The QCD Analysis of the Structure Functions and Effective Nucleon Mass

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On the basis of the target mass corrections to structure functions of deep-inelastic scattering of leptons, we evaluate effective nucleon mass that turns out to be twice $M_{\text{nuc.}}$ for deep-inelastic scattering on the nucleus target and equals $M_{\text{nuc.}}$ for the hydrogen target.

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Deep-inelastic scattering of leptons provides a precise information on structure functions (SF) of a nucleon. It is well known that when target mass corrections (TMC) are taken into account, the QCD description of the SF of deep-inelastic scattering is improved. This effect is of order $\frac{A^4 z_{\text{clus.}}}{Q^2}$. In this article, we are going to consider the question whether the mass of a nucleon is the best value for the description of data or in order to make the fit better, one has to use another value $M_{\text{eff.}}$ which could differ from the mass of nucleon.

The Nachtmann moments $[1]$ of SF $F_2$ and $F_3$ are defined as:

$$M_2^{QCD}(N, Q^2) = \int_0^1 \frac{dx \xi^{N+1}}{x^3} F_2(x, Q^2) \frac{3 + 3(N + 1)V + N(N + 2)V^2}{(N + 2)(N + 3)},$$

(1)

$$M_3^{QCD}(N, Q^2) = \int_0^1 \frac{dx \xi^{N+1}}{x^3} F_3(x, Q^2) \frac{1 + (N + 1)V}{(N + 2)},$$

(2)

where

$$\xi = 2x/(1 + V), \quad V = \sqrt{1 + 4M_{\text{nuc.}}^2 x^2/Q^2}.$$ 

(3)

Equations (1,2), could be expanded into a series in powers of $M_{\text{nuc.}}^2/Q^2$. Retaining only the terms of the order $M_{\text{nuc.}}^2/Q^2$ one could obtain:

\[ M_2(N,Q^2) = M_2^{QCD}(N,Q^2) + \frac{N(N-1)}{N+2} \frac{M_2^{QCD}(N+2,Q^2)}{Q^2}, \]
\[ M_3(N,Q^2) = M_3^{QCD}(N,Q^2) + \frac{N(N+1)}{N+2} \frac{M_2^{QCD}(N+2,Q^2)}{Q^2}. \]

\( M_2(N,Q^2) \) and \( M_3(N,Q^2) \) are the Mellin moments of the measured SF \( F_2 \) and \( zF_3 \):

\[ M_2(N,Q^2) = \int_0^1 dx x^{N-2} F_2(x,Q^2), \]
\[ M_3(N,Q^2) = \int_0^1 dx x^{N-2} zF_3(x,Q^2), \quad N = 2,3,... \]

The \( Q^2 \) - evolution of the moments \( M_2^{QCD}(N,Q^2) \) and \( M_3^{QCD}(N,Q^2) \) is given by QCD [2,3]. For the nonsinglet SF:

\[ M_3^{QCD}(N,Q^2) = \left[ \frac{\sigma_3(Q^2)}{\sigma_3(Q_0^2)} \right]^{4N} M_3^{QCD}(N,Q_0^2), \quad N = 2,3,... \]
\[ d_N = \gamma_N^{[0]NS} / 2 \beta_0, \quad \beta_0 = (11 - \frac{2}{3} f). \]
\[ \gamma_N^{[0]NS} = \frac{8}{3} \left[ 1 - \frac{2}{N(N+1)} + 4 \sum_{j=2}^{N} \frac{1}{j} \right]. \]

The unknown coefficients \( M_3^{QCD}(N,Q^2) \) in (8) could be parametrised as the Mellin moments of some function:

\[ M_3^{QCD}(N,Q^2) = \int_0^1 dx x^{N-2} A x^\alpha (1-x)^\alpha \gamma_x (1 + \gamma_x), \quad N = 2,3,... \]

where the constants \( A, b, c \) and \( \gamma \) should be determined from the fit of data.

Having in hand the moments (5,8) and following the method [4,5], we can write the structure function \( zF_3 \) in the form:

\[ zF_3^{N_{max}}(x,Q^2) = x^\alpha (1-x)^\beta \sum_{n=0}^{N_{max}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) M_j^{N_{max}}(Q^2), \]

where \( \Theta_n^{\alpha,\beta}(x) \) is a set of Jacobi polynomials and \( c_j^{(n)}(\alpha,\beta) \) are the coefficients of the series of \( \Theta_n^{\alpha,\beta}(x) \) in powers of \( x \):

\[ \Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\beta) x^j. \]
The quantities $N_{\text{max}}, \alpha$ and $\beta$ have to be chosen so as to achieve the most fast convergence of the series on the r.h.s. of Eq. (11) and to reconstruct $zF_3$ with the accuracy required. Following the results of [5] we use $\alpha = 0.12, \beta = 2.0$ and $N_{\text{max}} = 12$. These numbers guarantee accuracy better than $10^{-3}$.

Eq. (11) could be applied to reconstructing SF $F_2(x, Q^2)$ for $0.3 \leq x$ with eq. (1,4) for TMC taken into account.

The parameters $A$, $b$, $c$, $y$ and the parameter $A$ are determined by fitting experimental data. We also consider $M^{*\text{eff}}$ as a free parameter. It should be noted that the parameters $a$, $b$, $c$ and $y$ depend on $Q_i$. We have used experimental points with $Q^2 > 5 GeV^2$ for fitting, in order to avoid high-twist effects and chosen $Q^2_0 = 10 GeV^2$.

The results of concrete calculations made for SF measured in experiments on different targets are presented in Table I.

For the hydrogen target $M^{*\text{eff}}$ reproduces the value of the proton mass. For the iron target the effective mass $M^{*\text{eff}}$ is twice the nucleon mass. The data of the SKAT collaboration on a target which consists of a mixture of Neon and Hydrogen are not precise enough to determine the value of $A$. So following [7] we have fixed $A = 200 MeV$ and found the value of $M^{*\text{eff}}$: a little bit higher than for the hydrogen target. The increasing effective mass of a nucleon on the nucleus target takes place for a nonsinglet fit both for $F_2$ and $zF_0$ SF. It also takes place both for the leading and next to leading order QCD (see results for $zF_3$ data of CCFR in Table I.). The large value of $M^{*\text{eff}}$ found in the QCD fit of data of DIS on nucleon targets could be considered as indirect evidence of the existence of multiquarks clusters [8] or a few-nucleon correlation in a nucleus [9]. It is also compatible with the measured SF at $x > 1$ on DIS of leptons on the nucleus target [10].

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<table>
<thead>
<tr>
<th>Collaboration Reaction</th>
<th>Ref.</th>
<th>$A$ [MeV]</th>
<th>$\chi^2_{f.i.}$</th>
<th>$M^{*\text{eff}}$ [GeV]</th>
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<tbody>
<tr>
<td>BCDMS ($\mu p$) $F_2$</td>
<td>[6]</td>
<td>0.35 &lt; x</td>
<td>130 ± 4</td>
<td>183/223</td>
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<td>SKAT ($\nu Ne,p$) $zF_3$</td>
<td>[8]</td>
<td>0.05 ≤ x</td>
<td>200 (fix.)</td>
<td>25.3/30</td>
</tr>
<tr>
<td>EMC ($\mu Fe$) $F_2$</td>
<td>[9]</td>
<td>0.30 &lt; x</td>
<td>106 ± 26</td>
<td>45.3/45</td>
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<td>CCFR ($\nu Fe$) $F_2$</td>
<td>[9]</td>
<td>0.275 ≤ x</td>
<td>146 ± 12</td>
<td>37.9/81</td>
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<td>CCFR ($\nu Fe$) $zF_3$</td>
<td>[9]</td>
<td>0.015 ≤ x</td>
<td>64.7 ± 21</td>
<td>81.8/81</td>
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<td>CCFR ($\nu Fe$) $zF_3$ NLO</td>
<td>[9]</td>
<td>0.015 ≤ x</td>
<td>116 ± 30</td>
<td>73.4/81</td>
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References