

Bose-Einstein Interference and Factorization at High Energies

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We consider the emission or absorption of n identical bosons from an energetic or a massive particle, in which the bosons and the source particle are allowed to be offshell. The Bose-Einstein symmetrized amplitude can be decomposed into sums and products of more elementary objects which we choose to call 'atoms'. The origin of this decomposition, its significance and some of its applications will be summarized.

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I. Introduction

It is a great pleasure for me to dedicate this article to Prof. Ta-You Wu. I first met Prof. Wu many years ago, when I was an undergraduate student and he as the head of the Theory Group in the National Research Council of Canada. His important contributions to physics, to the training of physicists, and his seminal role in the development of science and physics in China, can be found elsewhere so they will not be repeated here. On this happy occasion of his 90th birthday, the Physical Societies of Chinese Mainland, Taiwan, Hong Kong, and Overseas have jointly organized a Meeting in his honour. This Meeting was held in Taipei from August 11 to 15, 1997. I was fortunate to be involved in the organization of this five-day meeting, and am happy to report that over three hundred physicists from all over the world participated, including four ethnic Chinese Nobel Laureates and a Field Medalist, as well as many leading physicists, especially those of Chinese ancestry. Twelve plenary talks and more than two hundred papers in the parallel sessions were delivered. The success of this Meeting can undoubtedly be attributed to the great admiration and respect the participants have for Prof. Wu, which motivated them to come from afar to dedicate their work to him.

I will base this article on the talk I gave at that conference. It is about the 'magic' of Bose-Einstein (BE) symmetry in the context of high-energy scattering. The 'magic' stems from a *factorization* (or decomposition) theorem which I will discuss below.

The experimental demonstration of BE condensation in the last two years [1] revives the interest in this old but important subject. The condensation can be traced to an effective attraction of bosons at short distances, induced by the constructive interference in the symmetrization of bosonic wave functions. These effects are well known and I will not dwell on them any further. Instead, I would like to ask whether BE *destructive* interference

plays any role in physics, and if so where does it become important. Certainly not at high temperatures, where phase informations are lost, but interestingly it manifests itself in high-energy scattering processes. In fact, the presence of such destructive interference is crucial in preventing certain theoretical disasters to occur. The more bosons there are the more important this interference effect will be. I shall come back to illustrate what I mean by this with some examples.

Constructive and destructive interferences are often two sides of the same coin. Consider for example a simple system of two particles with product wave function $\phi_1(\vec{x}_1)\phi_2(\vec{x}_2)$, where ϕ_1 and ϕ_2 are normalized and orthogonal to each other. When BE symmetrized, the normalized wave function becomes $\Phi(\vec{x}_1, \vec{x}_2) = [\phi_1(\vec{x}_1)\phi_2(\vec{x}_2) + \phi_1(\vec{x}_2)\phi_2(\vec{x}_1)]/\sqrt{2}$. At $\vec{x}_1 = \vec{x}_2 = \vec{x}$, $\Phi(\vec{x}, \vec{x}) = \sqrt{2}\phi_1(\vec{x})\phi_2(\vec{x}) > \phi_1(\vec{x})\phi_2(\vec{x})$, producing a *constructive interference*, which in the case of many-body wave functions leads to the effective attraction and condensation observed at low temperatures. Since both Φ and $\phi_1\phi_2$ are normalized, to preserve probability this enhancement at $\vec{x}_1 = \vec{x}_2$ must produce a depletion somewhere else, and this is just the *destructive interference* mentioned earlier as being important in high-energy scatterings.

It should be clarified at this point that 'high energy' means high *total* energy, and not necessarily high kinetic energy. Large mass in the presence of low kinetic energy would qualify as high energy as well. The latter occurs in systems involving heavy baryons and heavy quarks. The second example discussed below is of this latter variety.

Let me now discuss two examples illustrating the importance of BE destructive interference at high energies.

Consider first high-energy (\sqrt{s}) near-forward elastic scattering. Since each loop-integration can potentially produce a $\ln(s)$ factor, the effective coupling constant is $g_{eff}^2 = g^2 \ln(s)$, which can be large even when the coupling g^2 is small. Consequently at high energies perturbation diagrams of all orders are to be included, resulting in many gauge bosons being exchanged. BE symmetrization of these virtual bosons is carried out by summing Feynman diagrams in which gauge vertices are permuted in all possible ways. The theoretical effect of BE symmetry can therefore be seen by comparing the sum with the individual Feynman diagrams. As mentioned before, individual diagrams can grow with a positive power of $\ln(s)$, and the power generally increases with the order of the perturbation diagram. This can easily lead to the violation of Froissart bound! Fortunately, in electron-electron scattering via multiple-photon exchanges, such a disaster is averted, and unitarity restored, when the Feynman diagrams are added together. This is so because all positive powers of $\ln(s)$ are cancelled in the sum. *What causes* such a magical *cancellation*? As we shall discuss later, it is the BE destructive interference.

In QCD things become considerably more complicated. Explicit calculation up to the sixth order [2] shows that, depending on the colour of the exchange channel, some but not all of these powers of $\ln(s)$ may be cancelled. In fact, it turns out that none of the $\ln(s)$ powers are cancelled in the colour-octet channel. Instead, their contributions pile up to form a reggeized gluon [2, 3], thereby using another mechanism to restore unitarity. This is in sharp contrast to the case of QED discussed above, where photons do not reggeize and the restoration of unitarity relies on cancellations.

Why do we have this drastic difference between two gauge theories? The answer

can be found in the different way BE symmetry is being implemented in the two cases. Gluons carry a new *nonabelian* quantum number, 'colour', so for them both colour and spacetime coordinates must be BE symmetrized. This produces the qualitative difference between QCD and QED noticed above, but it also makes nonabelian BE symmetrization considerably more difficult to analyse [4-7].

I will now discuss briefly a second example which deals with *real* rather than virtual bosons. In the pi-nucleon Yukawa theory derived from QCD with a large number of colours N_c , the Yukawa coupling constant is proportional to $\sqrt{N_c}$, so every Feynman tree diagram with n pions grows like $N_c^{n/2}$ [8]. One might conclude from this that the theory favours the production of a large number of pions, and that high-order loop diagrams are terribly important. Fortunately this is not so because in summing the $n!$ tree diagrams to enforce BE symmetry, the large amplitudes coming from individual diagrams get quite thoroughly cancelled out, leaving behind only a residue going down with N_c like $N_c^{1-n/2}$ [8]. This cancellation once again can be attributed to BE destructive interference [9]. Incidentally, this is a 'high-energy' process even though the pions carry little kinetic energy because the mass of the nucleon is proportional to N_c .

These two examples illustrate the theoretical consequences of BE destructive interference. I believe these interferences also have directly observable *phenomenological* consequences, but unfortunately I have not had time to work them out as yet.

The destructive interference phenomenon discussed above follows from a far more fundamental property: the high-energy BE-symmetrized tree amplitude satisfies a *factorization* (or decomposition) theorem [4,5]. This theorem allows the amplitude to be decomposed into sums of products of more primitive objects which I choose to call atoms. An atom may have any number of bosons, and it differs from a Feynman diagram only in that it carries the '*adjoint colour*'. Discussions on its precise meaning, as well as the significance of this decomposition, will be postponed to later sections.

For abelian amplitudes this theorem simply degenerates into the well known 'eikonul formula' [10]. We may therefore think of this decomposition as a nonabelian *eikonul* formula.

In the following sections we shall discuss these and other topics in a more quantitative way.

II. High-energy approximation

Consider the tree amplitude of Fig. 1 in which all components of the bosonic momenta k_i are much less than the energy p^0 of the source particle. In what follows the final momentum p is always taken to be onshell, $p^2 = m^2$, but the initial momentum $p' = p + \sum_{i=1}^n k_i$ may or may not be. The bosonic momenta are always arbitrary so they can be sewed up with other bosons to turn the tree diagram into a loop diagram. In that way the formulas developed below for tree amplitudes can be applied to loop diagrams as well.

In this kinematical regime, we can approximate the denominator of a propagator by ignoring the quadratic term K^2 in the expression $(p + K)^2 - m^2 + it \simeq 2p \cdot K + it \equiv \omega + i\epsilon$, where K is a sum over a number of k_i 's. We shall refer to this approximation as the *eikonul approximation*. It can be shown that this amounts to ignoring the effect of recoil and treating the trajectory of the source particle in configuration space as a straight line

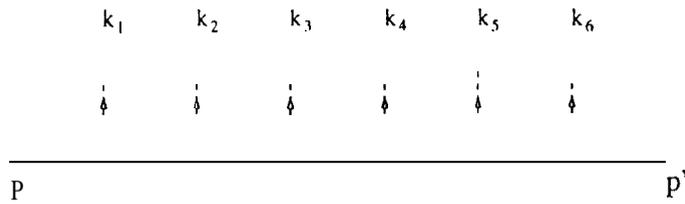


FIG. 1. (a) A Feynman tree diagram for the emission of n bosons from an energetic or massive particle of final momentum p and initial momentum p'

throughout. This approximation will be used in the rest of the discussions, with $\omega_i \equiv 2p \cdot k_i$.

We shall also assume the absence of numerators for the propagators of Fig. 1, so if λ_i represents the i th vertex, the offshell scattering amplitude for Fig. 1 is given by

$$A^*[123 \cdots n] = \prod_{i=1}^n \frac{1}{\sum_{j=1}^i \omega_j + i\epsilon} \cdot \lambda_1 \lambda_2 \lambda_3 \cdots \lambda_n \quad (2.1)$$

$$\equiv a^*[123 \cdots n] \cdot \lambda[123 \cdots n].$$

For easy reference, we shall generically refer to the vertex factors λ_i as 'colour matrices' and the nonabelian quantum numbers carried by the bosons as 'colours'. There is no implication in this terminology that λ_i has to be an $SU(3)$ matrix, nor even that they have to be the generators of any group.

For the onshell amplitude, $(p')^2 = m^2$ leads to $\sum_{i=1}^n \omega_i = 0$. The last propagator is absent but it is convenient to include explicitly in the amplitude the onshell δ -function, thus making

$$A[123 \cdots n] = -2\pi i \delta \left(\sum_{i=1}^n \omega_i \right) \prod_{i=1}^{n-1} \frac{1}{\sum_{j=1}^i \omega_j + i\epsilon} \cdot \lambda_1 \lambda_2 \lambda_3 \cdots \lambda_n \quad (2.2)$$

$$\equiv a[123 \cdots n] \cdot \lambda[123 \cdots n].$$

If $\{S\} = \{s_1 s_2 s_3 \cdots s_n\}$ is a permutation of $\{123 \cdots n\}$, $[s]$ is a Feynman diagram with the n boson lines similarly permuted, then its offshell amplitudes $A^*[s]$ and its onshell amplitude $A[s]$ can be obtained from (2.1) and (2.2) respectively by making this same permutation. BE symmetrization of the bosons are carried out by summing the $n!$ permuted Feynman diagrams over the symmetric group S_n :

$$M_n^* = \sum_{\{s\} \in S_n} A^*[s], \quad (2.3)$$

$$M_n = \sum_{\{s\} \in S_n} A[s].$$

The absence of a numerator factor in the propagator does not mean that the source particle cannot carry a spin. To illustrate this point let us assume the source particle to

carry spin $\frac{1}{2}$, and a momentum $p + K$. In the high energy approximation, the numerator factor $m + \gamma \cdot (p + K) \simeq m + \gamma \cdot p$ is equal to $\sum_{\lambda} u_{\lambda}(p) \bar{u}_{\lambda}(p)$, so if Γ_i is the true vertex factor for the diagram, it is equivalent to a theory without the numerator factor, but with $(\lambda_i)_{ab} = \bar{u}_a(p) \Gamma_i u_b(p)$ as the vertex. For example, for vector coupling (γ^{μ}) to a spin-1 particle, the effective vertex λ is proportional to $\bar{u}_a(p) \gamma^{\mu} u_b(p) = 2p^{\mu} \delta_{ab}$. Physically this simply says that the spin current is negligible compared to the translational current in the high energy limit. Similarly, for axial vector coupling ($\gamma^{\mu} \gamma_5 \vec{\tau} \partial_{\mu}$) to a pion, the effective vertex λ is proportional to $\sigma^i \vec{\tau} k_i$, so in this case 'colour' is really spin and isospin. Similar arguments can be applied to source particles of higher spins. In this way we are again invoking the familiar statement that spin is in some sense immaterial at high energies.

III. Abelian bosons

For abelian bosons we may assume all $\lambda_i = 1$, so $A[s] = a[s]$ and $A^*[s] = a^*[s]$. Let us first look at the simplest case, with two bosons.

III-1. $n = 2$

In this case

$$\begin{aligned} M_2^* &= a^*[12] \quad \mathbf{t} \quad a^*[21] = \frac{1}{\omega_1 + i\epsilon} \frac{1}{\omega_1 + \omega_2 + i\epsilon} \frac{1}{\omega_2 + i\epsilon} \frac{1}{\omega_1 + \omega_2 + i\epsilon} \\ &= \frac{1}{\omega_1 + i\epsilon} \frac{1}{\omega_2 + i\epsilon}, \end{aligned} \quad (3.1)$$

$$\begin{aligned} M_2 &= a[12] + a[21] = -2\pi i \delta(\omega_1 + \omega_2) \left\{ \frac{1}{\omega_1 + i\epsilon} \quad \mathbf{t} \quad \frac{1}{\omega_2 + i\epsilon} \right\} \\ &= (-2\pi i)^2 \delta(\omega_1) \delta(\omega_2). \end{aligned} \quad (3.2)$$

Note that factorization occurs in both the offshell and the onshell amplitudes. Moreover, for onshell amplitudes, the result of adding two $1/\omega$ distributions is to produce a sharply peaked interference pattern $\delta(\omega)$, with constructive interference at $\omega = 0$ and complete destructive interference at $\omega \neq 0$. The more $\&$ functions there are, the more sharply peaked the interference pattern will be. For that reason we shall measure the 'amount of spacetime interference' by the number of b-functions present.

III-2. Arbitrary n

Factorization persists to arbitrary n . The result is [10]

$$M_n^* = \sum_{\{s\} \in S_n} a^*[s] = \prod_{i=1}^n \frac{1}{\omega_i + i\epsilon}, \quad (3.3)$$

$$M_n = \sum_{\{s\} \in S_n} a[s] = \prod_{i=1}^n [-2\pi i \delta(\omega_i)]. \quad (3.4)$$

It is not difficult to understand why these sums factorize. The only difference between the $n!$ Feynman diagrams is the order of emission of the bosons. When summed over all possible orderings, each boson is allowed to be emitted independently at any place along the source, hence factorization results. Eikonal approximation is needed to derive factorization because we have implicitly assumed the source not to recoil in the above argument. Otherwise it does make a difference whether a boson is emitted before or after a recoil takes place.

IV. Nonabelian bosons

The algebra becomes more complicated in the nonabelian case because the vertex factors λ_i do not commute with one another. The case for $n = 2$ is still trivial to work out, but the combinatorials for an arbitrary n are fairly involved, so we will be content just to state the final result here.

IV-1. $n = 2$

By adding and subtracting $a^*[21]\lambda_1\lambda_2$ or $a[21]\lambda_1\lambda_2$, we get

$$\begin{aligned} M_2^* &= a^*[12]\lambda_1\lambda_2 + a^*[21]\lambda_2\lambda_1 \\ &= \frac{1}{\omega_1 + i\epsilon} \frac{1}{\omega_1 + \omega_2 + i\epsilon} \lambda_1\lambda_2 + \frac{1}{\omega_2 + i\epsilon} \frac{1}{\omega_1 + \omega_2 + i\epsilon} \lambda_2\lambda_1 \\ &= \frac{1}{\omega_1 + i\epsilon} \frac{1}{\omega_2 + i\epsilon} \lambda_1\lambda_2 + \frac{1}{\omega_2 + i\epsilon} \frac{1}{\omega_1 + \omega_2 + i\epsilon} [\lambda_2, \lambda_1], \end{aligned} \quad (4.1)$$

$$\begin{aligned} M_2 &= a[12]\lambda_1\lambda_2 + a[21]\lambda_2\lambda_1 \\ &= -2\pi i \delta(\omega_1 + \omega_2) \left\{ \frac{1}{\omega_1 + i\epsilon} \lambda_1\lambda_2 + \frac{1}{\omega_2 + i\epsilon} \lambda_2\lambda_1 \right\} \\ &= (-2\pi i)^2 \delta(\omega_1) \delta(\omega_2) \lambda_1\lambda_2 - 2\pi i \delta(\omega_1 + \omega_2) \frac{1}{\omega_2 + i\epsilon} [\lambda_2, \lambda_1]. \end{aligned} \quad (4.2)$$

111-2. Atoms

An *atom* A_c^* or A , is a Feynman amplitude whose product of vertices has been replaced by their multiple commutators:

$$A_c^*[s_1 s_2 \cdots s_n] = a^*[s_1 s_2 \cdots s_n] [\lambda_{s_1}, [\lambda_{s_2}, [\cdots [\lambda_{s_{n-1}}, \lambda_{s_n}] \cdots]]], \quad (4.3)$$

$$A_c[s_1 s_2 \cdots s_n] = a[s_1 s_2 \cdots s_n] [\lambda_{s_1}, [\lambda_{s_2}, [\cdots [\lambda_{s_{n-1}}, \lambda_{s_n}] \cdots]]]. \quad (4.4)$$

$$(4.5)$$

In terms of these, Eqs. (4.1) and (4.2) can be written

$$\begin{aligned} M_2^* &= A_c^*[1] A_c^*[2] + A_c^*[21], \\ M_2 &= A_c[1] A_c[2] + A_c[21]. \end{aligned} \quad (4.6)$$

It will be convenient to write a single notation for the products of atoms by merging the symbols together. For example, we shall write $A_c[1|2] = A_c[1]A_c[2]$, and $A_c[51|342|6] = A_c[51]A_c[342]A_c[6]$, with the vertical bar $|$ indicating where products should occur. We shall call the vertical bar a cut and these single A_c 's as *cut amplitudes* [5]. In terms of this notation we can rewrite Eq. (4.6) in terms of the cut amplitudes as follows: as

$$\begin{aligned} M_2^* &= A_c^*[1|2] + A_c^*[21], \\ M_2 &= A_c[1|2] + A_c[21]. \end{aligned} \tag{4.7}$$

IV-3. Decomposition theorem for arbitrary n

The atomic decomposition for a general BE-symmetrized amplitude M_n^* or M_n is

$$\begin{aligned} M_n^* &= \sum_{\{s\} \in S_n}^{n!} A_c^*[s_c], \\ M_n &= \sum_{\{s\} \in S_n}^{n!} A_c[s_c]. \end{aligned} \tag{4.8}$$

In other words, we simply replace the Feynman amplitudes $A^*[s]$ and $A[s]$ in (2.3) by the corresponding *cut* amplitudes $A_c^*[s_c]$ and $A_c[s_c]$, where $[s_c]$ is $[s]$ with some vertical bars inserted.

We must still specify how to put the vertical cuts inside the symbol $s = s_1 s_2 \cdots s_n$ to get s_c . The rule is simple though the proof is not [4]. Starting from the left, a vertical cut is put after s_i iff $s_i < s_j$ for all $j > i$. It is trivial to see that Eq. (4.7) satisfies this rule. For further illustration, the atomic decomposition for $n = 3$ is

$$M_3 = A_c[1|2|3] + A_c[1|32] + A_c[21|3] + A_c[231] + A_c[31|2] + A_c[321]. \tag{4.9}$$

IV-4. Abelian decomposition

For abelian vertices λ_i all commutators vanish, so only one-boson atoms survives. The decomposition (4.8) has only one term, $A_c^*[1|2|3|\cdots|n]$ or $A_c[1|2|3|\cdots|n]$. In other words, it factorizes in the way given by Eqs. (3.3) and (3.4).

For offshell amplitudes, this complete factorization leads to a Poissonian multiplicity distribution for the production of photons. It enables the amplitudes of all orders to be summed up to an exponential form, thus allowing an eikonal and a geometrical interpretation [2, 10, 11]. It is also this exponential form that allows the infrared divergences to be cancelled [12].

For onshell amplitudes, the appearance of $\delta(\omega_i) = \delta(2p \cdot k_i)$ indicates for example that the photons bremsstrahlung from electron-electron scattering must be emitted in the forward-backward directions.

Since the abelian cases are rather well known, we shall now pass on to the nonabelian situation.

IV-5. Spacetime and colour interferences

IV-5-1. Spacetime interference

Each onshell amplitude consists of a b-function (see Eq. (2.2)), whose argument is the sum of w 's of the bosons. When a BE-symmetrized *onshell* amplitude M_n is decomposed into the sum of Feynman amplitudes, as in (2.3), the dependence on the remaining $(n - 1)$ w 's is of the form $\prod(\sum \omega_i)^{-1}$ (see (2.2)), which has a broad distribution in the w variables. In contrast, when M_n is decomposed in terms of sums of products of atoms, as in (4.8), $(\alpha - 1)$ additional $\&$ functions appear in the term with α atoms. Just as in Sec. 3.1, these $\&$ functions may be thought of as peaked interference patterns produced by the coherent addition of the various broad Feynman amplitudes.

Constructive interference occurs when the argument of the S-function is zero, and complete destructive interference occurs whenever the argument is nonzero. In a high-energy scattering amplitude, is *the overall interference effect* constructive, or destructive? At least in the two examples being discussed in the Introduction, both of them appear to be destructive for the following reasons.

Schematically the $\ln(s)$ factors found in the elastic scattering Feynman amplitudes come from integrations of the form $\int^s d\omega/w = \ln(s)$. With interference, some $1/w$'s are changed to $\delta(w)$'s. Since $\int d\omega \delta(\omega) = 1$, the $\ln(s)$ dependence disappears. This shows that cancellation of $\ln(s)$ factors in elastic scattering is a BE *destructive interference* effect.

For QED (4.8) has $(n - 1)$ atoms and $(n - 1)$ additional b-functions. This is why a complete cancellation of $\ln(s)$ occurs in electron-electron scattering mediated by the multiple exchange of photons. For QCD atoms of all sizes appear, leaving behind fewer $\&$ functions and less $\ln(s)$ cancellations. In particular, there are no extra b-functions in the term with a single atom, so that term gives rise to no $\ln(s)$ cancellations. Eventually these uncanceled powers of $\ln(s)$ will sum up to be a power of s , bringing reggeization to the gluon. Since atoms always carry octet colour, this happens only in the octet channel, which is why the gluon may reggeize but the photon may not.

Other terms contain more than one atom so additional $\&$ functions are present and some amount of $\ln(s)$ cancellations occur [5,6].

IV-5-2. Colour interference

The multiple commutators appearing in an atom may be interpreted as exhibiting colour interference. To see how this comes about let us compare this with a Feynman amplitude, where the 'colour matrices' λ_i appear as ordinary products. If each matrix is coupled to a boson in the 'adjoint representation', then a Feynman amplitude with n vertices is coupled to n bosons whose total colour spreads over a wide range, a range that involves any colour obtainable by coupling n adjoint colours. In contrast, when these n colour matrices appear as a multiple commutator, only adjoint representation survives. The adjoint commutator may therefore be thought of as a *colour* interference pattern, obtained by coherently adding up several broad colour distributions to yield a pattern which is sharply peaked at the adjoint colour and zero at all other colours.

It is this colour interference that averts a potential disaster [9] in the inelastic pi-nucleon process $\pi + N \rightarrow (n - 1)\pi + N$ in large- N , QCD.

Imagine using Fig. 1 to describe this process. It is known that this Feynman diagram

is of order $N_c^{n/2}$. On the other hand, it is also known that the full amplitude, obtained by summing over the $n!$ permutations of the pions, must behave like $N_c^{1-n/2}$ [8]. What is the mechanism for $n-1$ powers of N_c to be cancelled out in the sum? The answer turns out to be 'colour interference', at least for tree diagrams [9]. A similar cancellation must take place for loop diagrams as well, but I have not yet been able to figure out how it works there.

A colourless nucleon is made up of N_c quarks. Even if all these quarks are in the S state, arbitrary spin and isospin alignments for the N_c quarks are allowed, thus producing many nucleon resonances with spins and isospins ranging from $\frac{1}{2}$ to $\frac{1}{2}N_c$. It turns out to be these high-spin/isospin resonances that give rise to the unacceptably large contribution $N_c^{n/2}$ in the amplitude. If somehow these contributions can be suppressed in the sum, then there is a chance to obtain the right answer $N_c^{1-n/2}$.

In the rest system of the heavy nucleon, whose mass M is proportional to N_c , $\omega_i = 2p \cdot k_i$ is $2M$ times the pion energy in this frame. Our notation is such that the w for the outgoing pions are positive and the w for the single incoming pion is negative. Since the pions are massive, it is impossible for the sum of any number of outgoing ω_i 's to vanish, so any term that involves a b -function with this sum as its argument would disappear. As a result, only single-atom terms survive in the atomic decomposition (4.8). However, the 'colour' (actually spin and isospin here) of a single atom is composed of multiple commutators, all of them having only the adjoint 'colour' (meaning singlet and triplet spins and isospins here). This makes it impossible for nucleon resonances of high spin/isospin to contribute, and it can be shown that this makes the total sum to behave correctly like $N_c^{1-n/2}$ [9].

IV-6. Complementarity between spacetime and colour interferences

The sharpness of spacetime interference can be measured by the number of additional δ -functions present. The sharpness of colour interference can be measured by the number of commutators around. Since in every term of the atomic decomposition (4.8), the sum of these two is always equal to $n-1$, a complementarity exists between the two kinds of interferences. If one is large, then the other is small, and vice versa.

IV-7. Advantage of atomic decomposition and nonabelian cut diagrams

The decomposition formula (4.8) can be considered as a resummation formula for Eq. (2.3), with both spacetime and colour interferences, and hence cancellations, automatically built in.

Most multiloop Feynman diagrams cannot be computed analytically, though in the presence of a large parameter (e.g., $\ln(s)$ or N_c) one might hope in certain cases to compute it approximately. Even so one can expect to compute it only up to the leading approximation, *viz.*, the leading power of $\ln(s)$ or N_c . If such leading contributions to individual Feynman diagrams cancel out in the sum, we would be left with no means to calculate other than tackling the subleading contributions, which is very difficult. That is not all, for the subleading and the sub-subleading contributions may also be cancelled. Such is the case in the elastic scattering of two electrons where all powers of $\ln(s)$ are cancelled, and is also the case for the πN inelastic scattering problem where $(n-1)$ powers of N_c are cancelled.

The atomic decomposition now offers a new way to do such calculations. Since all

interference and cancellation effects are already built into the atoms, or the factorization, no further unwanted cancellations will occur if we take the large $\ln(s)$ or large N_c limit. This atomic decomposition can actually be implemented in a graphical way, by making trivial changes to the Feynman diagrams to turn them into nonabelian *cut diagrams* [5,6] suitable for such calculations. As a matter of fact, besides not having to deal with cancellations at the end, individual nonabelian cut diagrams are actually easier to calculate than individual Feynman diagrams.

V. Conclusion

It is somewhat surprising that many seemingly unrelated phenomena can be understood by the factorization and *interference* effects inherent in BE symmetrization. It is interesting that by taking this symmetrization early in the calculation one can produce a new and powerful calculational scheme in terms of the *nonabelian cut diagrams*. The only approximation needed to obtain these generic and common results is to have sources to be energetic or massive. Many physical phenomena seem to fall into this category, so we are hopeful that the technique and the understanding developed here can be used to study a variety of unsolved nonabelian problems.

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