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Anti-Synchronization of a Class of Delayed Chaotic Neural Networks Via State or Non-State Coupling

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Based on an invariance principle of differential equations, an anti-synchronization method of a class of delayed chaotic neural networks is developed. This anti-synchronization method employs a much smaller feedback strength compared to some known results, and can be achieved without prior knowledge of the connection matrices. Moreover, it is robust against the effect of noise and multi-parameter mismatch. The state and non-state coupling in delayed chaotic neural networks are rigorously analyzed. Some numerical simulations are given to demonstrate the validity of the proposed method.

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I. INTRODUCTION

Twenty years ago, Pecora and Carroll, in their seminal work [1], proposed the concept of synchronization and showed that synchronization in chaotic systems is possible. Since then, this field has blossomed [2, 3] and has been applied like a domino effect in numerous fields, including secure communication [4–10], nano-oscillators [11], neurocomputing [12], biological neurons and oscillators [13–15], human brain and heart activity [16–21], and so on. Due to the above-mentioned and many other potential applications, various methods have been developed to achieve chaos synchronization, such as adaptive synchronization [22–26], complete synchronization [1, 27–29], phase synchronization [30–32], lag synchronization [33–36], generalized synchronization [37–40], anticipating synchronization [41, 42], time-scale synchronization [43, 44], fuzzy synchronization [45–47], impulsive synchronization [48, 49], etc. In the literature, synchronization means that two (or many) chaotic systems adjust their motion to a common behavior.

In fact, the first observation of synchronization by Huygens was an anti-synchronization (also called inverse sychronization) between two oscillators [50]. Anti-synchronization is a process wherein two state vectors of the synchronized systems have the same absolute values but are of opposite signs. Anti-synchronization also has important applicational significance, especially in improving the performance of semiconductor lasers [51, 52], secure communication [53], and human brain activity [21]. Recently, some approaches for chaos anti-synchronization have been proposed [54–58]. Adaptive anti-

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synchronization of chaotic systems without time delay was considered in Refs. [54] and [55]. The authors presented an anti-synchronization method for coupled time-delayed systems without parameter mismatch in Ref. [56]. In Ref. [57], the authors discussed complete synchronization and anti-synchronization in chaotic systems under only one parameter mismatch, but without time delay. In Ref. [58], a design of coupling for complete synchronization and anti-synchronization of a class of time-delayed systems with parameter mismatch and noise perturbation was established, but the added controller is very big.

Since noise and time delay influence are ubiquitous in both nature and artificial systems [59, 60], the study of chaotic anti-synchronization with time delay in noise-perturbed systems is of critical importance. The research in Ref. [61] showed that the noise and the correlation time can influence the threshold and the degree of synchronization, and the influence of different noises is also different. The interplay between the influence of noise and the time delay on the synchronization process in identical and non-identical oscillators was investigated in Ref. [62]. In addition, multi-parameter mismatch is inevitable in practical implementations of a chaos anti-synchronization system. So it is a very challenging task to achieve anti-synchronization in time-delayed chaotic systems with parameter mismatch and noise perturbation using a small controller or coupling strength.

On the other hand, there is increasing interest in the study of chaotic delayed neural networks [63–66]. Theoretical and experimental results have shown that the mammalian brain, which can be seen as a very large delayed neural network, not only can display chaos in its associative memories [67], but also can modulate neuronal synchronization by selective visual attention [68]. It is no longer disputed that the brain is the most complex organ in any living system, and there are various interactions between its millions of neurons. So we have reason to believe that the anti-synchronization of brain activity may have some potential applications.

Recently, Meng *et al.* [69] and Zhu *et al.* [70] researched the anti-synchronization of a class of chaotic delayed neural networks. Meng's method, utilizing the Halanay inequality lemma and Lyapunov stability theory, adds severe constraints between the weight matrices and the controller gain matrix. In Zhu's method, the weight matrices and the controller gain matrix must satisfy a complex LMI (linear matrix inequality). The controller gain matrices are negative definite in the two methods, which leads to conservative conditions. Moreover, they both use a fixed and large controller strength, which means a kind of waste in practice. Finally, parameter mismatch is not considered in their simulations. Huang [71] gave a simple method to achieve synchronization of identical chaotic systems, but he didn't consider multi-parameter mismatch, time delay, and non-state coupling.

Motivated by the above discussion, This paper presents a new anti-synchronization method for a class of chaotic delayed neural networks via state and non-state coupling. The simulation results show that this method is robust against noise-perturbation or multi-parameter mismatch. The rest of this paper is structured as follows. We start with preliminaries in Sec. II. Then, we give the anti-synchronization scheme and its proof in Sec. III. Numerical simulations are shown in Sec. IV. Our conclusion and discussion are given in Sec. V.

II. PRELIMINARIES

In this paper, we consider a class of chaotic delayed neural networks defined by the following delayed differential equations [69, 70]:

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_{ij})), \qquad i = 1, 2, \ldots, n. \tag{1}$$

System (1) can be written in compact form as follows:

$$\dot{\boldsymbol{x}}(t) = -\boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{A} f(\boldsymbol{x}(t)) + \boldsymbol{B} f(\boldsymbol{x}(t-\tau)), \tag{2}$$

where $\boldsymbol{C} = \mathrm{diag}(c_1, c_2, \ldots, c_n)$ with $c_i > 0$, $\boldsymbol{A} = (a_{ij})_{n\times n}$ is the weight matrix, $\boldsymbol{B} = (b_{ij})_{n\times n}$ indicates the delayed connection matrix within the networks with a constant delay τ; $\boldsymbol{x}(t) = [x_1(t), x_1(t), \ldots, x_n(t)]^T \in R^n$ is the state vector of the neural networks, $f(\boldsymbol{x}(t)) = [f_1(x_1(t)), f_2(x_2(t))], \ldots, f_n(x_n(t))]^T : R^n \to R^n$ denotes the activation functions of the neurons.

For the activation functions $f_i(x_i)$ $(i = 1, 2, \ldots, n)$ throughout the paper, we have the following assumption:

Assumption 1(A1): Each activation function $f_i : R \to R$ is an odd function and satisfies the Lipschitz conditions,

$$|f_i(x) - f_i(y)| \leq \quad k_i|x-y|, \quad i = 1, 2, \ldots, n \qquad \forall x, y \in R, \quad x \neq y.$$

It is obvious that this class of delayed neural networks includes some well-known neural networks, such as the Hopfield neural networks and cellular neural networks, with or without delays.

The following lemmas are needed in the proof of our results.

Lemma 1: For any vectors $\boldsymbol{x}, \boldsymbol{y} \in R^n$ and positive definite matrix $P \in R^{n\times n}$, the following matrix inequality holds: $2\boldsymbol{x}^T\boldsymbol{y} \leq \boldsymbol{x}^T P \boldsymbol{x} + \boldsymbol{y}^T P^{-1} \boldsymbol{y}$.

Lemma 2 (La Salle's Invariant Principle) [72]: Let Ω be a bounded closed (compact) set with the property that every solution of the system $\dot{\phi} = \omega(\phi)$ which begins in Ω remains for all future time in Ω. Suppose there is a scalar function $V(\phi)$ which has continuous first partials in Ω and satisfies: 1) $V(\phi) > 0$, for all $\phi \neq 0$; 2) $\dot{V}(\phi) \leq 0$, for all ϕ. Let E be the set of all points where $\dot{V}(\phi) = 0$, and let Γ be the largest invariant set contained in E. Then every solution orbit of the system converges asymptotically to the set Γ as $t \to \infty$.

Notation: In this paper, $\lambda_{\max}(Q)$ denotes the maximal eigenvalue for the symmetric matrix Q.

III. ANTI-SYNCHRONIZATION SCHEME

We take the delayed neural networks described by Eq. (2) as the master system, and the slaver system is given by the following equations:

$$\dot{\boldsymbol{y}}(t) = \quad -\boldsymbol{C}\boldsymbol{y}(t) + \boldsymbol{A} f(\boldsymbol{y}(t)) + \boldsymbol{B} f(\boldsymbol{y}(t-\tau)) + \boldsymbol{u} \otimes (\boldsymbol{x}(t) + \boldsymbol{y}(t)), \tag{3}$$

where $\boldsymbol{y}(t) = [y_1(t), y_1(t), \ldots, y_n(t)]^T$, coupling strength $\boldsymbol{u} = [u_1(t), u_2(t), \ldots, u_n(t)]^T$, and the feedback coupling $\boldsymbol{u} \otimes (\boldsymbol{x}(t) + \boldsymbol{y}(t)) = [u_1(x_1 + y_1), u_2(x_2 + y_2), \ldots, u_n(x_n + y_n)]^T$.

Now we define the anti-synchronization error signal between the master and slaver systems as $\boldsymbol{e}(t) = \boldsymbol{x}(t) + \boldsymbol{y}(t)$. Therefore, we obtain the error dynamical system

$$\dot{\boldsymbol{e}}(t) = -\boldsymbol{C}\boldsymbol{e}(t) + \boldsymbol{A}g(\boldsymbol{e}(t)) + \boldsymbol{B}g(\boldsymbol{e}(t-\tau)) + \boldsymbol{u} \otimes \boldsymbol{e}(t), \tag{4}$$

where $g(\boldsymbol{e}(t)) = f(\boldsymbol{x}(t)) + f(\boldsymbol{e}(t) - \boldsymbol{x}(t))$, $g(\boldsymbol{e}(t-\tau)) = f(\boldsymbol{x}(t-\tau)) + f(\boldsymbol{e}(t-\tau) - \boldsymbol{x}(t-\tau))$.

Next, we will give the anti-synchronization schemes of system (2) and system (3) with state coupling and non-state coupling, respectively.

III-1. With State Coupling

Theorem 1: Assume that (A1) holds, and the coupling strength $\boldsymbol{u} = [u_1, u_2, \ldots, u_n]$ satisfies the following update law:

$$\dot{\boldsymbol{u}}_i(t) = -\xi_i e_i^2(t), \qquad i = 1, 2, \ldots, n, \tag{5}$$

where $\xi_i > 0, i = 1, 2, \ldots, n$ are arbitrary constants, then the coupling master-slaver neural networks system (2) and system (3) can be anti-synchronized.

Proof: Define a Lyapunov function as

$$V(t) = \boldsymbol{e}^T(t)\boldsymbol{e}(t) + \sum_{j=1}^{n} \frac{1}{\xi_i}(u_i + l)^2 + \int_{t-\tau}^{t} g^T(\boldsymbol{e}(s))g(\boldsymbol{e}(s))ds, \tag{6}$$

where l is a constant to be determined.

By calculating the derivative of $V(t)$ along the trajectory of the error system (4), we obtain

$$\begin{aligned}
\dot{V}(t) = \;& 2\boldsymbol{e}^T(t)\dot{\boldsymbol{e}}(t) - 2\boldsymbol{e}^T(t)\boldsymbol{u}\boldsymbol{e}(t) - 2\boldsymbol{e}^T(t)l\boldsymbol{e}(t) \\
& + g^T(\boldsymbol{e}(t))g(\boldsymbol{e}(t)) - g^T(\boldsymbol{e}(t-\tau))g(\boldsymbol{e}(t-\tau)) \\
= \;& 2\boldsymbol{e}^T(t)\{-\boldsymbol{C}\boldsymbol{e}(t) + \boldsymbol{A}g(\boldsymbol{e}(t)) + \boldsymbol{B}g(\boldsymbol{e}(t-\tau)) \\
& + \boldsymbol{u} \otimes \boldsymbol{e}(t)\} - 2\boldsymbol{e}^T(t)\boldsymbol{u}\boldsymbol{e}(t) - 2\boldsymbol{e}^T(t)l\boldsymbol{e}(t) \\
& + g^T(\boldsymbol{e}(t))g(\boldsymbol{e}(t)) - g^T(\boldsymbol{e}(t-\tau))g(\boldsymbol{e}(t-\tau)) \\
= \;& -2\boldsymbol{e}^T(t)\boldsymbol{C}\boldsymbol{e}(t) + 2\boldsymbol{e}^T(t)\boldsymbol{A}g(\boldsymbol{e}(t)) \\
& + 2\boldsymbol{e}^T(t)\boldsymbol{B}g(\boldsymbol{e}(t-\tau)) - 2\boldsymbol{e}^T(t)l\boldsymbol{e}(t) \\
& + g^T(\boldsymbol{e}(t))g(\boldsymbol{e}(t)) - g^T(\boldsymbol{e}(t-\tau))g(\boldsymbol{e}(t-\tau)).
\end{aligned}$$

By Lemma 1, and taking P as the identity matrix, we have

$$\begin{aligned}
\dot{V}(t) \leq \;& -2\boldsymbol{e}^T(t)\boldsymbol{C}\boldsymbol{e}(t) + \boldsymbol{e}^T(t)\boldsymbol{A}\boldsymbol{A}^T\boldsymbol{e}(t) + \boldsymbol{e}^T(t)\boldsymbol{B}\boldsymbol{B}^T\boldsymbol{e}(t) \\
& + 2g^T(\boldsymbol{e}(t))g(\boldsymbol{e}(t)) - 2\boldsymbol{e}^T(t)l\boldsymbol{e}(t).
\end{aligned} \tag{7}$$

The following inequality can be easily deduced from (A1):

$$g^T(\boldsymbol{e}(t))g(\boldsymbol{e}(t)) \leq \boldsymbol{e}^T(t)K\boldsymbol{e}(t), \tag{8}$$

where $K = \max\{k_1^2, k_2^2, \ldots, k_n^2\}$.

Combining the inequalities (7) and (8), we get

$$\dot{V}(t) \leq -\boldsymbol{e}^T(t)\{2\lambda_{\max}(-\boldsymbol{C}) + \lambda_{\max}(\boldsymbol{A}\boldsymbol{A}^T) + \lambda_{\max}(\boldsymbol{B}\boldsymbol{B}^T) + K - 2l\}\boldsymbol{e}(t).$$

If we chose the constant l as follows,

$$l = \lambda_{\max}(-\boldsymbol{C}) + \frac{1}{2}\lambda_{\max}(\boldsymbol{A}\boldsymbol{A}^T) + \frac{1}{2}\lambda_{\max}(\boldsymbol{B}\boldsymbol{B}^T) + \frac{1}{2}K + \frac{1}{2},$$

then we obtain $\dot{V}(t) \leq -\boldsymbol{e}^T(t)\boldsymbol{e}(t)$.

Let $\Omega \subset R^n$ be the chaotic attractor set of the system, then Ω is a bounded closed set. The largest invariance set $\Gamma = \{(\boldsymbol{e}(t), \boldsymbol{u}) \in R^{2n} : \boldsymbol{e}(t) = 0, \boldsymbol{u} = \boldsymbol{u}_0 \in R^n\}$. According to Lemma 2, the orbit of the system converges asymptotically to the set Γ, i.e., $\boldsymbol{e}(t) \to 0$ as $t \to \infty$, which means the anti-synchronization is achieved. The proof is complete.

III-2. With Non-State Coupling

When the master and slaver systems are connected by non-state coupling, i.e., the slaver system has the following form:

$$\dot{\boldsymbol{y}}(t) = -\boldsymbol{C}\boldsymbol{y}(t) + \boldsymbol{A}f(\boldsymbol{y}(t)) + \boldsymbol{B}f(\boldsymbol{y}(t-\tau)) + \boldsymbol{u} \otimes (f(\boldsymbol{x}(t)) + f(\boldsymbol{y}(t))), \tag{9}$$

and the error dynamical system is

$$\dot{\boldsymbol{e}}(t) = -\boldsymbol{C}\boldsymbol{e}(t) + \boldsymbol{A}g(\boldsymbol{e}(t)) + \boldsymbol{B}g(\boldsymbol{e}(t-\tau)) + \boldsymbol{u} \otimes g(\boldsymbol{e}(t)), \tag{10}$$

then we have the following theorem:

Theorem 2: If assumption (A1) holds, the anti-synchronization between the system (2) and system (9) can be achieved, provided the coupling strength $\boldsymbol{u} = [u_1, u_2, \ldots, u_n]$ satisfies the following update law:

$$\dot{\boldsymbol{u}}_i(t) = -\xi_i K' e_i^2(t), \qquad i = 1, 2, \ldots, n, \tag{11}$$

where $\xi_i > 0,\ i = 1, 2, \ldots, n$ are arbitrary constants, $K' = \max\{k_1, k_2, \ldots, k_n\}$.

The proof of Theorem 2 is similar to that of Theorem 1, so we omit it.

Remark 1: This anti-synchronization is a nonlinear globally stability, which means that it is robust against the effect of noise. The constraints between the weight matrices and the coupling gain in Ref. [65] and [66] are too strong. There are no such constraints in our method. This implies that the method is effective under a slight multi-parameter mismatch.

Remark 2: Compared to the methods in Ref. [65] and [66], the boundedness of the activation function and negative definiteness of the controller matrix are not required here. In addition, the coupling strength $\boldsymbol{u}$ will be automatically adapted to a suitable strength to achieve the anti-synchronization, which is very different from that in Ref. [65] and [66] where a fixed strength is used.

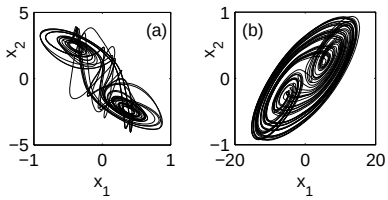

FIG. 1: Chaotic Attractors: (a) DHNN, (b) DCNN.

IV. NUMERICAL SIMULATIONS

In order to demonstrate the performance of the proposed method, a couple of examples are presented in this section.

Example 1: Consider a two-order delayed Hopfield neural network (DHNN) [64] described by the following equation:

$$\dot{\boldsymbol{x}}(t) = -\boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{A}f(\boldsymbol{x}(t)) + \boldsymbol{B}f(\boldsymbol{x}(t-1)), \tag{12}$$

where

$$\boldsymbol{x}(t) = [x_1(t), x_2(t)]^T, f(\boldsymbol{x}(t)) = [\tanh(x_1(t)), \tanh(x_2(t))]^T.$$

For the sake of comparing the results of the current study with those in Ref. [65] and [66], we use the same parameter matrices as follows:

$$\boldsymbol{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \boldsymbol{A} = \begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 3.0 \end{pmatrix}, \qquad \boldsymbol{B} = \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{pmatrix}.$$

Obviously, the system satisfies assumption (A1). Fig. 1(a) shows the chaotic behavior of the system with initial condition $[x_1(t_0), x_2(t_0)]^T = [0.4, 0.6]^T$.

First, consider the state coupling. The slaver system is defined by

$$\dot{\boldsymbol{y}}(t) = -\boldsymbol{C}\boldsymbol{y}(t) + \boldsymbol{A}f(\boldsymbol{y}(t)) + \boldsymbol{B}f(\boldsymbol{y}(t-\tau)) + \boldsymbol{u} \otimes (\boldsymbol{x}(t) + \boldsymbol{y}(t)). \tag{13}$$

In the following simulations, we take the initial conditions as $[x_1(t_0), x_2(t_0)]^T = [0.4, 0.6]^T$, $[y_1(t_0), y_2(t_0)]^T = [0.35, 0.55]^T$, $u_i(0) = 0,\ i = 1, 2$ and set $\xi_1 = 3.0,\ \xi_2 = 0.1$. Fig. 2 shows the evolution of the anti-synchronization errors $e_1,\ e_2$ and the coupling strengths $u_1,\ u_2$.

Remark 3: The maximal value of $\boldsymbol{u}$ are $u_1 = -1.66,\ u_2 = -1.86$. This is equivalent to the controller gain matrix in Ref. [65] and [66] being chosen as

$$\boldsymbol{\Omega} = \begin{pmatrix} -1.66 & 0 \\ 0 & -1.86 \end{pmatrix}.$$

It is easy to find that this $\boldsymbol{\Omega}$ fails to meet the condition of Theorem 1 in Ref. [65], and there is no strictly feasible solution to the LMI of Corollary 1 in Ref. [66]. Moreover, our coupling strength is smaller than the controller in Ref. [65] and [66].

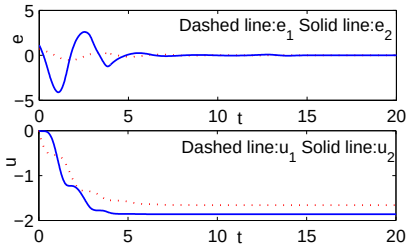

FIG. 2: (Color online) The anti-synchronization errors and coupling strength of DHNN.

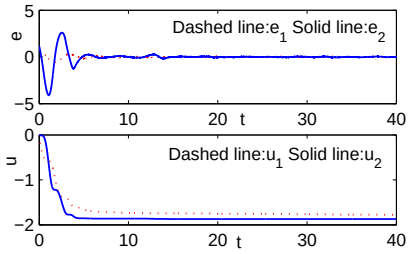

FIG. 3: (Color online) The anti-synchronization errors and coupling strength of DHNN under the effect of random noise with the signal-to-noise ratio of 5:3.

The additive noise with signal-to-noise ratio of 5:3 in time series x_1 and x_2 is added, and the corresponding numerical simulation is given in Fig. 3, which shows that the proposed method is robust against the effect of noise.

Now, we consider the slaver system with multi-parameter mismatch defined by

$$\begin{aligned}\dot{\boldsymbol{y}}(t) = & -\boldsymbol{C}\boldsymbol{y}(t) + (\boldsymbol{A} + \triangle\boldsymbol{A})f(\boldsymbol{y}(t)) \\ & +(\boldsymbol{B} + \triangle\boldsymbol{B})f(\boldsymbol{y}(t-\tau)) + \boldsymbol{u} \otimes (\boldsymbol{x}(t) + \boldsymbol{y}(t)),\end{aligned} \tag{14}$$

where $\triangle\boldsymbol{A}$ and $\triangle\boldsymbol{B}$ are the mismatches in the parameters. Take $\xi_1 = 3$, $\xi_2 = 0.6$, and

$$\triangle\boldsymbol{A} = \begin{pmatrix} 0.1 & -0.02 \\ -0.1 & 0.05 \end{pmatrix}, \qquad \triangle\boldsymbol{B} = \begin{pmatrix} -0.06 & -0.03 \\ -0.1 & -0.1 \end{pmatrix}.$$

As is clear from Fig. 4, the anti-synchronization can be achieved under multi-parameter mismatch by increasing the value of ξ_2. The maximal coupling strength $u_1 = -2.121, u_2 = -4.227$ is bigger than those in Fig. 2, which is a result of the multi-parameter mismatch.

Next, consider the non-state coupling. The slaver system without and with multi-parameter mismatch are given as

$$\dot{\boldsymbol{y}}(t) = -\boldsymbol{C}\boldsymbol{y}(t) + \boldsymbol{A}f(\boldsymbol{y}(t)) + \boldsymbol{B}f(\boldsymbol{y}(t-\tau)) + \boldsymbol{u} \otimes (f(\boldsymbol{x}(t)) + f(\boldsymbol{y}(t))), \tag{15}$$

$$\begin{aligned}\dot{\boldsymbol{y}}(t) = - & \boldsymbol{C}\boldsymbol{y}(t) + (\boldsymbol{A} + \triangle\boldsymbol{A})f(\boldsymbol{y}(t)) + (\boldsymbol{B} + \triangle\boldsymbol{B}) \\ & f(\boldsymbol{y}(t-\tau)) + \boldsymbol{u} \otimes (\boldsymbol{f}(x(t)) + f(\boldsymbol{y}(t))).\end{aligned} \tag{16}$$

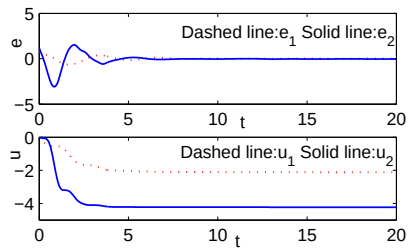

FIG. 4: (Color online) The anti-synchronization errors and coupling strength of DHNN with multi-parameter mismatch.

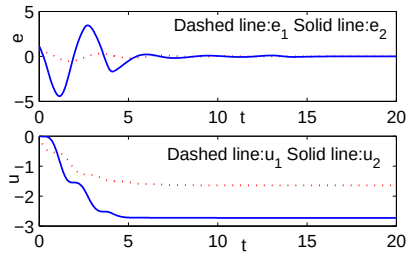

FIG. 5: (Color online) The anti-synchronization errors and coupling strength of DHNN via non-state coupling.

Using the same parameter set in the state coupling, we obtain the anti-synchronization process shown in Fig. 5. Although the coupling strength $u_1 = -1.643, u_2 = -2.728$ is a little bigger than those in state coupling, they still fail to meet the conditions in Ref. [65] and [66].

We consider uniform random noise with the range $[-0.6, 0.6]$ added to x_1 and x_2, the simulation results are shown in Fig. 6.

Let the system (16) have the same parameter mismatch value as the system (14), and set $\xi_1 = 6, \xi_2 = 0.6$, the anti-synchronization errors and coupling strength are shown

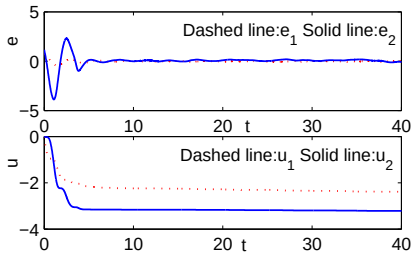

FIG. 6: (Color online) The anti-synchronization errors and coupling strength of DHNN via non-state coupling under the effect of noise.

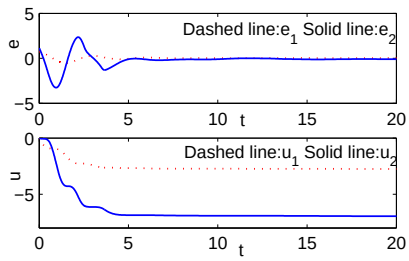

FIG. 7: (Color online) The anti-synchronization errors and coupling strength of DHNN via non-state coupling under multi-parameter mismatch.

in Fig. 7.

Remark 4: We find that the values of the coupling strength and the ξ_i in anti-synchronization via non-state coupling are somewhat bigger than those for state coupling. The reason may be that the signals are compressed by the activation function $\tanh(x)$, and thus have smaller magnitudes than the original signals.

Example 2: Consider a two-order delayed cellular neural network (DCNN) [63] as below:

$$\dot{\boldsymbol{x}}(t) = -\boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{A}f(\boldsymbol{x}(t)) + \boldsymbol{B}f(\boldsymbol{x}(t-1)), \tag{17}$$

where

$$\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \qquad f(\boldsymbol{x}(t)) = \begin{bmatrix} (|x_1+1| - |x_1-1|)/2 \\ (|x_2+1| - |x_2-1|)/2 \end{bmatrix}.$$

Take the same parameter matrices as in Ref. [65] and [66]:

$$\boldsymbol{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \boldsymbol{A} = \begin{pmatrix} 1+\pi/4 & 20 \\ 0.1 & 1+\pi/4 \end{pmatrix},$$

$$\boldsymbol{B} = \begin{pmatrix} -\sqrt{2}(\pi/4)1.3 & -0.1 \\ 0.1 & -\sqrt{2}(\pi/4)1.3 \end{pmatrix}.$$

The chaotic attractor can be seen in Fig. 1(b). The corresponding slaver systems have the same form as that in the forward example 1. The initial conditions of the master and slaver system are $[x_1(t_0), x_2(t_0)]^T = [0.1, 0.1]^T$, $[y_1(t_0), y_2(t_0)]^T = [0.15, -0.15]^T$.

The simulation results of this example are provided in Figs. 8–13.

The parameter mismatch in Fig. 9 and Fig. 12 are taken as follows respectively:

$$\triangle\boldsymbol{A} = \begin{pmatrix} 0.02 & 0.5 \\ 0.03 & 0.03 \end{pmatrix}, \qquad \triangle\boldsymbol{B} = \begin{pmatrix} -0.02 & -0.05 \\ 0.04 & -0.02 \end{pmatrix},$$

$$\triangle\boldsymbol{A} = \begin{pmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{pmatrix}, \qquad \triangle\boldsymbol{B} = \begin{pmatrix} -0.02 & -0.02 \\ 0.02 & -0.02 \end{pmatrix}.$$

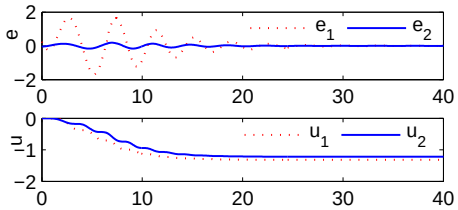

FIG. 8: (Color online) The anti-synchronization errors and coupling strength of DCNN via state coupling, setting $\xi_1 = 0.1,\ \xi_2 = 8$.

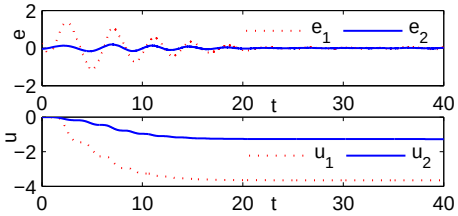

FIG. 9: (Color online) The anti-synchronization errors and coupling strength of DCNN via state coupling under uniform random noise with the range $[-0.6, 0.6]$, setting $\xi_1 = 0.6,\ \xi_2 = 8$.

V. CONCLUSION AND DISCUSSION

1) An anti-synchronization method of a class of delayed chaotic neural networks is presented. This anti-synchronization method employs a small feedback strength which fails to meet the conditions in some previous literature. The method is robust against the effect of noise and multi-parameter mismatch. The state and non-state coupling in delayed chaotic neural networks both are effective to anti-synchronize the chaotic neural networks. Two numerical simulations demonstrate that the proposed method is valid.

2) The parameter $\boldsymbol{\xi}$ used in the two examples may not be optimal. The small coupling strength may be obtained by adjusting appropriately the value of $\boldsymbol{\xi}$. However, how $\boldsymbol{\xi}$ affects the convergence rate of the error system and the coupling strength still requires further study.

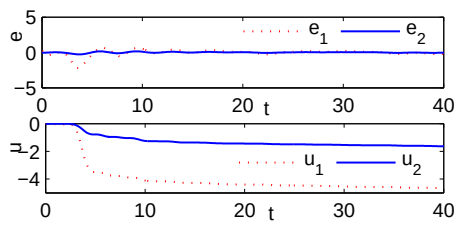

FIG. 10: (Color online) The anti-synchronization errors and coupling strength of DCNN via state coupling under multi-parameter mismatch, setting $\xi_1 = 0.6,\ \xi_2 = 8$.

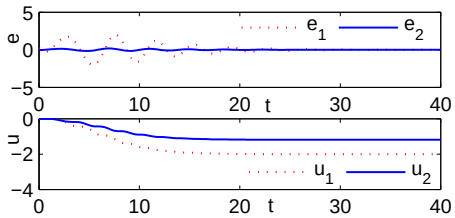

FIG. 11: (Color online) The anti-synchronization errors and coupling strength of DCNN via non-state coupling, setting $\xi_1 = 0.1,\ \xi_2 = 8$.

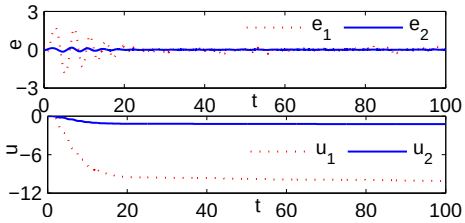

FIG. 12: (Color online) The anti-synchronization errors and coupling strength of DCNN via non-state coupling under uniform random noise with the range $[-0.2, 0.2]$, setting $\xi_1 = 0.6,\ \xi_2 = 10$.

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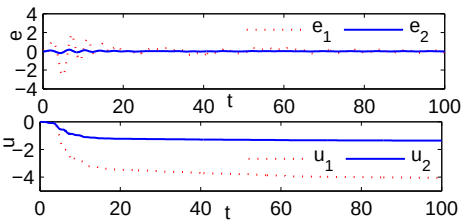

FIG. 13: (Color online) The anti-synchronization errors and coupling strength of DCNN via non-state coupling under multi-parameter mismatch, setting $\xi_1 = 0.2,\ \xi_2 = 10$.

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