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Error Analysis According to the Polarization Ratio on Measuring the Magnetic Domain of a Magnetic Material Measured by Polarized Neutrons

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A statistic error occurred corresponding to the polarization ratio during analysing the measured data of a magnetic domain by using neutron depolarization is presented. The error bar calculated of a fitting curve is strongly related to the polarization ratio but the relevant reports are rare. Therefore, we tried to understand how the polarization ratio affects the error and find out the optimal polarization ratio for the polarized neutron experiments in order to avoid non-necessary efforts on updating the polarization ratio of a polarized neutron beamline, especially for a ^{3}He polarizer. The accuracy of the data analysis is increased as the polarization ratio increases. However, it's still not necessary to increase the polarization ratio higher than 95% due to the statistical error is already acceptable to the experiments. The experimental result is still barely acceptable (within 5-10% error) after modifying the statistical error within 85% polarization ratio.

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I. INTRODUCTION

In polarized neutron experiments, such as polarized neutron reflectivity (PNR) or neutron depolarization (ND), the polarization ratio of the incoming neutron beam plays an important role on the data analysis of a sample [1–6]. Ideally, a neutron beam with 100% polarization ratio is preferred to avoid the error incurred during data correction. However, in most setup of polarized neutron beamlines, the goal to achieve a very high polarization ratio might not be easily due to the limitation of polarizer operated at non-optimal wavelengths, high harmonics contamination, defects of optical elements, and stray fields from the surrounding materials. To estimate the statistical error due to the insufficient polarized ratio of a beamline is important, especially, when a ^{3}He polarizer is used. In the recent developed ^{3}He polarizer, a polarization ratio of only 70-80% was achieved [7, 8]. For PNR and ND experiments, many previous works [9–11] discussed how to improve the polarization ratio of the incoming neutron beam in order to improve the precision of the physical parameters acquired. A polarization ratio of 99.98% has been achieved already [11]. Several reports also discussed about the importance of the corrections of polarization ratio in the PNR and ND experiments [12–14]. In this work, we will focus on the statistical error

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analysis of ND experiments to understand the certainty of deduced magnetic domain size affected by the limited polarization ratio.

A neutron depolarization technique gives the information of the average magnetic induction and the average magnetic domain size of a magnetic sample [3, 4]. Depolarization factor $D = P/P_0$ is an experimental parameter for analyzing the experimental data, where P_0 and P are the polarization vectors before and after neutrons transmitting the sample. The change of polarization vector during neutrons transmitting the samples is due to the influence of fluctuation in $B(r)$ in the local magnetic induction ΔB around the mean magnetic induction $< B >$. ΔB is denoted as $\Delta B = B(r) - < B >$ [15, 16]. The neutron intensity, the neutron polarization ratio, and the neutron wavelength affect the precision of the experimental data and the measurable domain sizes. The range of magnetic domains that can be measured by ND is from 10 nm up to mm's theoretically, if a good statistical data can be collected [3]. The statistical errors coming from the neutron intensity can be reduced by extending the data collecting time. However, a prolong data collection time is not practical in a low power multipurpose research reactor. In the following, we will take a ND measurement on the W3-ND beamline in the Tsing Hua Open Pool Reactor (THOR, 2 MW) as an example [17]. This ND beamline allows three selected neutron wavelengths, $\lambda =$ 0.16 nm, 0.237 nm, and 0.40 nm to do experiments. The typical error of the neutron wavelength is about 1%. The depolarized neutrons are collected by a cylinder ^{3}He proportional counter. The sample is a Ni-ferrite powder with grain size of about 2 μm.

II. ERROR ANALYSIS

The depolarization factor D in one-dimensional ND experiments related to the polarization ratio can be expressed as following [18]:

$$D = \frac{1 - (It/I_s t)}{P_0}, \tag{1}$$

where I is measured neutron intensity after neutrons propagate through samples; I_s is the intensity of a fully depolarized neutron beam; and t is the data collection time. Since the counting of neutrons distributed as a Poisson distribution function, the statistical error δD of D can be written as [18]:

$$(\delta D)^2 = \frac{(1 - DP_0)(2 - DP_0)}{(P_0)^2 I_s t}. \tag{2}$$

For a 3-dimensional ND experiment, the δD can also be calculated from the depolarization matrix. However, more data collection time is needed to complete each element of a depolarization matrix.

The magnetic domain, d can be calculated from the equation below [19]:

$$d = \frac{-\ln D}{c\lambda^2 (\Delta B)^2 L_W}, \tag{3}$$

where L_W is the effective sample thickness along the neutron trajectory. Error propagation through Eq. (3) is performed.

III. RESULTS AND DISCUSSION

The precision of D in Eq. (1) increases as the data collected time increases. Fig. 1 shows the calculated $\delta D/D$ versus different D under different data collected time if we assume $P_0 = 0.85 \pm 0.01$, $I_s = 400$ counts/s and $I = 220$ counts/s. From Fig. 1, the most suitable data collected time should be at more than 1000 s in order to achieve a 0.05 precision for D from 0.02 to 1.00. The inset of Fig. 1 shows the data collection time needed to obtain $\delta D/D$ to be within 0.05 and 0.10 as functions of polarization ratios. The smaller the polarization ratio, the longer the data collection time is needed to achieve the same error bar.

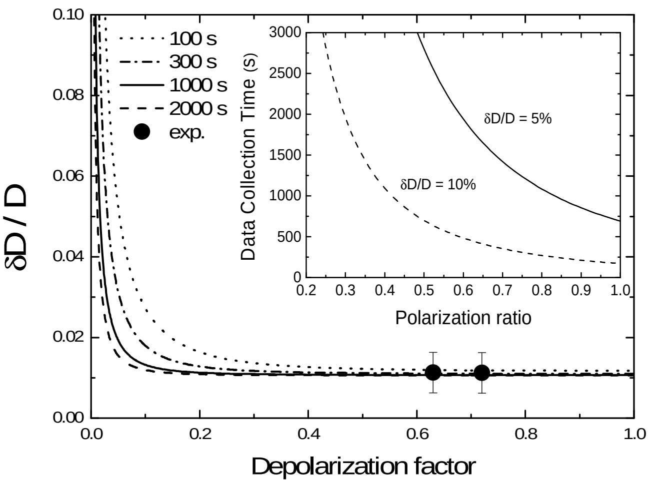

FIG. 1: The calculated $\delta D/D$ versus depolarization factor D under different data collection time. The inset shows the relation between data collection time and polarization ratio at $\delta D/D = 0.05$ and 0.10, respectively.

On the other hand, the data collection time is fixed at 1000 s, and then to observe the $\delta D/D$ changes as a function of polarization ratio. The result is shown in Fig. 2. After rearranging the data, the inset of Fig. 2 shows the statistical error of depolarization factor as functions of polarization ratios. We can see that the precision of D is almost levelling off in the range of polarization ratio from 0.8 to 1.0 within an acceptable (below 0.05) statistical error. The polarization ratio up to 0.8 is already sufficient for a typical ND experiment. In case of neutron experiment with low polarization ratio, the data collection time can be longer to achieve the same statistical uncertainty as those at higher polarization ratios.

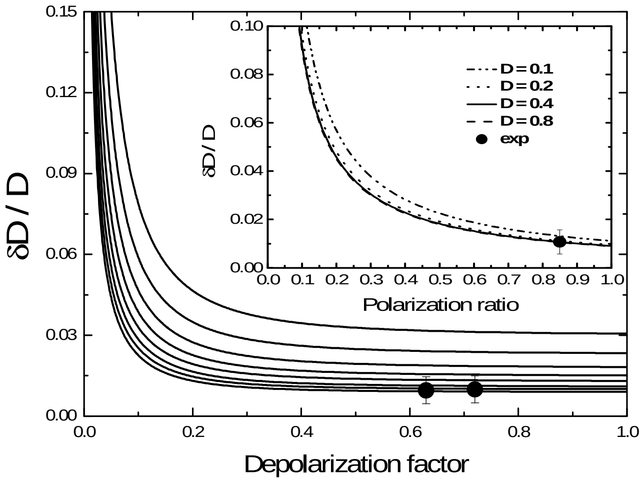

FIG. 2: The calculated $\delta D/D$ versus depolarization factor D under polarization ratios from 0.3 (top curve) to 1.0 (bottom curve). The polarization ratios of each curve from top one to the bottom one are separated by 0.1. The inset shows the relation between $\delta D/D$ and polarization ratio at a constant D.

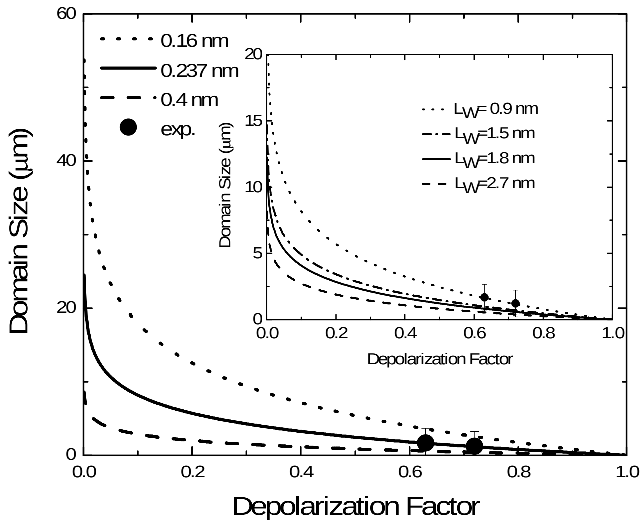

FIG. 3: The calculated magnetic domain versus depolarization factor as a function of wavelength and sample thickness (inset).

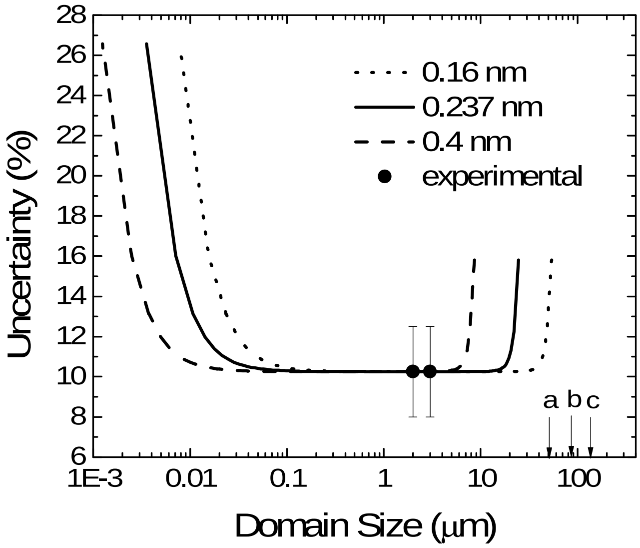

FIG. 4: The uncertainty of the measurable magnetic domain size under different wavelengths. The three arrows in the right-lower part of the figure indicate the upper limits of domain size for a conventional depolarization theory corresponding to λ = (a) 0.40 nm, (b) 0.237 nm, and (c) 0.16 nm, respectively.

An experiment was performed as an example. In this experiment, the thickness of Ni-ferrite powder sample L_W is 0.900 ± 0.009 mm, and the spontaneous induction ΔB is 0.160 ± 0.008 T. The relation between the depolarization factor and the measurable magnetic domain d is shown in Fig. 3 with three selected wavelength, 0.16 nm, 0.237 nm, and 0.4 nm, respectively. The uncertainty of the magnetic domain size is shown in Fig. 4 with data collection time of 1000 s. From Fig. 4, we can see that the measurable magnetic domain size is $0.04 - 20$ μm if the wavelength is 0.237 nm. If we assume that in a conventional depolarization theory, the neutron spin rotation angle is less than 90 degrees in each magnetic domain. Then, the upper limit of the measurable domain size deduced under this restriction is plotted as the arrow sign in Fig. 4.

The measurable magnetic domain size is strongly related to the neutron wavelength in a ND experiment. For example, from Eq. (3), if a neutron wavelength was reduced by a factor 10, the minimum measurable magnetic domain size would be reduced by 100 folds. In fact, from Eq. (3), we can see that the sample thickness can also be changed to extend the measurable domain size if other parameters are fixed. The thicker the sample, the smaller minimum measurable magnetic domain size can be measured provided that the increase of neutron absorption and the resulting small depolarization factor are not an issue (see inset of Fig. 3). In summary, a suitable magnetic domain size can be obtained by choosing a suitable neutron wavelength, sample thickness and typical data collection time under a polarization ratio of larger than 80% in an ND experiment.

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