

## Stochastic Stimulation-induced Organization in a Network of Chaotic Units with Variable Connection Strengths

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We investigate the influence of stochastic stimulations on a network of globally coupled logistic maps with temporally varying connection strengths. The network, for a fixed map parameter, exhibits a kind of organized inter-unit connectivity within certain parameter regimes of the coupling strength and the stochastic stimulation strength. Within these regimes, units separate into two groups whose distinguishing feature is that the first group possesses many outwardly directed connections to the second group, while the second group possesses only a few outwardly directed connections to the first. We show that such organization can be induced by stochastic stimulations.

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There has been much interest in the effect of stochastic stimulations on various nonlinear dynamics systems. Noise (a kind of stochastic stimulation) and the phenomena in relation to noise have become hot topics in this context [1–9]. It is known that noise tends to amplify chaoticity when the system is close to the bifurcation or to the crisis [1–3] and via stochastic resonance (SR) noise can enhance weak signal transduction in sensory neurons [4]. Also, noise-enhanced sensitivity has been observed in several biological neural systems such as (i) periodically stimulated crayfish neurons [5], (ii) information encoding in the primary auditory nerve of the squirrel monkey [6], (iii) prey targeting of paddle fish [7], and (iv) heart rate compensation for blood pressure regulation [8]. In recent years, H. Busch and coworkers have studied the effect of colored noise on spatiotemporal cluster formation. In their paper [9] they considered exponentially correlated Gaussian noise as a model for fluctuations. Within this model, the “noise color” denotes the correlation time. They found that the homogeneity of patterns (or the average cluster size) in a system of coupled nonlinear differential equations strongly depends on the temporal correlation of additive spatially incoherent noise. For chaotic networks, an increased correlation time increases the homogeneity of patterns, whereas in the nonchaotic case, a clear minimum of the homogeneity at an intermediate correlation time is observed. Furthermore, Kim and Kantz found that the strength of the noise plays an important role in the determination of the topological structure of their growing network models [10].

In the present work, we introduce stochastic stimulations into a model of a temporally evolving network which was introduced by Ito and Kaneko [11] and show that even without any inter-unit synchronization of dynamics the network exhibits a kind of organization, reflected by the collective behavior of units. Within this organization, units will

separate into two groups whose distinguishing feature is that the first group possesses many outwardly directed connections to the second group, while the second group possesses only a few outwardly directed connections to the first. In some cases such organization can be induced by the stochastic stimulations. In addition to being a novel phenomenon in the study of nonlinear networks under the influence of “noise”, this effect may be of relevance to applications in life sciences, as it suggests a possible mechanism of structure.

The model which we study is a network of globally coupled logistic maps with temporally varying connection strengths, previously introduced by Ito and Kaneko [11]. It is given by the set of equations

$$x_{n+1}^i = f \left[ (1-c)x_n^i + c \sum_{j=1, j \neq i}^N w_n^{ij} x_n^j \right], \quad (1)$$

where  $x_n^i$  is the state variable of the  $i$ th unit ( $1 \leq i \leq N$ ) at the  $n$ th time step. The coupling  $c$  ( $0 \leq c \leq 1$ ) represents the strength of the influence of the other units on the dynamics of unit  $i$ , and  $w_n^{ij}$  is the time-dependent weight of the connection from the unit  $j$  to  $i$  at time step  $n$ . The function  $f(x)$  providing the dynamics of the units takes the logistic map form  $f(x) = ax(1-x)$ . At the next time step  $n+1$ ,  $w_n^{ij}$  becomes

$$w_{n+1}^{ij} = \frac{[1 + \delta \cdot g(x_n^i, x_n^j)] w_n^{ij}}{\sum_{j=1, j \neq i}^N [1 + \delta \cdot g(x_n^i, x_n^j)] w_n^{ij}}, \quad (2)$$

with the rule  $g(x_n^i, x_n^j) = 1 - 2|x_n^i - x_n^j|$  which is regarded as an extension of Hebb’s rule. Here  $\delta$  is a parameter that represents the plasticity of the connection strengths.

To investigate the influence of stochastic stimulations on this network, we add stochastic stimulations to the state variable of each network element, as follows

$$x_{n+1}^i = f \left[ (1-c)x_n^i + c \sum_{j=1, j \neq i}^N w_n^{ij} x_n^j + \eta \right], \quad (3)$$

where  $\eta$  is any single type of stochastic stimulation. M. Amos has investigated the synchronization behavior between two logistic maps influenced by the stochastic variable  $\eta$  [12]. Each one is described as  $x_{n+1} = ax_n(1-x_n) + \eta$ , where  $\eta = W \times (\text{random})$ .  $W$  acts as the amplitude of  $\eta$ . ( $\text{random}$ ) is a random number chosen uniformly from the interval  $(d-1, 1)$  ( $0 \leq d < 1$ ) or  $(0, 1)$ , with the constraint that  $0 < x_{n+1}^i < 1$ , i.e., if a chosen number of ( $\text{random}$ ) violates the bound  $0 < x_{n+1}^i < 1$ , it is discarded and a new number is chosen. Following paper [12], we let  $\eta = W \times (\text{random})$  and constrain ( $\text{random}$ ) to meet  $0 < x_{n+1}^i < 1$  in our model Eq. (3), but we believe that qualitatively similar results would be displayed by the system for any type of stochastic stimulation.

In the course of our numerical simulation of this model, the number of units  $N$  and the parameter  $\delta$  are set to 100 and 0.1, respectively. The initial values of all connection strengths  $w_0^{ij}$  ( $i \neq j$ ) are determined to be  $1/(N-1)$ . For the state variables, the initial values are randomly chosen from the interval  $(0,1)$  with a uniform sampling measure. In

letter [11], without external stimulations, the parameters  $a$  and  $c$  range from 3.7 to 4.0 and 0 to 0.2, respectively, in which the network is in the desynchronized phase. In this phase the behavior of network is complicated, but not completely random, and the network can exhibit a kind of organization in some cases. Within this organization, units separate into two groups whose distinguishing feature is that the first group possesses many outwardly directed connections to the second group, while the second group possesses only a few outwardly directed connections to the first. In the present work, we focus on the influence of stochastic stimulations on the network, and therefore fix the parameter  $a$  (for example, here  $a = 3.85$ ) and set  $c$  and  $W$  in (3) to be in the intervals  $(0,0.2)$  and  $(0,0.5)$ , respectively.

To characterize the influence of stochastic stimulations on the global behavior of the network in the parameter space, we define an average variation of connection strength per step  $A$ , which is called the ‘‘activity of the network’’, and stipulate that the characteristic quantity  $w_{out}^i$  describes the sum of the average connection strengths emanating from one unit. They are written as,

$$A = \frac{1}{(N-1)^2} \frac{1}{\tau_m} \sum_{i \neq j} \sum_{n=\tau_t}^{\tau_t+\tau_m} |w_n^{ij} - w_{n-1}^{ij}|, \quad (4)$$

$$w_{out}^i = \frac{1}{\tau_m} \sum_{j=1}^N \sum_{n=\tau_t}^{\tau_t+\tau_m} w_n^{ij}, \quad (5)$$

where  $\tau_t$  is the length of the transient period and  $\tau_m$  is the length of the measuring period.

Similar to the discussion of (5) in paper [12], here we consider first the (*random*) which is asymmetric, in particular randomly generated from the interval  $(0,1)$  with a uniform sampling. Other asymmetric ones and the symmetric one (i.e., (*random*)) is replaced by (*random*)-0.5.) will be discussed at the end of this paper.

Choosing  $\tau_t = 1 \times 10^6$  and  $\tau_m = 1 \times 10^3$ , we compute the activity  $A$  and the quantity  $V_{ar} = (\sum_{i=1}^N |w_{out}^i - \langle w_{out}^i \rangle|)/N$ , characterizing the variance of  $w_{out}^i$  over  $i$ , where  $\langle w_{out}^i \rangle = \sum_{i=1}^N w_{out}^i/N$ . In Fig. 1,  $V_{ar}$  and  $A$  are plotted with respect to the parameters  $c$  and  $W$  on gray scales. An island of high activity corresponding to the bright region is seen at the bottom of Fig. 1(a). Just in this island a region of large  $V_{ar}$  (the high bright region) is found in Fig. 1(b). As mentioned in [11], we also found that there exists a desynchronized phase (i.e., there is no synchronization between the dynamics of the units) within some regions in the parameter space shown in Fig. 1(a). In contrast to the case where  $3.7 \leq a \leq 4.0$ ,  $0 \leq c \leq 0.2$  and  $W = 0$  (i.e., noise-free), which was studied by Ito and Kaneko, in our case, in addition to the desynchronized phase there is an ordered phase, observed often in many globally coupled maps (GCM), according to the degree of synchronization and clustering among units. In this ordered phase, the units split into a few clusters and all the units within each such cluster oscillate synchronously. Due to the formation of clusters of units, connections between units that belong to the same cluster have similar finite values, determined by the size of the clusters, while connections between

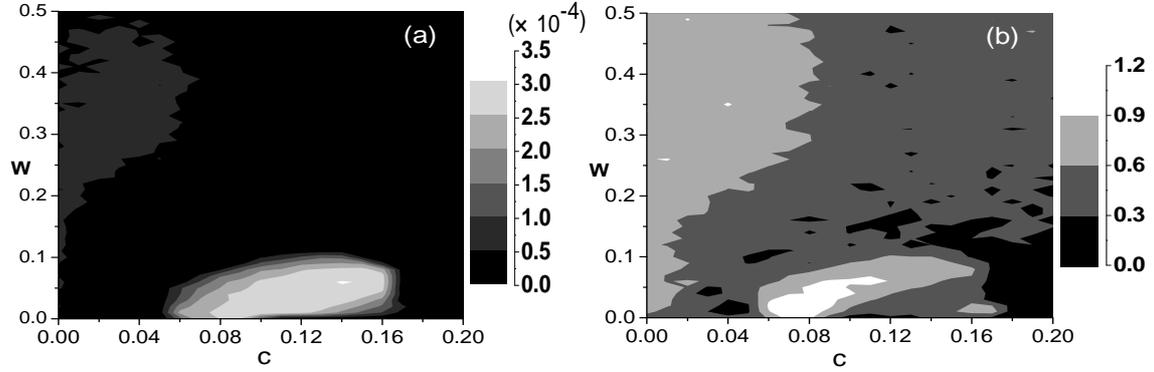


FIG. 1: Gray scale plots of network quantities  $A$  and  $V_{ar}$  with respect to the parameters  $c$  and  $W$ , with discretization of 0.01 for both. (a) for  $A$ . (b) for  $V_{ar}$ . The brighter shades correspond to large  $A$  or  $V_{ar}$ .

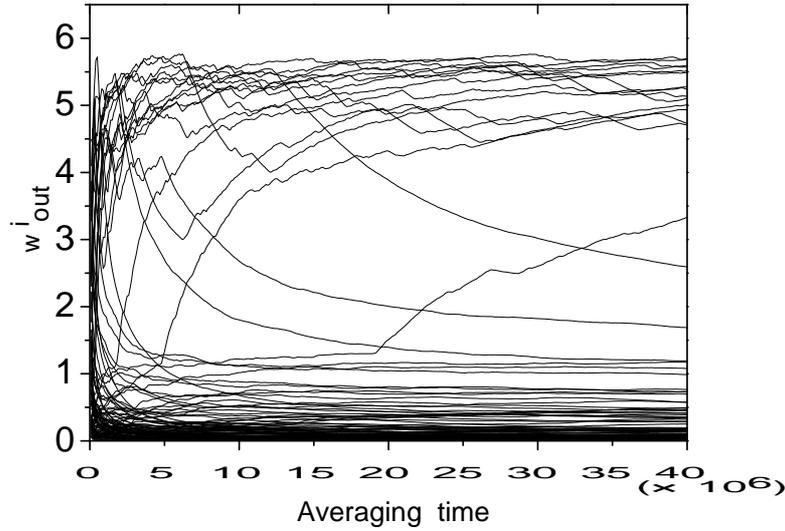


FIG. 2: A kind of organization of the network in the noise-free case, coming from Fig. 2 in [11].  $w_{out}^i$  is a function of the measuring time  $\tau_m$ . Each curve corresponds to single unit and represents a series of  $w_{out}^i$  with different measuring time. The series of  $w_{out}^i$  for all units are superimposed. The transient period is chosen as  $\tau_t = 0$ , and the parameters  $a$  and  $c$  are 3.97 and 0.12, respectively.

two units from different clusters tend towards 0. In this ordered phase, the network is finally static [11, 13, 14].

In regions of high activity  $A$  and large  $V_{ar}$ , the network exhibits a kind of organization that can be demonstrated by Fig. 2. From Fig. 2 one see that units separate into two groups: one with large values of  $w_{out}^i$  and the other with small values. The separation becomes more distinct as the measuring time increases. In terms of the criterion that if

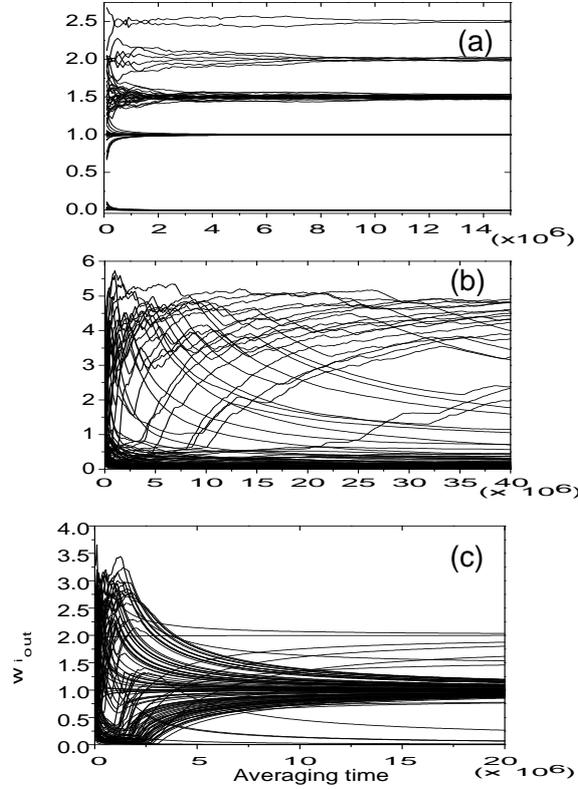


FIG. 3: A series of  $w_{out}^i$  as functions of the measuring time  $\tau_m$  (with fixed transient time  $\tau_t = 0$ ) are plotted for the network being both in and out of the region of large  $V_{ar}$ , respectively. Fig. 3(a) and Fig. 3(c) are for the case of that the network is out of this region, Fig.3(a) is for ( $c = 0.08$ ,  $W = 0.11$ ) and Fig. 3(c) is for ( $c = 0.08$ ,  $W = 0.0$ ). Fig.3b is for the case of that the network is in this region, here ( $C = 0.08$ ,  $W = 0.019$ ).

$w_n^{ij} > w_0^{ij} = 1/(N - 1)$  (the value  $1/(N - 1)$  is equal to the connection value in the case that a element uniformly connects to all the others) a connection from unit  $j$  to unit  $i$  is assigned, otherwise no connection is assigned, the group with the large value of  $w_{out}^i$  corresponds to the “core group” possessing many outwardly directed connections to the group with a small value of  $w_{out}^i$  which is named the “peripheral group”. The peripheral group possesses only a few outwardly directed connections to the core group. It means that the dynamics of the core group strongly influence the peripheral group, while the dynamics of the peripheral group have almost no influence on the core group [11].

Here, in this region of large  $V_{ar}$  in Fig. 1(b), the network shows such organized behavior, such as the case where  $a = 3.85$ ,  $c = 0.08$  and  $W = 0.019$  (with the presence of external stimulation, Fig. 3(b)), being in the high bright region of Fig. 1(b). Fig. 3(b) obviously shows a group with large  $w_{out}^i$  and an other with a small  $w_{out}^i$  occurs. In terms of the above connection criterion, the group with a large  $w_{out}^i$  corresponds to the core group

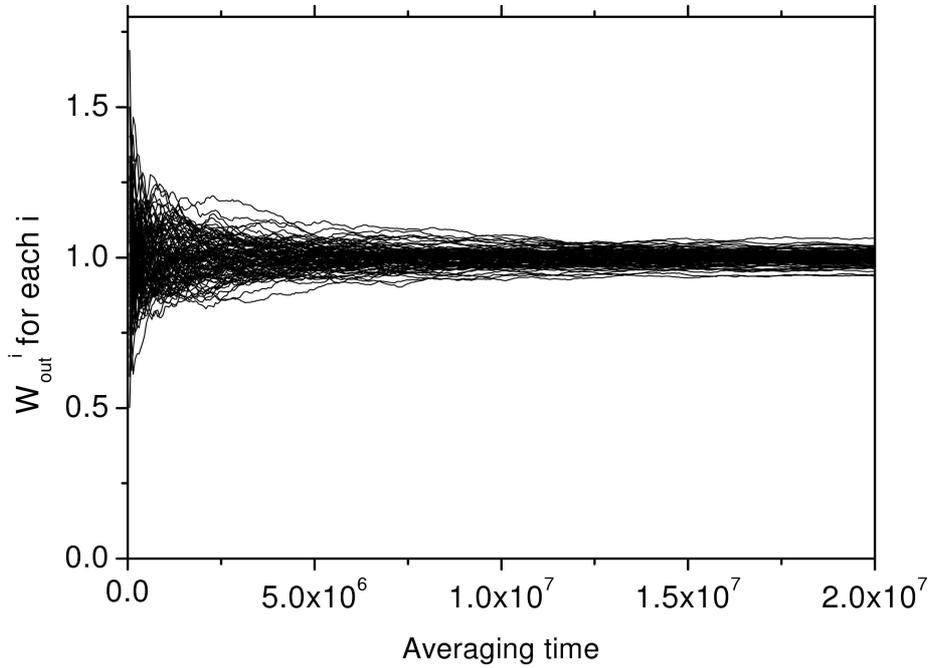


FIG. 4: A series of  $w_{out}^i$  as functions of the measuring time  $\tau_m$  (with fixed transient time  $\tau_t = 0$ ) are plotted for the network being out of the region of large  $V_{ar}$  and large  $A$  ( $c = 0.12$ ,  $W = 0.02$  and  $a = 3.85$ ).

and the other with a small  $w_{out}^i$  is the peripheral group. In the case of  $a = 3.85$ ,  $c = 0.08$ , and  $W = 0.0$  (with the absence of external stimulation, Fig. 3(c)) and in the case  $a = 3.85$ ,  $c = 0.08$  and  $W = 0.11$  (Fig. 3(a), being out of the high bright region of Fig. 1(b), with smaller  $V_{ar}$ ), there is no  $w_{out}^i \geq 3$  ( $1 \leq i \leq 100$ ) when the transient period  $\tau_t > 5 \times 10^6$ . In contrast to the case of  $a = 3.97$ ,  $c = 0.12$ , and  $W = 0.0$  (Fig. 2, with the absence of external stimulation), one can see that in these two cases such organization does not appear at all.

In regions where the activity  $A$  is quite low and the value of  $V_{ar}$  is very small, most of the units exist in pairs, with the units in each such pair having nonzero connections only between each other, forming fixed pairs not synchronized but highly correlated [11]. In the regions where the activity  $A$  is quite low and  $V_{ar}$  is smaller but not very small, the network exhibits the ordered phase shown in Fig. 3(a). In this ordered phase, the units split into a few clusters, and finally all the units within each such cluster oscillate synchronously. Connections between units that belong to the same cluster have similar finite values, determined by the size of the clusters, while connections between two units from different clusters tend towards 0. In this ordered phase, the network is finally static. In most other cases of the low bright region of Fig. 1(b), the network exhibits the behaviors shown as Fig. 3(c) or Fig. 4. Most of the units'  $w_{out}^i$ , are about  $(1/(N-1)) \times (N-1) = 1$ , with tiny fluctuation, and the network structure is nearly the same as initially.

Fig. 5 further illustrates that the network exhibits this organized behavior when it is

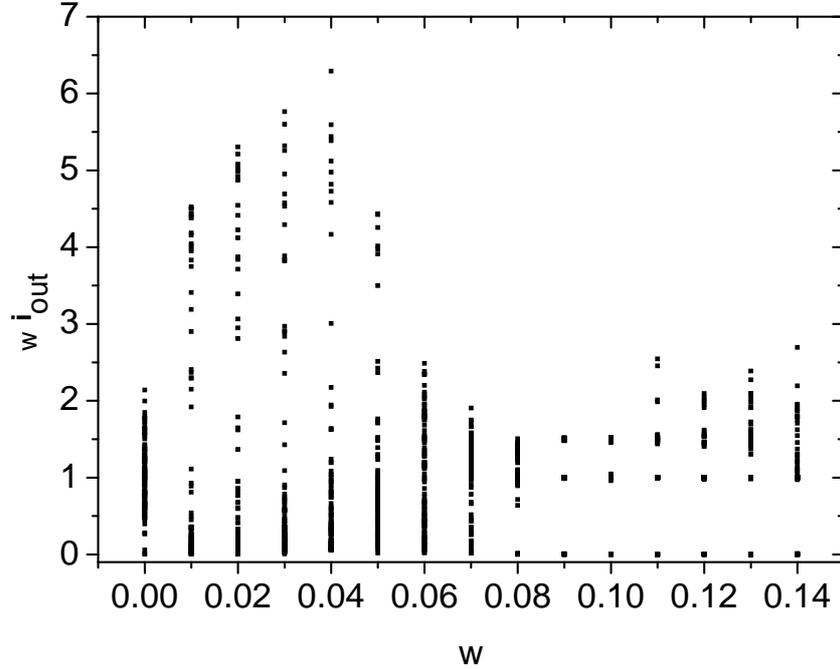


FIG. 5: A series of  $w_{out}^i$  as functions of the parameter  $W$ , with  $c = 0.08$ ,  $\tau_t = 0$ , and  $\tau_m = 5 \times 10^6$ , are plotted to show that only in the region of large  $V_{ar}$  and large  $A$  such organizations with a core group and a peripheral group appear. Because the computation is very time consumptive, we calculate it only in  $0 \leq W \leq 0.14$  with a discretization of 0.01.

within the region of large  $V_{ar}$  and large  $A$ , and does not when it is out of this region in Fig. 1. Furthermore, In the cases of  $a = 3.85$ ,  $c \geq 0.08$ , and  $W = 0.0$ , the network was reported not to exhibit this organization because in the plane of  $a$  and  $c$ , the coordinates ( $a = 3.85$ ,  $c \geq 0.08$ ) are above the boundary line  $c = 0.52 \times (a - 3.7)$  between the regions of large  $V_{ar}$  and small  $V_{ar}$  [11]. So we conclude that within the region of large  $V_{ar}$  and large  $A$  in the plane  $W$  and  $c$  ( $a$  is fixed to be 3.85) this organization, with the core group and peripheral group, is certainly induced by the stochastic stimulation when  $c \geq 0.08$ . We note that the description  $c = 0.15 \times (a - 3.7)$  for the boundary line between the regions of large  $V_{ar}$  and small  $V_{ar}$  in [11] is obviously wrong. According to it, the coordinate ( $a = 3.97$ ,  $c = 0.12$ ) is above and far away from rather than below this line. Based on our numerical simulation, this line can be written as  $c = 0.52 \times (a - 3.7)$ .

There must be inter-dependency of the unit dynamics and connection dynamics for the structure formation (i.e., the formation of this organization) discussed above to occur. In the following, we simply discuss some relationship between the unit dynamics and connection dynamics for the structure formation. We therefore consider the “mean distance” of units  $\bar{d}_{ij} = \sum_{n=\tau_t}^{\tau_t+\tau_m} |x_n^i - x_n^j| / \tau_m$ , where  $i \neq j$ , based on which the degree of synchronization between units  $i$  and  $j$  during the period from  $\tau_t$  to  $\tau_t + \tau_m$  can be characterized as well.

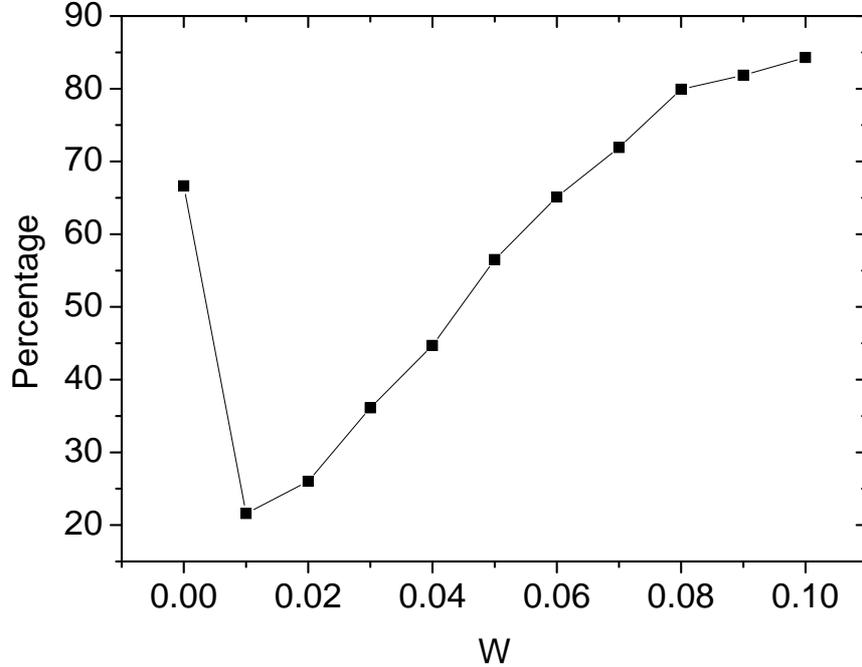


FIG. 6: The number of pairs (on a percentage scale) meeting  $\overline{d_{ij}} \geq 0.31$  as function of the parameter  $W$  with  $a = 3.85$ ,  $c = 0.08$ ,  $\tau_t = 5 \times 10^6$ , and  $\tau_m = 100$ .

For  $\tau_t = 5 \times 10^6$  and  $\tau_m = 100$ , the number of pairs of units meeting  $\overline{d_{ij}} \geq 0.31$  is calculated in Fig. 6. Fig. 6 indicates that, with the parameters  $a$  and  $c$  fixed as 3.85 and 0.08, respectively, the number of such pairs is a function of the strength of the stochastic stimulation. In interval  $0 < W < 0.05$ , just within the region for which the network exhibits the stochastic stimulation-induced organization (see Fig. 1, Fig. 5 and Fig. 6), the pairs meeting  $\overline{d_{ij}} < 0.31$  are more than those when  $W = 0$  (being in the absence of external stimulation) or  $W > 0.05$  (being out of this region). In light of Hebb's rule that the less the difference between  $x_n^i$  and  $x_n^j$ , the more strongly unit  $i$  connects unit  $j$  to each other. This implies that, with the appearing of many pairs whose mean distance are small when  $0 < W < 0.05$ ,  $a = 3.85$ , and  $c = 0.08$ , many units'  $w_{out}^i$  (stipulated by (2) and (5)) of a large value must appear simultaneously. In addition, in the ranges  $0 < c < 0.2$  and  $0 < W < 0.5$  with  $a = 3.85$  at any time step,  $\langle w_{out}^i \rangle$  is equal to 1 (which we found by numerical calculation) when  $\tau_t = 0$  and  $\tau_m > 2 \times 10^5$ . Thus, with the presence of many units of large  $w_{out}^i$  (say  $w_{out}^i > 3$ ), many more units of small  $w_{out}^i$  (say  $w_{out}^i < 0.5$ ) appear, i.e, the units separate into a core group and a peripheral group. Such organization of the core group and peripheral group is observed in the desynchronized phase. In our present desynchronized phase, there is no synchronization between any two units because their connections in such a case are random, reflected by the large fluctuation of  $w_{out}^i$  [11]. Thus, the stochastic stimulations here, in some cases, decrease the "distance" of many unit pairs

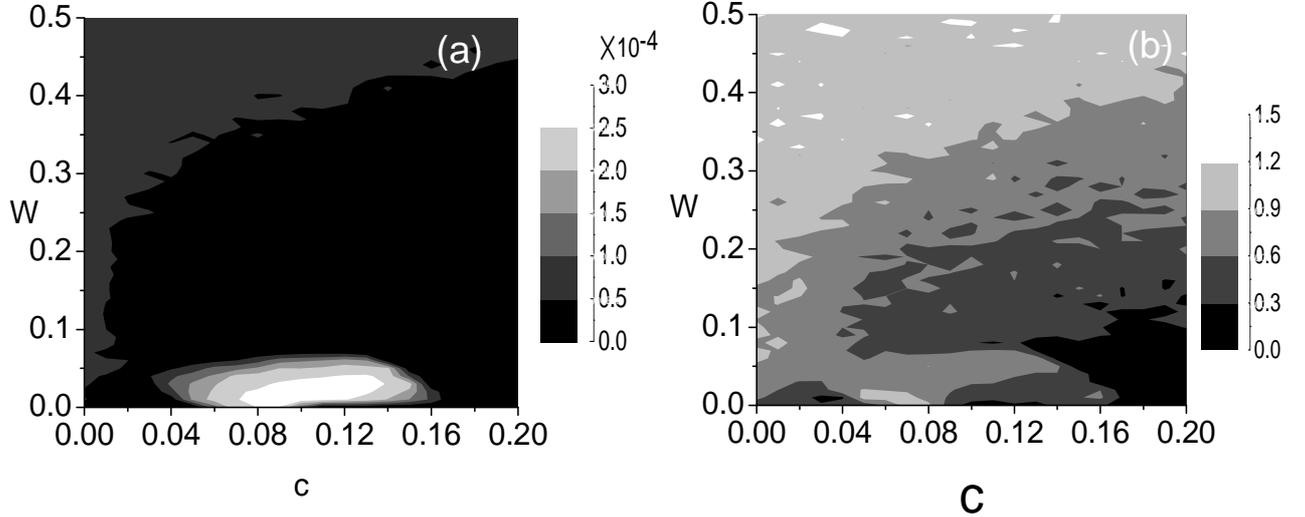


FIG. 7: For the symmetric stochastic stimulation ( $(random)$  is replaced by  $(random)-0.5.$ ), gray scale plots of network quantities  $A$  and  $V_{ar}$  with respect to the parameters  $c$  and  $W$  with discretization of 0.01 for both are shown. (a) for  $A$ . (b) for  $V_{ar}$ . The brighter shades correspond to large  $A$  or  $V_{ar}$ .

as well as at the same time decrease the correlation and even the degree of synchronization between units, consequentially, inducing networks exhibiting such organized behavior. This is why in the case of  $0 < W < 0.05$ ,  $a = 3.85$ , and  $c = 0.08$  the network shows a core group and a peripheral group (owing to the decrease of “distances” of many unit pairs) and all  $w_{out}^i$  fluctuate much strongly than it does in the other cases (owing to the decrease of the correlation and even the degree of synchronization between units) (see Fig. 3).

The above discussion is for the stochastic stimulation with the highest asymmetric degree, i.e.,  $(random)$  being in  $(0,1)$ . We find that: (1) for the other asymmetric stochastic stimulation ( $(random)$  is replaced by  $(random)-d_r$ , where  $0 < d_r < 0.5.$ ), similar results exist, but the region of large  $V_{ar}$  shrinks with the increasing of  $d_r$ ; (2) for the symmetric stochastic stimulation ( $(random)$  is replaced by  $(random)-0.5.$ ), the island of large  $A$  holds, while the part corresponding to  $c \geq 0.08$  in the region of large  $V_{ar}$  almost vanishes (see Fig. 7), and the organization mentioned above is not obvious.

In summary, we have studied the influence of stochastic stimulations on a model of a temporally evolving network. For the stochastic stimulations made asymmetric, there exists certain parameter regimes of the connectivity weight  $c$  and the stochastic stimulation strength  $W$  for which such stochastic stimulation can induce the network obviously exhibiting a kind of organization of two groups: the first group possesses many outwardly directed connections to the second group, while the second group possesses only a few outwardly directed connections to the first group. We must point out that for any map parameter

$a$  in the interval (3.7, 4.0) a similar result holds, and as mentioned in [11], it would be displayed by the system for any form of  $f(x)$  that exhibits chaos. These results are based on the network size  $N = 100$  and  $\delta = 0.1$  which represents the plasticity of connection strengths, but over a wide range of values of  $\delta$  and for large size systems they do not change qualitatively except for the existence of a longer transient behavior.

It is known that, in the early stage of development, axons arborize excessively and eventually are trimmed under the influence of neuronal activity [15]. More evidence shows that synaptic change occurs over a wide range of time scales, from hundreds of milliseconds to months or years [16]. In addition, a real biologic system or biologic neural network is always influenced by environmental circumstances, and some times this influence is stochastic. So our results may be of relevance for applications in life sciences, as it suggests a possible mechanism of structure.

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## References

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- [1] J. P. Eckmann, Rev. Mod. Phys. **53**, 643 (1981).
  - [2] J. C. Sommerer, W. I. Ditto, C. Grebogi, E. Ott, and M. L. Spano, Phys. Rev. Lett. **66**, 1947 (1991).
  - [3] H. Herzel and B. Pompe, Phys. Lett. A **122**, 121 (1987).
  - [4] a) V. Annishchenko, F. Moss, A. Neiman, and G. L. Schimansky, Phys. Usp. **42**, 7 (1999); b) D. Wiesenfeld and F. Moss, Nature (London), **373**, 33 (1995).
  - [5] A. Longtin, A. Bulsara, and F. Moss, Phys. Rev. Lett. **67**, 656 (1991).
  - [6] M. C. Teich, S. M. Khanna, and P. Guiney, J. Phys. **70**, 257 (1997).
  - [7] D. Russel, L. Willkens, and F. Moss, Nature (London) **402**, 291 (1997).
  - [8] I. Hidaka, D. Nozaki, and Y. Yamamoto, Phys. Rev. Lett. **85**, 3740 (2000).
  - [9] H. Busch, M. -Th. Hütt, and F. Kaiser, Phys. Rev. E **64**, 021105 (2001).
  - [10] J. W. Kim and H. Kantz, Phys. Rev. E **68**, 026110 (2003).
  - [11] J. Ito and K. Kaneko, Phys. Rev. Lett. **88**, 028701 (2002).
  - [12] M. Amos and R. B. Jayanth, Phys. Rev. Lett. **72**, 1451 (1994).
  - [13] K. Kaneko, Physica (Amsterdam) **41D**, 137 (1990).
  - [14] K. Kaneko, Physica (Amsterdam) **54D**, 5 (1991).
  - [15] D. Purves and J. W. Lichtman, Science **210**, 153 (1980).
  - [16] W. Maass and A. M. Zador, in *Pulsed Neural Networks*, (MIT Press, Cambridge, 2001).