Heterodyne Interferometric Measurement of the Thermo-Optic Coefficient of Single Mode Fiber

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The thermo-optic coefficient of a single mode fiber is determined by a heterodyne interferometric technique. The heterodyne beat signal, resulting from the beating of the carrier and two side-band frequencies, is related to the phase shift difference between the carrier and the side-band waves in a fiber Fabry-Perot interferometer. This phase shift difference is determined with a phase sensitive detection technique and is found to be temperature dependent. From the temperature dependence, the thermo-optic coefficient of the single mode fiber is determined to be $9.2 \times 10^{-6}$ $\degree C^{-1}$. The experimental result agrees well with a theoretical simulation using the temperature dependent Sellmeier equation for fused silica.

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I. Introduction

The thermo-optic coefficient $(dn/dT)$ is an important parameter for optical fibers used in high-speed transmission systems [1-3]. A small portion of the transmitted laser power absorbed by the fiber, and/or a change of ambient temperature, can cause temperature variations in the fiber and consequently changes in its refractive index. Variation in the refractive index shifts the zero dispersion wavelength of the fiber system, and broadens the transmitted laser pulse leading to a decrease in the signal transmission rate. In addition, the spatial variation of the refractive index in the fiber caused by the thermo-optic effect can enhance the scattering of the transmitted laser and hence reduces the transmission power. Therefore, knowledge of the thermo-optic properties of a fiber plays a vital role in designing high-speed fiber communication systems.

The thermo-optic coefficient of optical materials such as an optical fiber or nonlinear crystal is commonly determined by the method of the angle of minimum deviation (AMD) [3-5]. Since the AMD technique gives a value for the refractive index accurate only to the fourth digit after the decimal point, the thermo-optic coefficient is also limited to the same accuracy [6]. However, the thermo-optic coefficients of most materials including optical fibers are on the order of $10^{-5}$-$10^{-6}$. Thus the thermo-optic coefficients determined with AMD technique are
not accurate enough. Recently, we have introduced a more accurate technique, the heterodyne interferometric technique, to determine thermo-optic coefficients of nonlinear crystals [7-8]. In this technique, a multiple-reflected laser from a Fabry-Perot KNbO$_3$ monolithic resonator was mixed with a reference radio-frequency (rf) signal resulting in a heterodyne beats signal. The heterodyne beats signal was related to the phase shift difference between the carrier wave and the two side-band waves of the laser in the monolithic resonator. The phase shift difference was found to be temperature dependent and the thermo-optic coefficients of the KNbO$_3$ crystal along different crystallographic axes were determined from the temperature dependence [8]. In this study, we apply the heterodyne interferometric technique to measure the thermo-optic coefficient of the Corning 28** single mode fiber. The experimental result is compared with that of a simulation using the temperature dependent Sellmeier equation.

II. Experiment

The experimental set up is shown in Fig. 1. The light source is a strained layer quantum well DFB diode laser module (GEC-Marconi LD6439). It provides a stable output wavelength at 1558 nm with a 0.8 MHz line width, and includes an isolator for blocking reflection from the optics down stream. The laser is driven by a dc current and simultaneously modulated by a very weak rf current, $\omega_m = 280$ MHz, from an rf oscillator. Consequently, the diode laser spectrum contains three frequencies: the carrier ($\omega_0$) and two side bands ($\omega_0 \pm \omega_m$). The laser with these three frequencies is first directed through a fiber connected to the laser (pigtail fiber) and then coupled into one end of a $2 \times 2$ coupler with 50% coupling ratio. One of the output ends of the coupler is connected to a fiber Fabry-Perot interferometer (FFPI) sensor; the other is in contact with index-matching oil so that literally no light reflected back to the coupler from this and face. Light reflected from the FFPI propagates through the coupler and falls on the photo-detector (New Focus 1611). The FFPI is fabricated from the Corning 28** single mode fiber, 12 mm in length. Its two end faces are coated with single layer TiO$_2$ with $3 \sim 4\%$ reflectivity. The FFPI is surrounded by a stainless tube, in contact with a thermo-electric cooler for temperature control with a controller (Newport 325).

![Diagram](image-url)

**FIG. 1.** A layout for the heterodyne interferometric measurement of the thermo-optic coefficient of the fiber Fabry-Perot interferometric (FFPI) resonator. Laser traversing the fiber is coupled to the FFPI and then back to the photo-detector.
The photo-detector detects the light signal arising from the heterodyne beating of the multiple-reflected laser beam in the FFPI. A rf amplifier amplifies the $\omega_m$ signal from the photo-detector. The amplified $\omega_m$ signal is first allowed to pass through a band-pass filter and then mixed with a reference signal of the same frequency in a rf mixer. The dc output of the mixer, which is related to the phase shift difference between the carrier and the side bands, is obtained through a low-pass filter. The dc signal is monitored and recorded by an oscilloscope while the temperature of the FFPI sensor is being changed continuously.

III. Results and discussion

Since the modulation depth of the dc current that drives the diode laser is kept small, the spectrum of the diode laser contains a strong carrier at $\omega_0$, two weak side bands at $\omega_0 \pm \omega_m$, and negligibly weak higher order side bands. The optical field of the diode laser output can then be approximated with [9]

$$E(t) \approx E_0 \{ J_0(\beta)e^{i\omega_0 t} + J_1(\beta)e^{i(\omega_0 + \omega_m)t} + J_{-1}(\beta)e^{i(\omega_0 - \omega_m)t} \},$$  

where $E_0$ is the amplitude of the optical field. $J_n(\beta)$ ($n = 0, \pm 1$) is the Bessel function and $\beta$, the frequency modulation index, is experimentally controlled at a very low level, $\beta \ll 1$. For the laser optical field that enters the FFPI (Fig. 1) multiple reflections occur at its end faces of finite reflectivity. The multiple-reflected optical field that leaves the front face of the FFPI is expressed as

$$E_r(t) \approx E_0 \{ T_0(\omega_0)J_0(\beta)e^{i\omega_0 t} + T_1(\omega_0 + \omega_m)J_1(\beta)e^{i(\omega_0 + \omega_m)t}$$

$$+ T_{-1}(\omega_0 - \omega_m)J_{-1}(\beta)e^{i(\omega_0 - \omega_m)t} \}. \quad (2)$$

In this equation $T_n(\omega_n)$ are the complex reflection functions that we write as

$$T_n(\omega_n) = \exp(-\delta_n - i\varphi_n), \quad (3)$$

where $\delta_n$ is the amplitude attenuation and $\varphi_n$ is the optical phase shift at $\omega_n$, and

$$\omega_n = \omega_0 \pm n\omega_m, \quad n = 0, \pm 1. \quad (4)$$

With $\beta \ll 1$ and the $T_n(\omega_n)$ given by equation (3), the optical field that leaves the front face of the FFPI and enters the photo-detector is simplified to be

$$E_r(t) \approx E_0 \{ T_0(\omega_0)e^{i\omega_0 t} + T_1(\omega_1)\frac{\beta}{2}e^{i\omega_1 t} + T_{-1}(\omega_{-1})\frac{-\beta}{2}e^{i\omega_{-1} t} \}. \quad (5)$$

For the optical field impinging upon the photo-detector only the $\omega_m$ frequency component is selected from the detector output by the rf amplifier and the band-pass filter downstream (Fig. 1). As a result, the terms need to be considered in the calculation of the light intensity $I(t)$ detected by the photo-detector are

$$I(t) \approx E_0^2 \beta e^{-\delta_0} \{ e^{-\delta_1} \cos(\varphi_0 - \varphi_1) - e^{-\delta_{-1}} \cos(\varphi_1 - \varphi_0) \} \cos \omega_m t$$

$$+ e^{-\delta_{-1}} \sin(\varphi_1 - \varphi_0) - e^{-\delta_1} \sin(\varphi_0 - \varphi_1) \} \sin \omega_m t \}. \quad (6)$$
This equation signifies two harmonic waves of frequency $\omega_m$, which are mixed with the reference signal (arising from the rf oscillator and the electric phase shifter, Fig. 1). The detected signal output $V(t)$ from the mixer is then

$$V(t) \propto E_0^2 \beta e^{-\delta_0} \left\{ [e^{-\delta_1} \cos(\varphi_0 - \varphi_1) - e^{-\delta_{-1}} \cos(\varphi_{-1} - \varphi_0)] \cos \omega_m t \sin \omega_m t \right.$$  
$$+ [e^{-\delta_{-1}} \sin(\varphi_{-1} - \varphi_0) - e^{-\delta_1} \sin(\varphi_0 - \varphi_1)] \sin^2 \omega_m t \}.$$  

(7)

Of the signal coming from the mixer a low pass filter blocks the ac electrical signal and hence only the dc signal $V(\varphi_n, \delta_n)$ in equation (7) is monitored (by the oscilloscope),

$$V(\varphi_n, \delta_n) \propto \frac{1}{2} E_0^2 e^{-\delta_0} [e^{-\delta_{-1}} \sin(\varphi_{-1} - \varphi_0) - e^{-\delta_1} \sin(\varphi_0 - \varphi_1)].$$  

(8)

This equation shows that the dc signal $V(\varphi_n, \delta_n)$ is related to the optical phase shift difference between the carrier and the two side bands in the FFPI resonator, $\varphi_{-1} - \varphi_0$ and $\varphi_0 - \varphi_1$. Had we chosen the rf reference signal to be $\cos \omega_m t$ we would have to consider the first, rather than the second, term of equation (7) for $V(\varphi_n, \delta_n)$ that would be still related to $\varphi_{-1} - \varphi_0$ and $\varphi_0 - \varphi_1$. It is easy to see from equation (8) that $V(\varphi_n, \delta_n)$ becomes zero when $\delta_{-1} = \delta_1$ and $\sin(\varphi_{-1} - \varphi_0) = \sin(\varphi_0 - \varphi_1)$. The latter are true when the FFPI resonator’s resonance frequency is temperature-tuned to agree with the carrier frequency.

As the laser of frequency $\omega_n$ reflects off the end faces of the FFPI, its amplitude and phase undergo changes. The changes are expressed, respectively, as

$$\exp(-\delta_n) = \frac{r[(1 - \cos \theta_n)^2 + (\sin \theta_n)^2]^{1/2}}{[(1 - r^2 \cos \theta_n)^2 + (r^2 \sin \theta_n)^2]^{1/2}}$$  

(9)

and

$$\varphi_n = \tan^{-1} \left[ \frac{\sin \theta_n}{1 - \cos \theta_n} \right] + \tan^{-1} \left[ \frac{r^2 \sin \theta_n}{1 - r^2 \cos \theta_n} \right].$$  

(10)

In these equations $r$ is the amplitude reflection coefficient. $\theta_n$ is the optical phase shift of the laser after one round trip in the FFPI resonator, and is written as

$$\theta_n = \frac{n_{\omega_n} L \omega_n}{c},$$  

(11)

where $L$ is the round-trip cavity length of the FFPI resonator at room temperature, and $c$ is the speed of light in vacuum. $n_{\omega_n}$ is the refractive index of the FFPI at $\omega_n$ and a given temperature, which we calculate from the temperature dependent Sellmeier equation of the fused silica fiber [2]. With $\theta_n$ given by equation (11) we calculate from equations (9) and (10), respectively, the amplitude attenuation $\delta_n$ and the optical phase shift $\varphi_n$ for a given $r$. Finally, we use equation (8) to calculate the dc beats signal. When the same procedure is repeated for various temperatures, the dc beats signal as a function of temperature results.

The line curve of Figure 2 shows the simulated dc beats signal as a function of temperature without considering thermal expansion effect on the fiber. In this simulation it is assumed that $r = 0.2$, $\beta = 0.1$, and $E_0 = 1.0$. As illustrated in this figure, the simulated dc beats signal
oscillates periodically with temperature. The oscillation period ($\Delta T$) is found to be 6.10 °C. The dots of Figure 2 are the experimental; they also oscillate with temperature with a slightly longer period ($\Delta T'$), 6.51 °C. The small discrepancy between the simulation $\Delta T$ and the experiment one may be due to the doping material difference in the fused silica fiber and the Corning 28** single mode fiber. The large shift between experimental data and simulation result at high temperature is also due to the small discrepancy between the simulation $\Delta T$ and the experiment $\Delta T'$. The oscillation period remains unchanged in the temperature range from 20 °C to 60 °C; so must be the thermo-optic coefficient of the fiber. As illustrated in Fig. 2, the experimental result is in general agreement with the theoretical simulation. We thus conclude that: (1) the temperature dependent Sellmeier equation of fused silica fiber is valid for a description of the thermo-optic property of the Corning 28** single mode fiber, and (2) leaving out thermal expansion contribution to the phase shift in the FFPI, as we do here, is not a bad approximation. Indeed, it is found below that the thermal expansion coefficient of fused silica fiber is one order of magnitude smaller than the thermal optic coefficient.

Since the laser circulating in the FFPI experiences a $2\pi$-phase change in one temperature-oscillation period $\Delta T$, the latter satisfies the following equation,

$$\frac{dn}{dT} \Delta T L + n\alpha L \Delta T = \lambda, \quad (12)$$

where $n$ is the refractive index of the FFPI fiber at $\omega_0$. $\lambda$ is the carrier wavelength of the diode laser. $\alpha$ is the thermal expansion coefficient of fiber. With $\alpha = 0.5 \times 10^{-6}, \degree C^{-1}$, [10] $\Delta T = 6.51 \degree C$, and $n = 1.4488$ at 1558 nm, [11] $dn/dT$ is determined to be $9.2 \times 10^{-6} \degree C^{-1}$ from equation (12). This $dn/dT$ value is very close to the previously reported ones [1, 10-15]. Thus, the heterodyne interferometric technique provides, with good accuracy, a more convenient way to determine the thermo-optic coefficient of optical fiber. The thermo-optic coefficient at other wavelength can be easily obtained by just changing the laser wavelength and coating on the FFPI.

![FIG. 2. The dc beats signal vs. temperature: line, theoretical simulation; dots, experimental result.](image)
IV. Conclusion

The thermo-optic coefficient of a single mode fiber is measured with a novel heterodyne interferometric technique. A theoretical simulation based on the temperature dependent fused silica Sellmeier equation supports the result, which is in close agreement with documented values. For determining the thermo-optic coefficient of fiber this technique is more convenient and accurate to use than the conventional angle of minimum deviation.

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