New Localized Excitations in a (2+1)-Dimensional Generalized Nozhnik-Novikov-Veselov System

Chun-Long Zheng[1,2,*] and Li-Qun Chen[2]

[1] *Department of Physics, Zhejiang Lishui University, Lishui 323000, China.*
[2] *Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, China*

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Using an extended homogeneous balance approach and a multilinear variable separation method, we obtain a new general variable separation excitation for the (2+1)-dimensional generalized Nozhnik-Novikov-Veselov(GNNV) system. Based on the derived solution, two new types of localized excitations, i.e., a bell-like loop soliton and a peak-like loop soliton, are constructed and some evolutional properties of these novel semifolded localized structures are briefly discussed.

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I. INTRODUCTION

In nonlinear science, soliton theory plays an essential role and has been widely applied in almost all of the physics fields such as fluid mechanics, plasma physics, nonlinear optics, condensed matter physics, etc. [1–4]. In particular, some localized coherent theories of higher dimensional soliton systems have proved to be of considerable interest. For instance the discovery, via the Bäcklund transformation by Boiti *et al.* [5, 6], of dromion type coherent solutions of the Davey-Stewartson I system provided renewed interest in higher dimensional soliton systems. In the past decade, in the study of higher dimensional physical models, although there are some reports on multiple valued solitary waves (folded in all directions) [7, 8], the main effort has been focused on single valued localized excitations. However, in various cases, real natural phenomena are too intricate to be described only by virtue of single valued functions or only by multiple valued functions, since nature is very greatly colorful and may exhibit quite complicated structures, such as semifolded ones. For example, some localized structures such as ocean waves may fold in one direction, say x, and localize in the usual single valued way in another direction, say y. In a previous letter [9], we successfully constructed a new type of semifolded localized coherent structure. Now an interesting and important problem is: are there similar semifolded localized coherent structures in other physical models? Can we obtain new types of localized excitations? How about their evolutional behavior? In this work, our concern is with some semifolded localized coherent structures and their evolution properties for the following (2+1)-dimensional generalized Nozhnik-Novikov-Veselov system (GNNV) [10]

$$v_t + av_{xxx} + bv_{yyy} + cv_x + dv_y - 3a(uv)_x - 3b(vw)_y = 0\,, \tag{1}$$

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$$v_x = u_y\,, \quad v_y = w_x\,, \tag{2}$$

where a, b, c, and d are arbitrary constants. When $c = d = 0$, the (2+1)-dimensional GNNV system degenerates to the usual (2+1)-dimensional Nozhnik-Novikov-Veselov (2DNNV) equation [11]:

$$v_t + av_{xxx} + bv_{yyy} - 3a(uv)_x - 3b(vw)_y = 0\,, \quad v_x = u_y\,, \quad v_y = w_x\,, \tag{3}$$

which is an isotropic Lax extension of the well known (1+1)-dimensional NNV equation. Many types of soliton solutions of the 2DNNV equation have now been studied by many authors. For instance, Boiti, Leon, Manna, and Pempinelli [10] solved the 2DNNV equation via the inverse scattering transformation. Tagami [12] and Hu and Li [13] obtained its soliton-like solution by means of the Bäcklund transformation. Hu [14] has also given a nonlinear superposition formula for the 2DNNV equation. Ohta [15] obtained the Pfaffian solutions for this mode. Radha and Lakshmanan [16] constructed its dromion solution from its bilinear form after analyzing its integrability aspects. Lou [17] obtained some new special types of multi-soliton solutions of this equation, by using the standard truncated Painlevé analysis and localized excitations by means of a variable separation approach.

However, the (2+1)-dimensional GNNV system has not been studied so well even though Boiti, Leon, Manna, and Pempinelli [10] solved this system via the inverse scattering transformation, and Radha and Lakshmanan have constructed its multidromion solutions [16], while Zhang *et al.* [18] obtained an exact solution for this system based on an extended homogeneous balance approach, and Zheng *et al.* [19] derive its localized chaotic and fractal excitation patterns by the standard truncated Painlevé approach. However, as Huang [20] recently pointed out, the GNNV system yields many interesting localized structures that have not yet been found. In this paper, we investigate the GNNV system further. More attention is paid to some new localized coherent structures, such as semifolded coherent structures and their interesting evolutional properties.

II. GENERAL SOLUTION TO THE (2+1)-DIMENSIONAL GNNV SYSTEM

In this section, we will present a general solution to the (2+1)-dimensional GNNV system. As is known, in the search for solitary wave solutions to a given nonlinear partial differential model, one can utilize different approaches. One useful and powerful method is an extended homogeneous balance approach (EHBA). In accord with the EHBA principle (which can also be obtained through the standard truncated Painlevé expansion), let us begin with a Painlevé-Bäcklund transformation to the GNNV system

$$u = -2(\ln f)_{xy} + u_0\,, \quad v = -2(\ln f)_{xx} + v_0\,, \quad w = -2(\ln f)_{yy} + w_0\,; \tag{4}$$

the function f is set to

$$f = A + \sum_{k=1}^{N} (P_k(x,t) + Q_k(y,t))\,, \tag{5}$$

here the variable separated functions, $P_k(x,t) \equiv P_k$ and $Q_k(y,t) \equiv Q_k$ $(k = 1, 2, \cdots, N)$, are only functions of $\{x,t\}$ and $\{y,t\}$, respectively, and A is an arbitrary constant. Then a general variable separation excitation for the GNNV system yields

$$u = \frac{2\sum_{k=1}^{N} P_{kx} \sum_{k=1}^{N} Q_{ky}}{(A + \sum_{k=1}^{N}(P_k + Q_k))^2}, \tag{6}$$

$$v = \frac{-2\sum_{k=1}^{N} P_{kxx}}{A + \sum_{k=1}^{N}(P_k + Q_k)} + \frac{2(\sum_{k=1}^{N} P_{kx})^2}{(A + \sum_{k=1}^{N}(P_k + Q_k))^2} + p_0, \tag{7}$$

$$w = \frac{-2\sum_{k=1}^{N} P_{kyy}}{A + \sum_{k=1}^{N}(P_k + Q_k)} + \frac{2(\sum_{k=1}^{N} P_{ky})^2}{(A + \sum_{k=1}^{N}(P_k + Q_k))^2} + q_0, \tag{8}$$

where p_0, q_0, P_k, and Q_k satisfy the following Eqs. (9) and (10):

$$P_{kt} = (3ap_0 - c)P_{kx} - aP_{kxxx} + \sum_{l=1}^{M} C_{kl}(t), \tag{9}$$

$$Q_{kt} = (3bq_0 - d)Q_{ky} - bQ_{kyyy} - \sum_{l=1}^{M} C_{kl}(t), \tag{10}$$

and $C_{kl}(t)$, $(k = 1, 2, \cdots, N;\ l = 1, 2, \cdots, M)$, are arbitrary functions of time t.

Based on the above general variable separation excitation, we can derive a special variable separation excitation of the (2+1)-dimensional GNNV system by choosing $N = M = 1$, $\{P_1, Q_1\} \equiv \{P, Q\}$ and $C_{11}(t) \equiv \tau(t)$:

$$u = \frac{2P_x Q_y}{(A + P + Q)^2}, \tag{11}$$

$$v = \frac{-2P_{xx}}{A + P + Q} + \frac{2P_x^2}{(A + P + Q)^2} + \frac{P_t + cP_x + aP_{xxx} - \tau(t)}{3aP_x}, \tag{12}$$

$$w = \frac{-2P_{yy}}{A + P + Q} + \frac{2P_y^2}{(A + P + Q)^2} + \frac{Q_t + dQ_y + bQ_{yyy} + \tau(t)}{3bQ_y}, \tag{13}$$

with three arbitrary functions $P(x,t)$, $Q(y,t)$, $\tau(t)$ and one arbitrary constant A.

III. SOME SPECIAL EXAMPLES: SEMIFOLDED LOCALIZED STRUCTURES IN THE GNNV SYSTEM

Here we do not study the general field (7), but only discuss the special field u expressed by (11). From Eq. (11), we know that for general selections of P and Q there may be some singularities for the special field u. We have to choose the functions P and Q carefully to avoid these singularities. However, even in this special situation, one can still find abundant localized structures for the (2+1)-dimensional GNNV system, such as dromions, peakons, compactons, and foldons, by selecting the functions P and Q appropriately [3, 9, 19, 21].

From the above brief analysis, one may deduce that the field u allows some novel localized excitations that have not been revealed. Now we discuss some possible new coherent structures for the field u, and focus our attention on some three-dimensional semifolded localized structures which may exist in certain situations: when the function Q is a t-independent single valued function, and P is selected via the relations [9]

$$P_x = \sum_{j=1}^{M} h_j(\zeta + d_j t)\,, \quad x = \zeta + \sum_{j=1}^{M} X_j(\zeta + d_j t)\,, \quad P = \int^{\zeta} P_x x_\zeta d\zeta\,, \tag{14}$$

where d_j, $\{j = 1, 2, \cdots, M\}$ are arbitrary constants and h_j, X_j are localized excitations with the properties $h_j(\pm\infty) = 0$, $X_j(\pm\infty)$ =consts. From Eq. (14), one sees that ζ may be a multi-valued function in some suitable regions of x for appropriate functions X_j. Therefore, the function P_x may be a multi-valued function of x in these areas, although it is a single valued function of ζ. Moreover, in general terms, if the functions P or Q are taken as multiple localized excitations that possess some phase shifts of (1+1)-dimensional models then the (2+1)-dimensional localized excitations involving the representation (11) will inherit phase shift structures. For example, when

$$P_x = \mathrm{sech}(\zeta)^2 + 0.5\mathrm{sech}(\zeta - 0.5t)^2\,, \quad x = \zeta - 1.5\tanh(\zeta) - 1.5\tanh(\zeta - 0.5t)\,, \tag{15}$$

$$Q = 1 + \tanh(y)\,, \quad A = 10\,, \tag{16}$$

we can obtain a new type of semifolded localized excitation, i.e., a bell-like loop soliton. Some interesting structures are depicted in Fig. 1. From Fig. 1 one can find that the two semifolded localized structures possess some novel properties, which fold in the x direction and localize in a single valued way in the y direction like a bell, so we call them bell-like loop solitons. Moreover, one can also find the interaction between the two loop solitons is completely elastic, since their amplitudes, velocities, and wave shapes do not change after their collision. Meanwhile, one will find there are some phase shifts in the procedure of the two loop solitons' interaction. To see this more clearly, here the velocity of the larger of the two loop solitons has been set to zero; the position located by the large static loop soliton is still altered from about $x = -1.5$ to $x = 1.5$ and stopped after the interaction.

Along the above lines, one can obtain some peak-like loop solitons for the field u (11)

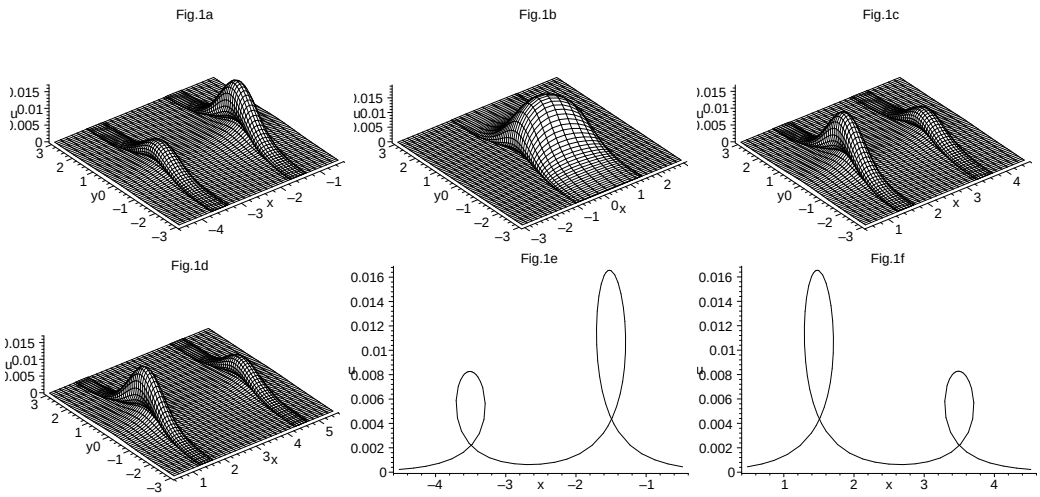

FIG. 1: The evolution of the interactions of two bell-like loop solitons for the field u expressed by (11) with conditions (15) and (16) at times (a) $t=-10$, (b) $t=0$, (c) $t=10$, (d) $t=12$. (e) A sectional view related to (a) at $y=0$. (f) A sectional view related to (c) at $y=0$.

when P is chosen as an appropriate piecewise smooth function of (x,t):

$$P = 1 + \sum_{i=1}^{M} \begin{cases} \psi_i(x+d_it), & x+d_it \le 0\,, \\ -\psi_i[-(x+d_it)] + 2\psi_i(0)\,, & x+d_it > 0\,, \end{cases} \tag{17}$$

while Q is chosen as an appropriate multivalued function of y:

$$Q_y = \sum_{j=1}^{N} g_j(\eta)\,, \quad y = \eta + \sum_{j=1}^{N} Y_j(\eta)\,, \quad Q = \int^{\eta} Q_y y_\eta d\eta\,, \tag{18}$$

where g_j, Y_j are localized functions. From Eq. (18), one knows that η may be a multivalued function in some suitable regions of y by selecting the functions Y_j appropriately. Correspondingly, the function Q_y, which is obviously an interaction solution of N localized excitations since $\eta\,|_{y\to\infty}\to\infty$, may be a multivalued function of y in these areas, although it is a single valued functions of η. For instance, when choosing

$$Q_y = \mathrm{sech}^2(\eta)\,, \quad y = \eta - 2\tanh(\eta)\,, \quad Q = \int^{\eta} Q_y y_\eta d\eta\,, \tag{19}$$

and setting

$$M = 2\,, \quad \psi_1 = \exp(x+t)\,, \quad \psi_2 = 2\exp(x-2t)\,, \tag{20}$$

in Eq. (17), $A=20$ for the field u (11), then we can obtain another new type of semifolded localized excitation, i.e., a peak-like loop soliton presented in Fig. 2. From Fig. 2, one can find that the two peak-like loop solitons possess some interesting properties: they fold in

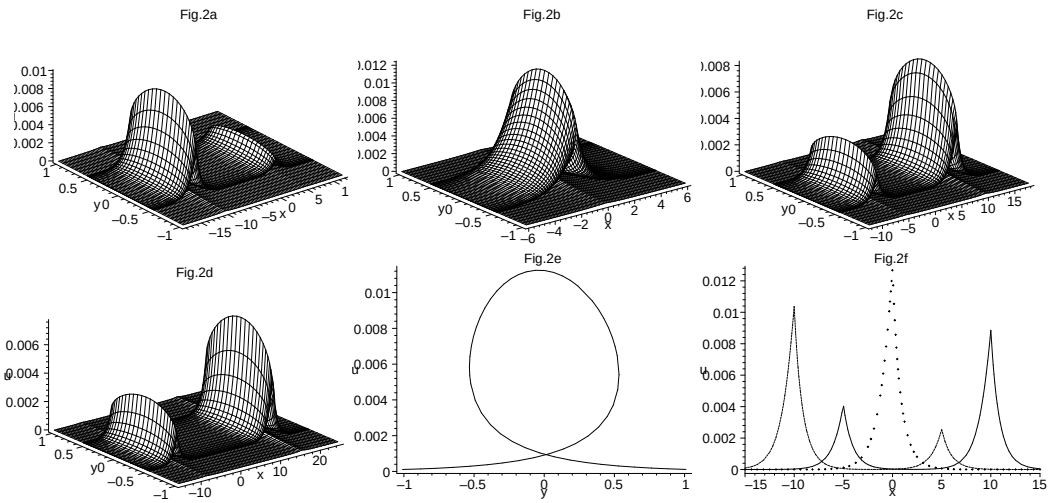

FIG. 2: The evolution of the interactions of two peak-like loop solitons for the field u expressed by (11) with conditions (17), (19), and (20) at times (a) $t = -5$, (b) $t = 0$, (c) $t = 5$, (d) $t = 6$. (e) A sectional view related to (b) at $x = 0.1$. (f) A sectional view related to (a), (b) ,and (c) at $y = 0$: dashed line (a), dotted line (b), and solid line (c).

the y direction, and localize in a single valued way in the x direction like a peak, so we call them peak-like loop solitons. Moreover, by an analysis similar to Fig. 1, one can find that the interaction between the two peak-loop solitons is not completely elastic, since their amplitudes and wave shapes are changed after their collision even though their velocities do not undergo any changes.

Certainly, if we consider other selections for the arbitrary functions P and Q, such as Jacobian functions or the solutions of the well-known Lorenz chaotic system, then we may derive some solitary wave solutions with double periodic properties or chaotic behavior which are neglected in our present paper. Actually, because there exist the arbitrary characteristic functions P and Q in the special field u, many new localized excitations may be further constructed and many exotic behaviors may be engendered along the above lines, when we choose some appropriate functions having physical meaning.

IV. SUMMARY AND DISCUSSION

In summary, by means of an extended homogeneous balance approach and a multilinear variable separation method, the (2+1)-dimensional GNNV system is successfully solved. Based on a special solution of the derived general solution with arbitrary functions, we list two semifolded localized excitations as simple examples, bell-like and peak-like loop solitons for the (2+1)-dimensional GNNV system. Meanwhile some interesting evolutional properties for the selmifolded localized excitations are briefly investigated. To our knowledge, general localized excitations, especially semifolded structures, for the (2+1)-dimensional

GNNV system were not reported in the previous literature. However, our present short note is merely an initial work. Due to the wide potential applications of soliton theory, to learn of more new selmifolded localized excitations and their related evolutional properties, particularly, their applications in reality is worthy of further study.

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* Corresponding author; Electronic address: zjclzheng@yahoo.com.cn

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