scalar field. The eigenvalue of the new operator is the scalar field charge [48, 53], namely
\[ \partial \chi \Phi = i \ell e \Phi, \] up to an undetermined parameter \( \ell \) correlated with the ambiguity in the central charge. Therefore, the radial equation can be realized as the Casimir operator (32) acting on \( \Phi \) with the following identifications, including the CFT temperatures:

\[ T_L = \frac{\ell (\beta_+ + \beta_-)}{2\sqrt{3} \pi q^2}, \quad T_R = \frac{\ell (\beta_+ - \beta_-)}{2\sqrt{3} \pi q^2}, \]

(34)

\[ n_L = \frac{r_+ + r_-}{2q}, \quad n_R = \frac{r_+ - r_-}{2q}. \]

(35)

As the first evidence, one can easily verify the agreement of the microscopic and macroscopic entropies:

\[ S_{CFT} = \frac{\pi^2}{3} (c_L T_L + c_R T_R) = \frac{\pi^2}{2} \beta_+ = S_{BH}. \]

(36)

**IV. ABSORPTION CROSS SECTION**

For a further support of the holographic Q-picture, we will show that the absorption cross section for the probe scalar field (with the assumptions \( m_1 = m_2 = 0 \)) scattered in the near region of the black hole matches with the two point function of the dual operator in the CFT with identified, left and right, central charges and temperatures. The absorption cross section can be written as [48, 63]

\[ P_{\text{abs}} \sim \sinh(2\pi \gamma_Q) |\Gamma(a_Q)|^2 |\Gamma(b_Q)|^2, \]

(37)

where the three coefficients can be straightforwardly read out from Equation (23):

\[ \gamma_Q = \frac{\beta_+ \omega - \sqrt{3}eqr_+}{2(r_+^2 - r_-^2)}, \]

\[ a_Q = 1 + \frac{l}{2} - i \frac{(\beta_+ - \beta_-) \omega - \sqrt{3}eq(r_+ + r_-)}{2(r_+^2 - r_-^2)}, \]

\[ b_Q = 1 + \frac{l}{2} - i \frac{(\beta_+ - \beta_-) \omega - \sqrt{3}eq(r_+ - r_-)}{2(r_+^2 - r_-^2)}, \]

(38)

leading to the relation \( a_Q + b_Q = 2 + l - 2i \gamma_Q \). In order to explicitly check that the \( P_{\text{abs}} \) really matches with the microscopic greybody factor of the dual CFT, one needs to identify the related parameters of the dual operator. Firstly, the conformal weights of the dual operator are

\[ (h_L, h_R) = \left( 1 + \frac{l}{2}, 1 + \frac{l}{2} \right). \]

(39)

Moreover, from the first law of black hole thermodynamics,

\[ T_H \delta S_{BH} = \delta m - \Omega_1 \delta J_1 - \Omega_2 \delta J_2 - \mu_0 \delta q, \]

(40)
one can identify the conjugate charges as
\[ \delta S_{BH} = \delta S_{CFT} = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R}. \]

In the Q-picture description, one should assume \( \delta m = \omega, \delta q = e, \) and \( \delta J_1 = \delta J_2 = 0, \) and the solution of the conjugate charges is
\[ \delta E_L = \tilde{\omega}_L = \omega_L - q_L \mu_L, \quad \delta E_R = \tilde{\omega}_R = \omega_R - q_R \mu_R, \]

where
\[
\begin{align*}
\omega_L &= \frac{\ell(\beta_+^2 - \beta_-^2)}{2\sqrt{3}q^2(r_+^2 - r_-^2)} \omega, \\
\mu_L &= \frac{\ell(\beta_+ + \beta_-)}{2q(r_+ + r_-)}, \\
\omega_R &= \frac{\ell(\beta_+^2 - \beta_-^2)}{2\sqrt{3}q^2(r_+^2 - r_-^2)} \omega, \\
\mu_R &= \frac{\ell(\beta_+ - \beta_-)}{2q(r_+ - r_-)}.
\end{align*}
\]

Finally, the absorption cross section can be expressed as
\[
P_{abs} \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh \left( \frac{\tilde{\omega}_L}{2T_L} + \frac{\tilde{\omega}_R}{2T_R} \right) \left| \Gamma \left( h_L + i \frac{\tilde{\omega}_L}{2\pi T_L} \right) \right|^2 \left| \Gamma \left( h_R + i \frac{\tilde{\omega}_R}{2\pi T_R} \right) \right|^2,
\]
in agreement with the two point function of the dual operator.

V. CONCLUSION

The “microscopic hair conjecture” proposed in [48] claims that for each macroscopic hair parameter, in addition to the mass, of a black hole there should exist an associated holographic CFT\(_2\) description. For the charged rotating black holes in five dimensional minimal supergravity, the J-picture descriptions associated with two angular momenta has been studied previously in [57] for the extremal case and in [58] for the hidden conformal symmetry. In this paper, we have explored the supplementary holographic description, the Q-picture, based on the electric charge of the black hole. The central charge of the Q-picture CFT actually is independent of the angular momenta, so it can be computed simply from the non-rotating counterparts of the black hole. Unlike the J-picture, the central charge of Q-picture CFT is encoded both in the metric and the gauge potential. The charge contribution cannot be obtained directly from the central extension of the asymptotic symmetry group [60]. Generically, the near horizon geometry of an extremal non-rotating charged black hole has only an AdS\(_2\) structure, and the \( U(1) \) fiber of the fundamental AdS\(_3\) is held by the gauge potential, which can be revealed by a Kaluza-Klein uplifting.

Specifically, we considered the wave equation of a massless charged scalar field in the background of a black hole in five dimensional minimal supergravity. It turns out that under certain low frequency and low charge limits, the radial part of the “near region”
KG equation is equivalent to a Casimir operator of the $SL(2, R)_L \times SL(2, R)_R$ group. The CFT temperatures then can be identified. The macroscopic entropy and the absorption cross section of the probe scalar field match precisely to the microscopic CFT entropy and the corresponding two point function. All our results provide evidence for the validity of the holographic Q-picture description of the black holes in five dimensional minimal supergravity.

A recent result in [61] has given a remarkable microscopic realization of the J-picture holographic description for the extremal, but non-BPS, charged rotating black hole in 5-dimensional minimal supergravity. It would be interesting to construct the associated microscopic realization for the Q-picture description. Moreover, the J- and Q-pictures provide two “orthogonal” descriptions of the dual CFTs. It is natural to expect that there should exist a certain duality among these different pictures. Further clarification of these important issues is desirable.

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The J- and Q-pictures are two basic “orthogonal” descriptions in which the central charges depend only on either the angular momentum or on the charge. Surely, there are other possible descriptions which provide mixed versions of these two pictures.

For the purpose of deriving the corresponding central charge, one only has to focus on the spacetime geometry and can simply neglect the gauge field part.

In such limits, the separation constant $\lambda$ reduces to $l(l + 2)$ corresponding to the spherical harmonics of the $S^3$. 