Yang-Mills, Yang-Baxter, and Local Quantum-Group in 4-d Quantum Field Theories; Reminiscences and Reflections

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Through the quantization of the self-dual Yang-Mills system, a prototype of four-dimensional (4-d) field theory, we show that Yang-Baxter equation and local quantum-group (Mopf algebra) are basic structures in the realm of 4-d quantum field theories. We have obtained the nonperturbative results of nontrivial critical exponents. These results are obtained from quantizing a group-valued gauge-invariant local field \( J \), which was first introduced by C. N. Yang in self-dual Yang-Mills system. The quantized \( J \) fields form noncommutative vector-spaces.

On this occasion of celebrating Professor Yang's 70th birthday in Taiwan, I shall reminisce, reflect, and pay tribute.

The universal problems that theoretical physicists confront today are to find nonperturbative solutions; for example: what is the mass-generating mechanism in the electroweak theory? How do we formulate quantum gravity? What is the mass spectrum in Yang-Mills theory? (This question was already raised in the original paper by Yang and Mills and is still unanswered. For the interesting account of an encounter between Yang and Pauli on this question, see Yang's Selected Papers, 1945-1980; with Commentary.) What are the mechanisms for high \( T_c \) superconductivity?

We need to find a better way to do quantum field theory since the conventional way, as done for QED and QCD, fails to give nonperturbative results. The path I have chosen can be called the integrable-system approach. This brings back the memory of the late 1970's. I was impressed with the elegant results in two-dimensional integrable systems which were very much discussed at Professor Yang's Institute at Stony Brook. Ever since then I have been a member of this group of theorists, trying to see if we can bring the lessons learned from 2-d integrable systems to 4-d field theories. I am very happy that we have in time these new quantum-field results to celebrate Professor Yang's 70th birthday. On this occasion of celebrating Professor Yang's 70th birthday in Taiwan, I shall reminisce and reflect while briefly highlight the recent

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results obtained with Itaru Yamanaka. For details, I would refer the reader to our papers and my talk at a conference sequel to this one at Academia Sinica, Taipei.

The main difference between our (Yamanaka and I) current way of doing quantum field theory and the conventional way is that we quantize a group-valued gauge-invariant local-field $J$ rather than the Lie-algebra-valued gauge-dependent local-field+$ as in QED and QCD. The classical $J$-field was introduced in self-dual Yang-Mills system by Professor Yang in 1977. Everything he touches eventually turn into gold. This paper of his probably was not considered too highly by him. It was not collected in his Selected Paper, 1945-1980.

Based upon this $J$-field and Yang's 1977 paper, first with M. K. Prasad and A. Sinha (two postdoctorals then at Professor Yang's Institute; Prasad is now a senior scientist at Livermore and Sinha a professor at Ohio State University) and later with visitors at Professor Yang's Institute, we discovered in the self-dual Yang-Mills system many classical integrable-system characteristics like those in the two-dimensional (2-d) theories: Backlund transformations, non-local conservation laws, spectral-space affine algebra. This work opened for me the door to a wonderful field of research.

As it turned out this $J$-field formulation can be extended to classical full Yang-Mills theory and to classical full conformal supergravity; but one needs to use extended superfield ($n = 3$ for supersymmetric Yang-Mills; $n \geq 5, 6, 7, 8$ for conformal supergravity). My learning and conversion to superspace was not so straight forward in the late 1970's. We were exposed to a lot of discussions on supersymmetry at Professor Yang's Institute. Following the successful work of constructing supergravity by Freedman, van Nieuwenhuizen, and Ferrara, there were many talks on supergravity and supersymmetry. But most of the talks were very complicated and the seminar blackboard was full of unintelligible (at least to me) equations. I decided that was not for me. So when I read Witten's 1978 paper, my mind automatically shut off at the second-half of that paper because it used supersymmetry. In 1983, I was reminded by Volovich's paper about the importance of superfields and the second-half of Witten's 1978 paper. After spending a year working out several results for the full supersymmetric Yang-Mills and its $J$-field formulation, I was convinced that superfields are indeed essential for applying the integrable-system approach to the full Yang-Mills and the gravity theory. Then I had to make a decision whether I should go straight to find ways to quantize the $J$-field or first to study supergravity to see if the $J$-field formulation was possible for it. Retrospectively, I made the right decision to first settle the $J$-formulation for supergravity since the techniques needed to quantize the $J$-field were not ripe yet. After another two years learning superspace and supergravity, with C. S. Lim (then a postdoctoral at Brookhaven and now a professor at Kobe University), we showed that full conformal supergravity has similar structures as self-dual Yang-Mills and full supersymmetric Yang-Mills, in that their equations of motion are consequences of light-light integrability in extended superspace.

Soon after moving to Davis, in 1957 I gave the $J$-formulation to full conformal super-
These experiences made me understand the statements by Professor Yang that it was good that he and Mills did not entangle with the gravity equation, and gravity is a special case of Yang-Mills theory. In gravity all indices are related to derivatives while in the non-Abelian gauge theory of Yang-Mills, some indices are Lie-algebra-generator indices unrelated to those of derivatives. (i.e., gravity is a pure tangent-bundle gauge theory; yet Maxwell and Yang-Mills are principal-bundle gauge theories.) The confining of all indices to derivatives causes a lot of troubles and makes it much harder to construct the J-formulation and its linear system. Recently I learned from Professor Chern that in mathematics, Elie Cartan first introduced this liberation from the tangent bundle to principal bundle in 1932. Of course, Cartan did not have the Yang-Mills equations of motion, which as stated in Yang and Mills paper was purely motivated by physics reasoning. From these experiences I also realized why supergravity looked so impossibly complicated in the 1970's to an outsider like me; people were using components rather than superfields, which is like insisting on using real variables when working with functions of one-complex-variable.

I have since held and am still holding the view that, despite whether supersymmetry will manifest in particle spectra or not, superspace will be an essential language for physics. This is similar to complex number for physics; being an essential tool for quantum mechanics, complex number has no physical realization. (I always chuckle when I advocate this theme. Several years ago at Moscow, after such a statement by me, Professor Okun complained "what do you mean imaginary number has no physical realization. Is not the menu of a Moscow restaurant such a realization?) All these integrable-system approach for 4-d classical field theories has been reviewed in my 1987 Nankai talk.

After years of exploring different ways of doing quantum field theory, recently with Yamanaka, we have finally done it for the J-field in self-dual Yang-Mills system. The results are modern but the procedure used is Dirac's classic method described in his lectures on Quantum Mechanics, Yeshiva University, 1964. It is a classic. It deals with constrained dynamical systems. It is thin, less than one-hundred pages, and so very clearly written that it gives a misleading impression that the subject is simple. It took me many readings. I feel that I still have not mastered the full implications of the first-class constraint, though I have used it and published two PRL papers using Dirac's method. On this occasion of visiting the Academia Sinica, I learned from Professor T.-Y. Wu, President of the Academia Sinica, that Dirac spent years working on quantizing systems with constraints. In 1956, Dirac was one of the visitors at Professor T.-Y. Wu's Institute in Canada for three months. Professor Wu said that Dirac was completely absorbed in working on constrained systems and paid no attention to whatever else in physics was going on.

Another related book by Dirac is Lectures on Quantum Field Theory, Yeshiva University, 1966. In this book, Dirac severely criticized the usual treatment of quantum field theory: "The usual treatment should thus be considered a stopgap, without any lasting future."
a single consistent comprehensive theory. Any special theory that one sets up for dealing with a particular problem should be consistent with this general theory. I do not need to tell you that such a theory has not yet been attained. It is the ultimate goal, toward which all physicists are working. These lectures are intended to show you how far one can go toward this goal at the present time.” I had read through the book. I did not get the deep sense why he was so upset about the usual treatment and how much he had done to his own satisfaction. My personal journey in wanting to develop this integrable-system approach for 4-d quantum field theories had been and is still being motivated by the beauty and power of 2-d integrable systems and the need of finding non-perturbative results in 4-d quantum field theories. With Yamanaka, we have obtained the non-perturbative results of nontrivial critical exponents (fractal dimensions, in more popular terminology; or Hausdorff dimensions, in mathematical terminology); shown that Yang-Baxter equation and quantum-group (Hopf algebra) are basic structures in the realm of 4-d quantum field theory.

Yang-Baxter equations and quantum groups were discovered in 2-d solvable models and integrable systems. Professor Yang discovered the Yang-Baxter equation in the 1-d system of bosons with repulsive b-function interaction. Later, Baxter found similar relations in 2-d solvable models. Now with Yamanaka, we have found in 4-d self-dual Yang-Mills system the Yang-Baxter equation being satisfied by the structure matrix $R$ in the permutation relations (which we call quantum-exchange-algebras following the terminology used by Faddeev in 2-d integrable-systems) of the quantized $J$-fields. We are happy to have, for Professor Yang’s 70th birthday, this paper with a title containing both Yang-Mills and Yang-Baxter. The Yang-Baxter equation satisfied by the our $R$-matrix of 4-d quantum-field theory contains generalizations of those in 1-d and 2-d solvable models. I refer you to our papers for details.

Quantum-group was found and named by L. D. Faddeev and his co-workers, and by Jimbo in 2-d lattice models. It has been identified to be the Hopf algebra by V. Drinfeld, which brings out the general importance of the quantum-group. The quantum-group structure we have discovered in 4-d quantum field theory originates from this general mathematical structure. The quantum $J$-fields form elements of a non-commutative vector-space and their tensor-product vector-spaces have to follow the quantum-group (Hopf algebra) rules. The details of the emergence of our 4-d quantum-group structure are different from those in 2-d lattice models. The quantum group structure we find in the 4-d self-dual Yang-Mills is truly a quantum-field effect of quantizing this group-valued local field $J$. The origin of the quantum group is related to a parameter composed of the planck constant $\hbar$ and other constants which specify the 4-d theory; making this is a new quantum deformation. Whether it is the third deformation, coined by Faddeev after the planck constant $\hbar$ and the light velocity $c$, only time and action will tell.

That $A_{\mu}$ is not a gauge-invariant quantity and causes complications in quantization is only a minor problem. Mandelstam had demonstrated that one can quantize a gauge-invariant loop-
field $\psi[C] = \exp\left( i \oint_C A_\mu d^4 \! x \right)$, though the loop-field $\psi(C)$ is nonlocal and complicated to work with. The more important issue is that quantizing $A_\mu$ so far only leads to perturbative results.

Our results have demonstrated that the integrable-system approach via the local J-field is gauge-invariant and can lead to nonperturbative results. We are certain that Yang-Baxter equations and quantum-group structures will play important roles in obtaining other 4-d nonperturbative physical results, as they have in 2-d theories. We are actively pursuing them.

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