Entropic Stochastic Resonance with Dichotomous Noise and White Noise

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The entropic stochastic resonance (ESR) in a confined system driven by a periodic force as well as dichotomous noise and white noise is investigated. For the case of low frequency and weak amplitude of the periodic force, we obtain the expression of the output signal-to-noise ratio (SNR). Results shows that the SNR is a non-monotonic function of the strengths of the dichotomous noise and the white noise; the SNR varies non-monotonically with an increase of the parameters ε and b of the confined structure. The influence of the correlation rate of the dichotomous noise and the frequency of the periodic force on the SNR is also discussed.

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I. INTRODUCTION

The phenomenon of stochastic resonance began with the study of climatic dynamics. Benzi *et al.* [1] invoked this phenomenon to explain the change of the Earth's climate: the eccentricity of the Earth's orbit varies periodically in time, and if the amplitude of this variation is too small to explain the succession of ice ages and relatively warm periods, the periodical phenomenon is amplified by some perturbations. This cooperative effect between the coherent 'signal' and the 'noise' was called stochastic resonance (SR). SR has been investigated by many researchers. McNamara *et al.* [2–3] observed SR by a bidirectional ring laser and obtained the expression of the signal-to-noise (SNR) in the adiabatic limit. Linear-response theory and perturbation theory were introduced to investigate SR [4, 5]. Zhou and Moss [6] employed the residence time distribution to explain SR as a resonance synchronization phenomenon. SR is considered as a cooperative result of a periodic signal and noise in a nonlinear system. The SR phenomenon has been studied in a variety of nonlinear systems with additive and multiplicative noises [1–6]. It was concluded that nonlinearity with a periodic and a random force were the essential ingredients for the onset of SR. On the other hand, the existence of SR was also found in linear systems [7–8]. It was suggested that noise multiplicativity and time correlation are the necessary conditions for the SR to occur in linear systems.

The study of SR has mainly been focused on systems with purely energetic potentials [1–8]. However, there are also situations frequently found in systems, such as soft

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condensed matter and biological systems, in which particles move in constrained regions such as small cavities, pores, or channels whose presence and shape play an important role for the SR dynamics [9–13]. Burada *et al.* [14–15] presented a novel scheme for the appearance of stochastic resonance when the dynamics of a Brownian particle takes place in a confined medium. The presence of uneven boundaries giving rise to an entropic contribution to the potential may, upon application of a periodic driving force, result in an increase of the spectral amplification at an optimum value of the ambient noise level. They called this phenomenon entropic stochastic resonance (ESR). Zhao *et al.* [16] investigated the phenomenon of ESR in a two-dimensional confined system driven by a transverse periodic force when colored fluctuation is included in the system.

In actual systems, additive white noise and dichotomous noise may simultaneously exist. Additive thermal fluctuations exist inherently at nonzero temperatures of the system, while dichotomous noise can be formed by quasiparticles (defects, impurities, spins, etc.) jumping between two-level systems [17–19]. For example, a digital circuit may be perturbed by Gaussian white noise resulting from the background noise around the circuit; meanwhile, the digital circuit can also be disturbed by random telephone noise (one form of dichotomous noise), which is induced by other digital circuits close to it. Dichotomous noise is widely studied in physical, chemical, biological, etc. problems, which can be generated by a two-state Poisson process. Because of its relatively simple calculation and well-defined limiting procedures leading to both Gaussian white noise and white shot noise, colored dichotomous noise has been paid more and more attention [20–22]. Therefore, the study of a system with white noise and dichotomous noise is of practical significance.

The non-monotonic effect of external force and the strength of additive white noise on the output signal of a confined system have been investigated [14–16], however, to the best of our knowledge, little research has been focused on how dichotomous noise and white noise, as well as the parameters of a confined structure, influence the output signal in a confined system. Therefore, the motivation of this paper is to investigate the ESR phenomenon in a confined system with dichotomous and additive white noise driven by a transversal constant force and a longitudinal periodic force.

II. THE CONFINED SYSTEM AND ITS OUTPUT SIGNAL-TO-NOISE RATIO

When a periodic force $F(t)$ along the x-axis of the structure and a constant force G in the transversal direction y are imposed on a Brownian particle in a confined geometry, as depicted in Fig. 1, the overdamping dynamics of the system can be described by the Langevin equation

$$\gamma\frac{d\vec{r}}{dt} = -G\vec{e}_y - F(t)\vec{e}_x + \sqrt{\gamma k_B T}\vec{\xi}(t) + B_0\vec{\eta}(t), \tag{1}$$

where $\vec{r}$ means the position of the particle, γ is the friction coefficient, k_B is the Boltzmann constant, T denotes the temperature of the heat bath, and $\vec{e}_x$ and $\vec{e}_y$ the unit vectors along the x and y directions, respectively. $F(t)$ is the longitudinal force, $F(t) = A\cos(\Omega t)$, with amplitude A and frequency Ω. $\vec{\xi}(t)$ is a Gaussian white noise with zero mean and variance

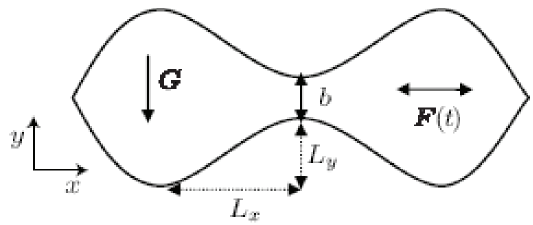

FIG. 1: Schematic diagram of the two-dimensional structure, a Brownian particle moves in the confined medium.

$\langle \xi_i(t)\xi_j(t_1)\rangle = 2\delta_{ij}\delta(t-t_1)$ for $i, j = x, y$. $\vec{\eta}(t)$ is a dichotomous noise, uncorrelated with $\vec{\xi}(t)$, with unit amplitude $\eta_\pm = \pm 1$ and correlation rate λ. The white noise $\vec{\xi}(t)$ is the thermal noise of the system, which is responsible for stochastic switching between metastable states. The dichotomous noise $\vec{\eta}(t)$ denotes an external stochastic force, influencing the switching events between states of the system. We assume that the magnitude of the dichotomous noise is small enough so that it cannot induce transitions between metastable states of the system by itself. It was shown that the addition of dichotomous noise leads to dramatic changes in a bistable system [23, 24]. In particular, dichotomous noise can synchronize the switching events, so that the mean switching rate of the system is locked and equal to the flipping rate of dichotomous noise in a finite range of thermal noise intensity.

For the structure depicted in Fig. 1, the walls are defined by

$$w_l(x) = L_y\,(x/L_x)^4 - 2L_y\,(x/L_x)^2 - \frac{b}{2} = -w_u(x), \tag{2}$$

where w_l and w_u correspond to the lower and the upper boundary functions, respectively, L_x denotes the distance between bottleneck position and position of maximal width, L_y refers to the narrowing of the boundary functions, and b the remaining width at the bottleneck; $2w(x) = w_u(x) - w_l(x)$ gives the local width of the structure. For the sake of a dimensionless description, we scale all lengths by the characteristic length L_x, i.e., $\tilde{x} = x/L_x$, $\tilde{y} = y/L_x$, $\tilde{b} = b/L_x$, and $\tilde{w}_l = w_l/L_x = -\tilde{w}_u$. Supposing an irrelevant reference temperature T_R and time in the units of $\tau = \gamma L_x^2/k_B T_R$, then the particle takes twice the time to diffuse a distance L_x at temperature T_R, i.e., $\tilde{t} = t/\tau$ and $\tilde{\Omega} = \Omega\tau$. We scale forces by $F_R = \gamma L_x/\tau$, so the orthogonal force is $\tilde{G} = G/F_R$ and the sinusoidal force $\tilde{F}(\tilde{t}) = F(t)/F_R$. In the following we shall omit the tilde symbols for better legibility. In dimensionless form the Langevin equation (1) and the boundary function (2) read:

$$\frac{d\vec{r}}{dt} = -G\vec{e}_y - F(t)\vec{e}_x + \sqrt{D}\vec{\xi}(t) + B\vec{\eta}(t), \tag{3}$$

$$w_l(x) = -w_u(x) = \varepsilon x^4 - 2\varepsilon x^2 - b/2, \tag{4}$$

where we defined the aspect ratio $\varepsilon = L_y/L_x$ and the dimensionless temperature $D = T/T_R$ and dichotomous noise amplitude $B = B_0/T_R$. Making use of the symmetry of the structure, i.e., $w_u(x) = -w_l(x)$ and the definition of the half width function $w(x) = (w_u - w_l)/2$, the potential function becomes [14, 15]

$$U(x) = -D\ln\left[\frac{2D}{G}\sinh\left(\frac{Gw(x)}{D}\right)\right]. \tag{5}$$

The master equation of the system can be described as [23, 24]

$$\frac{d}{dt}\rho(x,\eta) = -W_0(x_+,\eta_+)\rho(x_+,\eta_+)+W_0(x_-,\eta_+)\rho(x_-,\eta_+)+\lambda[\rho(x_+,\eta_-)-\rho(x_+,\eta_+)], \tag{6}$$

where

$$W_0(x_\pm,\eta_\pm) = \exp\left(-\frac{\Delta U + x_\pm\eta_\pm B}{D}\right) \tag{7}$$

and $x_\pm = \pm 1$ indicates the position of the minimum of the symmetric potential $U(x)$.

The mean switching frequency of the output $x(t)$ is given by [23, 24]

$$\langle W\rangle_{\mathrm{out}} = \frac{\pi}{2}\left[a_1 + a_2 - \frac{(a_2-a_1)^2}{a_1+a_2+2\lambda}\right], \tag{8}$$

with the rates

$$a_{1,2} = \exp[-(\Delta U \pm B)/D]. \tag{9}$$

For the case of a sufficiently slow and weak harmonic force, one can obtain the modified rates [23, 24]

$$W(x,\eta) = W_0(x,\eta)\exp[-A\eta\cos(\Omega t)/D]. \tag{10}$$

From the master equation (6) with the modified rates (10), the autocorrelation function can be obtained as

$$\begin{aligned}\tfrac{d}{dt}\langle x(t)x(t_1)\rangle &= -(a_1+a_2)\langle x(t)x(t_1)\rangle + (a_2-a_1)\langle \eta(t)x(t_1)\rangle\\ &\quad + [(a_1+a_2)\langle x(t_1)\rangle - (a_2-a_1)\langle x(t)\eta(t)x(t_1)\rangle]A\cos(\Omega t)/D.\end{aligned} \tag{11}$$

The equation for the cross-correlation function reads

$$\frac{d}{d}\langle \eta(t_1)x(t_2)\rangle = -2\lambda\langle \eta(t_1)x(t_2)\rangle. \tag{12}$$

After a Fourier transform, one obtains the power spectrum

$$S(\omega) = N(\omega) + A^2\pi M\delta(\omega-\Omega). \tag{13}$$

Here $N(\omega)$ is the power spectrum of the background

$$N(\omega) = -\frac{a_1 + a_2}{(a_1 + a_2)^2 + \omega^2}\left[1 + \frac{(a_2 - a_1)^2}{4\lambda^2 + \omega^2}\right] - \frac{4(a_2 - a_1)^2}{(a_1 + a_2 + 2\lambda)(4\lambda^2 + \omega^2)}, \tag{14}$$

and M is the spectral power amplification

$$M = \frac{4}{\pi^2 D^2}\frac{\langle W\rangle_{\mathrm{out}}^2}{(a_1 + a_2)^2 + \Omega^2}. \tag{15}$$

The signal-to-noise ratio is defined as

$$SNR = \frac{\pi M}{N(\Omega)}. \tag{16}$$

III. DISCUSSION AND CONCLUSION

Up to now, we have obtained the expression of the signal-to-noise ratio. Now let us discuss the influence of the noises and the parameters of the symmetric structure on the signal-to-noise ratio and draw some conclusions.

For a bistable system with additive noise, when a small periodic modulation is applied to the potential at a modulation frequency much smaller than the intrawell relaxation rate, the thermal activation rates are modulated periodically in time. For some optimal amount of noise level, the transitions between the two wells also occur almost periodically in time, the timescale associated with the Kramers rate equals approximately half the signal period. In this case the SR takes place. Entropic stochastic resonance is a kind of SR. Several quantifiers have been used to characterize SR in noisy, continuous systems. The average output amplitude, or the spectral amplification (SPA), has been studied in [25, 26] and the phase of the output average in [27–29], respectively. The signal-to-noise ratio (SNR) is usually used in physical systems [2, 3], thus we use the SNR to characterize the ESR in our model.

The ESR phenomenon is observed in a confined structure subject to white noise and colored noise [14–16]. It is demonstrated that the spectral amplification shows non-monotonic behavior when it is plotted as a function of the intensity of the white or colored noise, and as a function of the amplitude of the periodic force. In the present paper, despite a white noise, we consider simultaneously a dichotomous noise for the confined system. From Fig. 2, we see that each of the SNR curves has a peak with the increment of the dichotomous noise amplitude B, this means that dichotomous noise can also cause resonant like behavior. In addition, the SNR decreases with an increment of the frequency of the driving force, a similar effect to that occurs in Refs. [14–16].

We analyze the influence of the intensity of the additive white noise on the system SNR in Fig. 3. From this figure, one can conclude that the ESR phenomenon occurs on the SNR curves, which is a similar result to that observed in Refs. [14, 15] . At the same time, the SNR increases with the increase of the correlation rate λ of the dichotomous noise. The

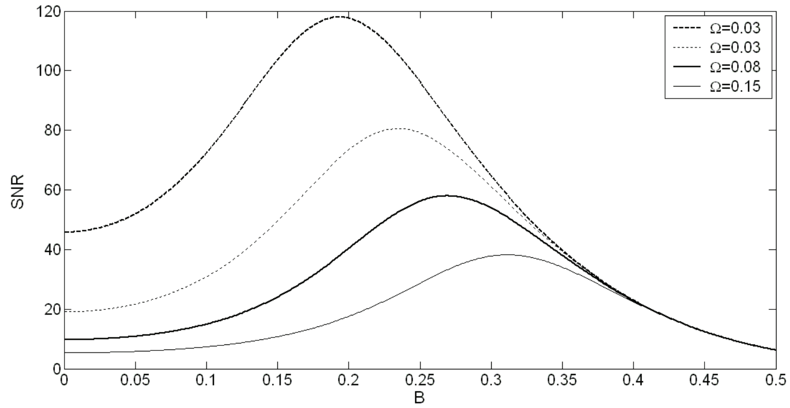

FIG. 2: The SNR as a function of the amplitude of the dichotomous noise for $G = 0.1$, $\varepsilon = 0.1$, $b = 0.1$, $D = 0.15$, and $\lambda = 1$ for different values of the frequency Ω of the driving force.

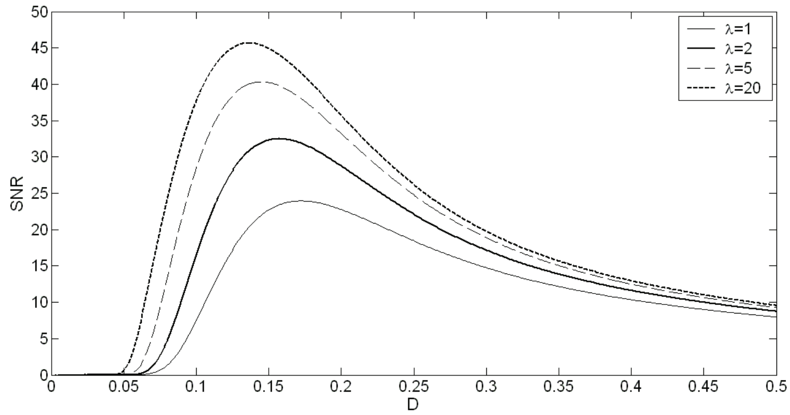

FIG. 3: The SNR as a function of the intensity of the additive white noise for $G = 0.1$, $\varepsilon = 0.2$, $b = 0.2$, $B = 0.4$, and $\Omega = 0.1$ for different values of the correlation rate λ.

effect of the parameter ε and b of the symmetric structure on the system signal-to-noise ratio SNR is investigated in Fig. 4 and Fig. 5. The phenomenon of ESR is obviously observed in these two figures. This is an interesting result, which was not studied in Refs. [14–16]. Therefore, in the present confined model, one can tune the parameters of the symmetric structure to improve the system signal-to-noise ratio.

In conclusion, we have studied the ESR phenomenon in a confined symmetric structure with dichotomous noise and white noise, driven by a periodic force and a constant force. Under the adiabatic approximation condition, the expression of the SNR is obtained. We observe two types of entropic stochastic resonance. One is in the case of the SNR plotted versus the intensity of the white noise and versus the strength of the dichotomous noise; the other is in the case of the SNR plotted versus the parameters of the symmetric

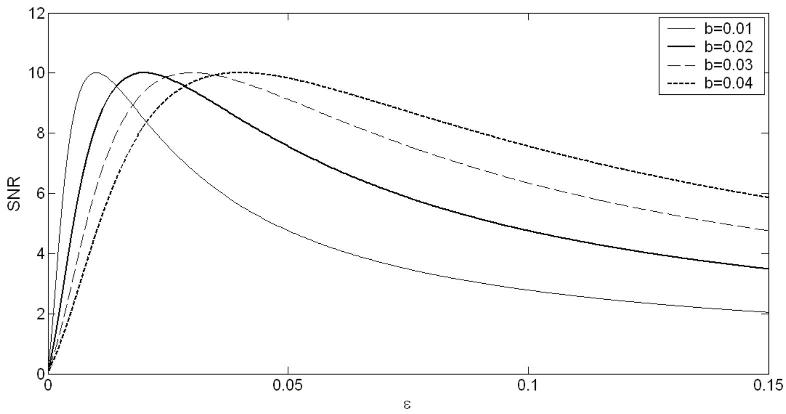

FIG. 4: The SNR as a function of the parameter ε of the symmetric structure for $G = 0.1$, $D = 0.2$, $B = 0.1$, $\Omega = 0.1$, and $\lambda = 1$ for different values of the remaining width b at the bottleneck of the confined structure.

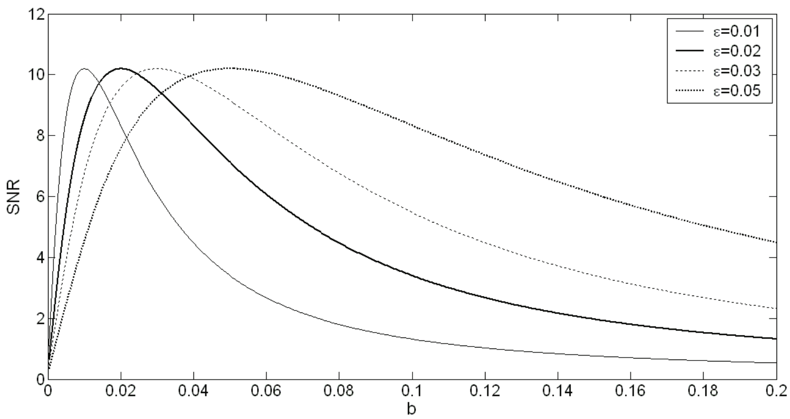

FIG. 5: The SNR as a function of the remaining width b at the bottleneck of the confined structure for $G = 0.2$, $D = 0.3$, $B = 0.1$, $\Omega = 0.1$, and $\lambda = 1$ for different values of the parameter ε of the symmetric structure.

structure. In addition, the SNR increases with an increment of the correlation rate of the dichotomous noise, while the SNR decreases with an increase of the frequency of the driving force. Therefore, the SNR depends on the external force and the shape of the region. With a proper choice of the geometric parameters, the response of the system can be optimized. It is believed that the results obtained in this paper can be helpful for the investigation of the ESR in confined systems, and for the use of ESR in controlling the properties of the confined structure.

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