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Solitons in the Crab Nebula

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High resolution z-ray observations of the Crab Nebula show the existence of a well-defined bright torus. A straightforward estimate suggests the inadequacy of the conventional model for the pulsar-wind terminal shock. A resolution to this difficulty is given in this report; it is proposed that the pulsar-wind shock can only be "thermalized" in a collisionless fashion and hence the shock transition is bound to be broad. A dynamical scenario is given regarding how the shock "thermalization" should proceed, and it employs the excitation of solitons that serve as the energy carriers in transporting the pulsar wind energy afar and depositing the energy when certain conditions in the background plasmas are met. A necessary condition for such an energy-deposition process to occur is that the charateristic wave speed decreases with distance, a condition that can hold in the Crab Nebula.

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I. Introduction

Crab Nebula presents itself as the only nearby cosmic object that exhibits clear evidences for housing a large accelerator producing ultra-relativistic electrons at energies exceeding tens of TeV [1]. The main evidence supporting the existence of these ultra-relativistic electrons comes from the large flux of linearly polarized $x-$ ray continuum emissions in the Crab Nebula. The linearly polarized photons can extend from the hard x-ray down to the optical frequency range in a power-law form, and the luminosity per decade of frequency has a nearly vanishing power index. The fact that a wide range of power-law photons exist is by itself a strong indication that these photons be originated from a single mechanism; moreover, the fact that these photons are polarized narrows the possibility of this mechanism to the synchrotron emissions. Self-consistent models of the Crab Nebula constrained by the observed luminosity and nebula expansion speed all point to an estimated magnetic field strength about 10^{-3} gauss in the inner part of the nebula where the intense x-ray emissions exhibit a torus-shape morphology [2]. At this magnetic field strength, the electrons must have an energy about 10 TeV in order to yield the hard x-ray synchrotron photons. Moreover, as the y-ray emissions are also observed in the nebula, it hence supports the existence of relativistic electrons of energy well beyond 10 TeV.

The ultimate question for such an high-energy accelerator is its energy source. The discovery of the pulsar as a rotating neutron star answers the question of the ultimate energy source, which delivers energy to the entire nebula at a rate of $10^{38}\,\mathrm{erg/sec}$; it is the rotational energy of a rapidly rotating neutron star located at the core of the Crab Nebula that serves as the energy source. A simple estimate shows that a young neutron star, such as the Crab pulsar, which has a surface magnetic field about $10^{13}--10^{12}$ gauss and a rotational frequency 30 Hertz, can indeed deliver a Poynting flux about $10^{38}\,\mathrm{erg/sec}$.

Detailed modelling of the $x-$ ray emissions requires the synchrotron photons to be produced in the post-shock region of an ultra-relativistic pulsar wind; furthermore, this pulsar wind immediately before its impact onto the standing shock must consist of an almost *unmagnetized* plasma [3,4,5]. In other words, the pulsar wind is initially accelerated by an electromagnetic drive, but must eventually manage to convert almost all the available electromagnetic energy into the flow kinetic energy before the fast wind impinges onto the shock located about one lightyear away from the neutron star. How the acceleration, satisfying the above requirement, may manage to operate is still an unsolved problem that has attracted the attention of many high-energy plasma astrophysicists over the past few decades.

With the advent of high-resolution x-ray imagers, the Rosat x-ray satellite has recently discovered that the x-ray emissions are actually confined within a well-defined torus, with its minor axis diameter about 1 lightyear [2]. As the $x-$ ray photons are produced immediately behind the shock when the electrons are thermalized, it is natural to estimate the lifetime of the x-ray producing high-energy electrons using the observed x-ray volume. The downstream flow speed of a strong relativistic shock is at most $1/\sqrt{3}$ of the speed of light c. Dividing the extent of the x-ray region in the torus by the downstream flow speed yields the lifetime of these 10-TeV electrons to be at least 1.7 years. However, a simple estimate for the synchrotron emission power for an optically thin (because of the photon polarization) hot plasma shows that the lifetime of the x-ray producing electrons should have been no longer than two months [1].

The two estimates differ by one order of magnitude, suggesting that the x-ray energy must be transported from the standing shock to fill the entire x-ray torus in an unconventional fashion. Thus, the energy must be temporarily stored in a coherent form and transported at a sizable distance before it is thermalized and yields the synchrotron x-ray. In addition to the above energetic argument, it has been noticed long ago in the optical continuum that long-lived, thin filaments perpendicular to the axis of the torus exist in the torus. They are dubbed as the wisps and their locations are changing on the time scale of years. These wisps can be large-amplitude nonlinear waves initiated at the front of the pulsar wind shock. If so, these waves can be the desired agents responsible for transporting the energy to fill the entire x-ray torus.

In this paper, I report a scenario that describes how part of the pulsar wind energy can be transported far away from the standing shock and dumps itself promptly to the background plasmas when certain conditions are met thereby producing the synchrotron x-ray. The physical picture under consideration has a close analogy to the wave dynamics near the sea shore. The shallow water wave has a dispersion relation that requires the wave speed to decrease when the water becomes increasingly shallow. A finite-amplitude wave has finite energy and momentum, and hence has a finite action associated with the wave. Since the wave action is an adiabatically invariant quantity as the wave progressively runs toward the beach, one can show that the wave amplitude grows and the wave steepens at the wavefront. When the wavefront becomes sufficiently steepened, the wave suddenly breaks and promptly converts its energy into heat.

In the above picture of wave steepening and breaking, two essential features are involved. First, waves become steepened when they propagate into an environment that has a decreasing wave speed according to the linear dispersion relation. Second, the wave steepening must be terminated at a certain threshold, beyond which the nonlinear waves rapidly become unstable and turbulent. These are the features that may occur within the z-ray torus embeded in the much larger Crab Nebula.

II. Initiation of finite-amplitude magnetosonic waves

The pulsar wind is of so low density that the plasma within it is collisionless. Even behind the standing pulsar-wind shock, the plasma can become also so hot that it is again collisionless within the time scale of several years. Strong collisionless shocks are known to be highly turbulent, especially for those that have a large ratio of thermal energy to magnetic energy. Concrete examples are found behind the earth bow shock when the high-speed solar wind impinges onto the earth. They include in regions of the bow shock where the shock normal directs perpendicular to the magnetic fields, the so-called perpendicular shock regions. Post-shock turbulence is manifested by the existence of abundant finite-amplitude waves propagating downstream. The primary reason for the excitation of these finite-amplitude waves is that the plasma can not promptly thermalize itself at the front of a collisionless shock. Therefore in order to satisfy the Rankine-Hugoniot relation, part of the energy and momentum downstream of a shock can only be stored in the form of waves, which then become dissipated far downstream. In this sense, one may regard the regions behind the collisionless shock, within which these waves are dissipated, as the transition layer of a collisionless shock, a layer that would have been of the diffusion length scale in a collisional shock.

These waves can propagate downstream at characteristic wave speeds in the reference frame of the bulk plasma, and furthermore since these waves are magnetosonic and of finite amplitude, they actually propagate at speeds faster than the linear phase velocity. Previous works [6, 7], studying relativistic plasmas containing electrons, positrons and some small amount of ions, show that these nonlinear waves can be localized in the forms of solitons, which have typical sizes about the ion gyro-radius. The solitons are related to the relativistic lower-hybrid waves in the linear regime, and in the long wavelength limit reduced to the familiar magnetosonic MHD waves. Moreover, in the small-amplitude limit, these solitons resemble the KdV solitons. They are characterized by two independent parameters, the Mach number and the Lorentz factor γ. For a given γ, the soliton amplitude becomes larger and width gets narrower as the Mach number increases. Beyond a critical Mach number, the tip of the soliton becomes singular and the soliton must break. In the relativistic-temperature regime, the solitons are, like a capacitor, charged with an electrostatic potential about $AT/m_i c^2$ GeV in the rest frame of the solitons, where T is the ion temperature, since the plasmas contain some small amount of ions of atomic number A. This large amount of energy will be released locally to heat the background plasmas when the solitons break.

In the framework of the aforementioned scenario of wave steepening and breaking, it is sufficient to show that these solitons propagate outwards into a background of an ever-decreasing magnetosonic speed

III. Background plasmas in the x-ray torus

Since the average magnetic fields downstream of the standing wind shock are toroidal and the average flow is radial, the two vector fields are perpendicular to each other. In simple configuration, the field lines are convected away as if they were scalar fields, obeying the frozen-in condition that

$$\frac{\partial}{\partial r}\left[\frac{B_0}{n_0 r}\right] = 0, \tag{1}$$

where no is the plasma density and the subscript "0" denotes the background quantities. It follows that B_0/n_0r is a distance-independent quantity. As the flow expands, the plasma density must decrease with distance and hence the magnetic field strength will also decrease outwards in the same way as the plasma density. The " classical " Alfven speed $B_0/\sqrt{n_0}$ hence will have to decrease with distance as $\sqrt{n_0}r$. In addition, as the flow expands, it must cost a plasma internal energy due to the PdV work, and hence the temperature decreases. For a relativistically hot plasma, the equation of state is polytropic with an index of adiabaticity equal to $4/3$. It then follows that the plasma temperature decreases as $n_0^{1/3}$. However, as long as the temperature is relativistically high, the sound speed will remain $c/\sqrt{3}$. The magnetosonic speed for a relativistically hot plasma in fact has a more delicated dependence [7] of T and B_0, and it is $[(\sigma+(1/3))/(\sigma$ t I)] ‘ / “ , where $\sigma \equiv 3B_0^2c^2/16\pi E$, the squared relativitic Alfven speed, and $E \sim n_0T$ the plasma internal energy. The magnetosonic speed can decrease with decreasing σ, which is proportional to $rn_0/T \sim rn_0^{2/3}$, or proportional to $(rv_0^2)^{-1/3}$ by virtue of the mass conservation $r^2v_0n_0 = const.$

IV. Wave steepening and breaking

The above evaluation for the scaling law of the magnosonic speed sets the stage for the discussions of wave steepening and eventual wave breaking to follow. In a test problem concerning the issue of wave steepening and breaking, Chiueh and Juang have recently studied the non-relativistic counterpart of the relativistic case in question [8]. In this work, a new type of non-relativistic magnetosonic solitons in a cold plasma are discovered, and they exist in the regime where the electron cyclotron frequency is much greater than the electron plasma frequency, an appropriate regime for which extension to the relativistic plasmas can be readily made. In this regime, the ion inertia is much greater than the electron inertia, and hence the evolution of solitons is dominantly governed by the ion dynamics.

As solitons are propagating in a slowly varying evironment, the strategy for the investigation of the soliton evolution can take advantage of the action of soliton being an adiabatic invariant. The adiabatic invariance of action stems from the fact that the actual solution itself is an extremum of action against any functional variation, and any first-order change of the solution arising from the change of environments only introduces a second-order change in the value of action. Thus, the original value of action remains unchanged to the first order. One may therefore evaluate the value of action for the original soliton solution in the fram of the background flow, as a function of the soliton speed V and the soliton location X. The X dependence arises from the spatial variation of the background and V dependence arises from the Mach number of the soliton. Actually, for the operational reason, one may instead evaluate the Lagrangian L of the soliton as a function of X and V, i.e, $L = L(X, V)$, and manages to variate the Lagrangian so that the action is an extremum. In so doing, one has cast the problem to a one-dimensional dynamical problem for a classical particle. Although it turns out that the soliton Lagragian can be formally expressed as if the soliton contains only the kinetic energy of the excess ions, i.e.,

$$L = \int (n_i - n_0)\mathbf{v}_i^2 dx, \tag{2}$$

where n_i, $\mathbf{v}_i$ and n_0 are the ion density and velocity and the background density respectively, the actual functional form of $L(X, V)$ is much more complicated since the electromagnetic fields are contained in the soliton. Though always integrable, the solitons trajectory as a function of time can not be expressed by a simple analytical form. Nevertheless, one can take further advantage of the Hamiltomian structure of this dynamical problem and constructs the appropriate Hamiltonian for such an $L(X, V)$. The resulting Hamiltonian is a conserved quantity. By great contrast, the total energy associated with the soliton is actually *not* a constant, since it can exchange energy with the background plasmas; the soliton mass and momentum are not constant, either, as the solitons propagate through the inhomogeneous background. Fig. (1) shows the result of the energy gain in the soliton as it propagates down the gradient of decreasing Alfven speed and gains its Mach number. The dashed lines show the equi-energy contours as a function of the Mach number and background density n_0, and the solid line shows the equi-Hamiltonian contour in the Mach number–density phase space. The two sets of contours cross, indicating that the soliton energy is not a conserved quantity. In particular, along the equi-Hamiltonian contour which traces the actual soliton trajectory, one finds that the soliton is gaining energy as its Mach number increases.

This test problem not only confirms the expectation that solitons become steepened when propagating into a background plasma of decreasing Alfven speed, but it also reveals a new insight that solitons can further extract energy from the background. This result can be pictured as follows. Upon sweeping across the region of decreasing Alfven speed, the solitons collect background magnetic flux and plasmas into themselves, and thus the downstream background magnetic fields and plasma density decrease after the passages of solitons.

Returning to the original problem concerning the solitons in Crab Nebula, we may follow the procedures of this illuminating test case. The main difference is that we will now have not only the Mach number as the sole charateristic of the soliton as in the non-relativistic case, but also the Lorentz factor γ. The Lagrangian so constructed will of the form, $L(M_A, \gamma, X, V_b)$, where M_A is the Mach number and V_f the decelerating background flow in the nebula. Nevertheless, the Lagrangian can still be expresses as $L(X, V)$. It is not clear, at this point, whether the soliton can also extract energy from the background, but it is much more certain that the soliton can become steepened as it travels outwards into the bulk of the nebula. Even if in the worse case, the solitons loss some energy to the background plasmas, the amount of electric potential, as seen by the background plasma,

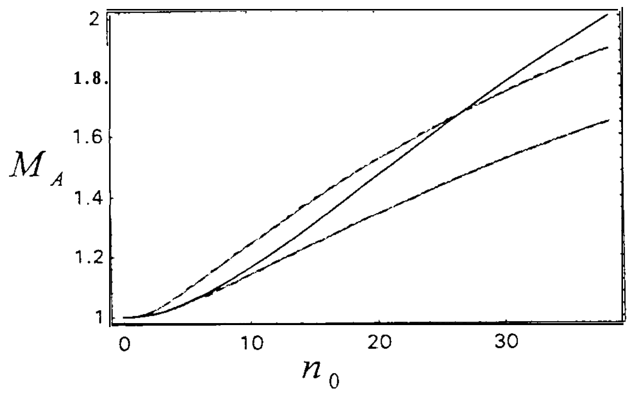

FIG. 1. Equi-Hamiltonian contour (solid line) and Equi-energy contours (dashed lines) of the solitons as functions of the instantaneous soliton Mach number M_A and background density n_0. Along the equi-Hamiltinian contour, it traces the soliton. trajectory and it is clear that the solitons are gaining energy.

will still be of order $\gamma Z\,\mathrm{GeV}$, which is dictated by the structure of the soliton solution; unless the soliton is significantly slowed down to a non-relativistic speed, which is unlikely if the initial γ is sufficiently large.

When the waves break, the soliton energy must be deposited into the background plasmas. Since the structure of solitons is governed by the ion inertia, one would have expected that it is ions that should be the primary receptor of the soliton energy. Indeed, this is true in the first phase of thermalization; more precisely, the soliton energy is primarily given to the ion motion perpendicular to the magnetic fields. However, the plasma must be locally turbulent and is prone to subject to various micro-instabilities, since it contains an ample amount of free energy. One thus expects that electrons can then easily tap the energy from the relativistically hot ions. The x-ray synchrotron emissions thus result from these energetized electrons.

In sum, I have pointed out an inconsistency between the x-ray observations and the conventionally accepted model of Crab Nebula. The resolution to it pertains to the recognition of the delayed thermalization for strong collisionless shocks. Only a fraction of the pulsar wind energy is dissipated at the inner edge of the x-ray torus, and the rest can only be deposited into the plasmas in much later time, of order a year or so, and across a much large distance. The wind energy can be carried outwards by energetic, localized solitary waves, and gets dumped rapidly when the solitory waves, or solitons, become steepened to such an extent that they must break. I have shown that wave steepening can occur because the effective magnetosonic speed in the nebula decreases outwards. In this regard, one should conceive the x-ray torus to be the transition region of the pulsar-wind terminal shock. The soliton model advocated in this report is also supported by the optical observations that show the existence of bright filaments, the wisps, which appear in motion on the time scale of years [9].

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References

[1] R. N. Manchester and J. H. Taylor, *Pulsars,* (Freeman, New York 1977).
[2] J. J. Hester *et.* al., Astrophys. J. 448, 240 (1995).
[3] M. J. Rees and J. E. Gunn, Mon. Not. Roy. Astron. Soc. 167, 1 (1974).
[4] C. F. Kennel and F. V. Coroniti, Astrophys. J. 283, 694 (1984).
[5] R. T. Emmering and R. A. Chevalier, Astrophys. J. 321, 334 (1987).
[6] T. Chiueh, Phys. Rev. Letts. 63, 113 (1989).
[7] T. Chiueh and T. C. Lai, Phys. Rev. A44, 6944 (1991).
[8] T. Chiueh and F. R. Juang, Phys. Rev. E55, 1002 (1997).
[9] J. D. Scargle, Astrophys. J. 156, 401 (1969).