

Penetration of Magnetic Flux and Electrical Current Density into Superconducting Strips and Disks[†]

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(Received September 1, 1995)

It has been known for over 30 years how to calculate magnetic-flux-density and current-density profiles in the critical state of type-II superconducting slabs and cylinders subjected to parallel applied magnetic fields. These profiles, which are parameterized by the critical current density J_c , are dependent on magnetic history. Recent theoretical developments have now made it possible to easily calculate the critical-state magnetic-flux-density and current-density profiles associated with vortex penetration into thin superconducting strips and disks subjected to perpendicular applied magnetic fields and transport currents. In this paper we briefly review what has been done in superconducting strips and disks, discuss the detailed behavior for a strip in an increasing applied perpendicular magnetic field, compare these results with those for a slab in an increasing parallel field, and finally present results for the measuring-circuit geometry dependence of the apparent hysteretic losses for a strip carrying an alternating current.

PACS. 74.60.Ge - Flux pinning; flux creep, and flux-line lattice dynamics

PACS. 74.60.Jg - Critical currents.

PACS. 74.76.-w - Superconducting films.

I. Introduction

There have been a number of recent theoretical advances in understanding the magnetic field profiles in superconducting strips and disks. These advances have been driven in part by the high degree of activity in the field of high-temperature superconductivity in visualizing and quantitatively analyzing the flux distribution by magnetic decoration [1-3], magneto-optics [4-12], Hall probes [13-16], scanning Hall probes [17-19], Hall-sensor arrays [20-21], Lorentz microscopy [22], magnetic force microscopy (MFM)[23], and scanning SQUID microscopy [24-28].

In this paper, we discuss several of these theoretical developments. In Sec. II we briefly review some critical-state profiles for a strip in a perpendicular applied field, and in Sec. III we report on a comparison of apparent and real ac losses measured by transport measurements with an alternating current. We give a brief summary in Sec. IV.

[†] Refereed version of the invited paper presented at the 1995 Taiwan International Conference on Superconductivity, August 8-11, 1995, Hualien, Taiwan, R.O.C.

II. Critical-State profiles

The critical-state theory [29-31], characterized by a critical current density J_c , has long been used to quantitatively understand quasi-static flux penetration into type-II superconducting slabs and cylinders in a parallel field. Despite the important early work of Norris [32], which did not receive the notice it deserved, until recently it was commonly thought that calculations of the flux penetration into flat samples subjected to a perpendicular magnetic field would be too complicated to calculate analytically. A number of recent papers [33-35], however, have extended the method of Norris [32] to derive analytical expressions for the magnetic-field (B) and current-density (J) profiles in long, thin superconducting strips whose critical current density obeys the Bean model [29-31] (J_c independent of B). An important theoretical development by Mikheenko and Kuzovlev [36] has permitted B and J profiles also to be calculated [36-38] for circular disks characterized by a field-independent J_c .

Figure 1 compares the critical-state current-density and magnetic-field profiles in an infinite slab [(a) and (b)] of thickness $2W$, subjected to a parallel magnetic field B_a with the corresponding profiles in a long strip [(c) and (d)] of width $2W$ in a magnetic field B_a applied perpendicular to the flat faces. In both cases, the applied magnetic field is along the z direction, the width of the sample is measured along the x direction, and the induced currents flow in the plus or minus y direction. The characteristic magnetic fields used to scale B_a in this figure are $B_s = (4\pi/c)WJ_c$ in Gaussian units or $B_s = \mu_0 WJ_c$ in mks units for slab geometry and $B_f = (4/c)dJ_c$ in Gaussian units or $B_f = \mu_0 dJ_c/\pi$ in mks units for film or strip geometry.

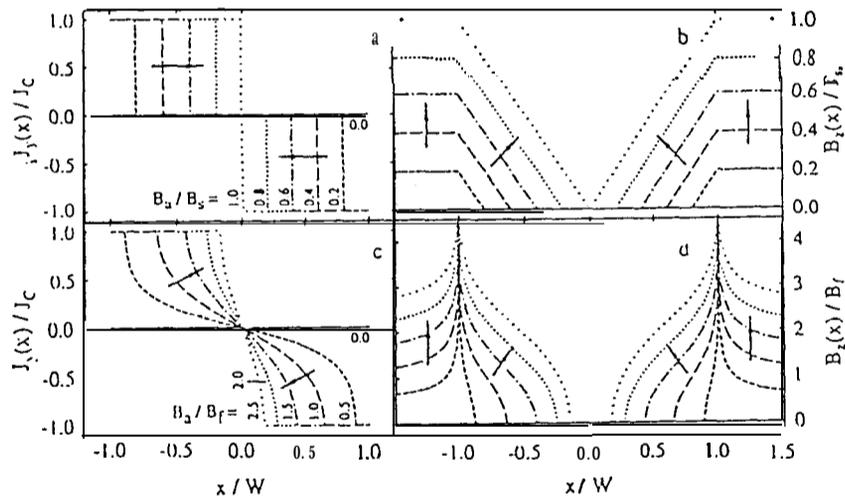


FIG. 1. Calculated critical-state behavior as the applied field B_a is increased for a sample initially in the virgin state (Ref. [35]). The current-density and magnetic-flux-density profiles for a slab are shown in (a) and (b), and the corresponding profiles for a strip are shown in (c) and (d). Arrows indicate the progression of the profiles as B_a increases.

The left panels [Fig. 1 (a) and (c)] show the evolution of the quasi-static current-density profiles $J_y(x)$ versus x as the field B_a is applied to a sample in the virgin state. For the case of the strip, $J_y(x)$ is the current density averaged over the sample thickness. In the slab (a), the magnitude of J_y is J_c in the flux-penetrated regions but drops discontinuously to zero in the vortex-free regions. (This drop actually occurs over a range characterized by the penetration depth λ , but here only samples for which $\lambda \ll W$ are being considered, so that the change in J_y appears to be discontinuous on the scale of W). In the strip (c), J_y is $\pm J_c$ in the flux-penetrated regions but varies smoothly from $+J_c$ to $-J_c$ over the vortex-free regions. Note that there is no discontinuity in J_y at the flux fronts. The nonzero value of J_y arises from the discontinuity of the tangential component of the magnetic field at the top and bottom surfaces of the sample. There are very strong demagnetizing effects in this geometry, as the magnetic field wraps around the sample, and the current density can be thought of as arising entirely from the tangential magnetic-field discontinuity [7,16,32-42].

The right panels [Fig. 1 (b) and (d)] show the evolution of the quasi-static magnetic-flux-density profiles $B_z(x)$ versus x as the field B_a is applied to a sample in the virgin state. For the case of the strip, $B_z(x)$ is evaluated in the plane of the strip. In the slab (b), the distributions are the well-known sandhill profiles, which have constant slope if J_c is independent of B , and B inside the superconductor is taken to be approximately equal to $\mu_0 H$. (The latter assumption is appropriate in high- κ superconductors when the local field H is well above the lower critical field H_{c1} .) In the strip (d), the distributions are similar to those in (b), but the slopes are not constant, even with constant J_c . An unphysical feature of the profiles is the infinite slope at the flux fronts; in the next level of approximation: this is smeared out over a length scale characterized by the sample thickness d . Another unphysical feature is the logarithmic divergence at the sample edges: which is a direct consequence of the Biot-Savart law in strip geometry. In the next level of approximation, this singularity, which is an artifact arising from taking the sample thickness to be zero, is rounded off over the length scale d .

As shown in Refs. [34] and [35], similar profiles can be obtained for a strip, initially in the virgin state, subjected to a transport current, as well as many other combinations of applied fields and currents. The quasi-static critical-state behavior of a strip in an ac field or current can be calculated most conveniently using a superposition approach, and convenient expressions for the hysteretic ac losses and complex ac permeability can be obtained ([34,35]).

Starting with the approach of Ref. [36], one may obtain similar results for a circular disk subjected to a perpendicular field [37,38]. As shown in Refs. [37] and [38], it is a straightforward matter to calculate the hysteretic ac losses and complex ac permeability.

III. Apparent and real transport ac losses

The theoretical expression [32] for the hysteretic losses in a strip carrying an alternating current $I(t) = I_0 \cos(\omega t)$ should be directly relevant to the self-field ac losses of multifilamentary tapes containing a uniform density of superconducting filaments. This is because the filaments are coupled, and the entire cross section behaves as a monolith with an effective critical current density J_e given by J_c of the individual filaments multiplied by the superconducting volume fraction. We do not discuss here the complication of the

possible presence of both intergranular and intragranular losses, which has been treated in recent papers by K.-H. Müller et al. ([43,44]).

An initially surprising experimental fact is the observed voltage-tap dependence of the apparent ac power loss of a strip, i.e., the time-averaged value of $I(t)V(t)$, where $V(t)$ is the voltage measured between a pair of contacts spaced along the length. The apparent power loss is found to depend on where the voltage taps are placed on the tape (along an edge or along the centerline) and on how far away the voltage leads are extended before they are brought together and twisted ([45,46]).

As explained by Campbell ([47]), it is important to account for the fact that when low-resistance leads are attached at contact points a and b on the conductor, the voltage V measured by a high-impedance voltmeter is the sum of an E-field integral term and a flux term. As shown in Refs. ([48,49,50]), this voltage is

$$V = \int E \cdot dl - (d/dt)\Phi, \quad (1)$$

where the line integral is to be carried out from a to b along a path C_s through the conductor, and Φ is the magnetic flux up through the loop bounded by the path C_s and the measuring circuit leads (which define the contour C_M). Because of magnetic hysteresis, the flux term, i.e., the second term on the right-hand side of Eq. (1), has terms in strip geometry that are both in phase and out of phase with the current $I(t)$. Only for a circular wire does the flux term have a vanishing in-phase component. As pointed out by Campbell ([47]), in general only when the leads are brought out to a large distance before bringing them together does the measurement give the true loss, i.e., the dissipated power delivered by the power supply to the segment between a and b .

Using Eq. (1) and the results of Refs. [34] and [35] discussed in Sec. II, we have calculated the power loss per cycle per unit length L_c , which is just the integral over one period of the product of the current $I(t)$ and voltage per unit length $V'(t)$ for two geometries. For one of these, which we call the *parallel* geometry, the contacts a and b are attached at the edge of the strip of width $2W$, and the leads are kept in the xy plane (the plane of the strip) as they are brought perpendicular to the strip to a distance X from the center of the strip, at which point they are brought straight together, then twisted, led out, and connected to the voltmeter. The results of this calculation can be expressed in terms of elementary functions, but the final expression is too long to give here. Instead, we show the results for this geometry as the dot-dash curves in Fig. 2. The apparent loss L_c in the parallel geometry is largest when $X/W = 1.0$, i.e., when the leads are brought along the edge of the strip. With increasing X , the apparent loss quickly approaches the real, or true loss per cycle per unit length [32] (solid curve in Fig. 2),

$$L = (\mu_0 I_c^2 / \pi) [(1 - F) \ln(1 - F) + (1 + F) \ln(1 + F) - F^2], \quad (2)$$

where $F = I_0/I_c$ and $I_c = 2dWJ_c$ is the critical current of the strip.

We also have calculated results for the apparent losses L_c in the *perpendicular* geometry, for which the contacts a and b are attached along the centerline of the strip, and the leads are kept in the yz plane as they are brought perpendicular to the strip to a height Z

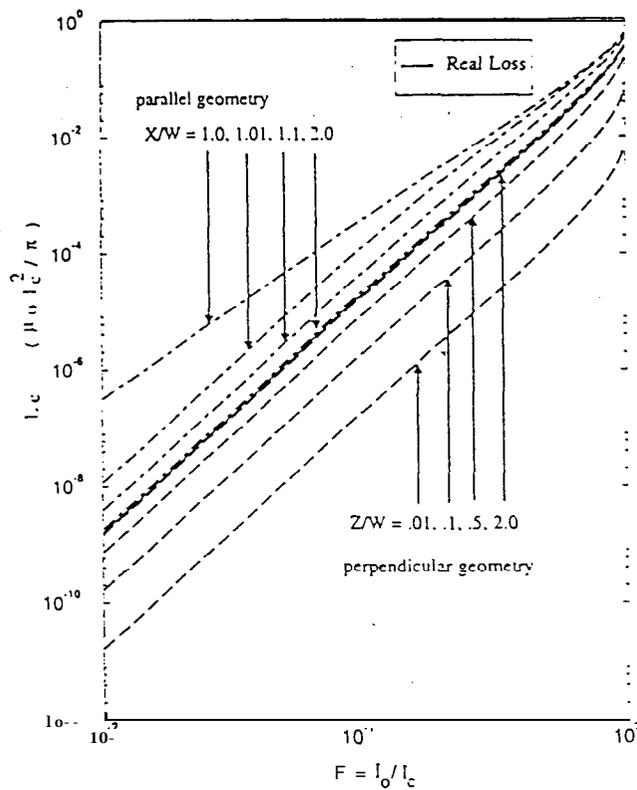


FIG. 2. Apparent loss per cycle per unit length for a strip of width $2W$, calculated as described in the text for parallel (dot-dashed curves) and perpendicular geometry (dashed curves). The real loss, representing the actual power dissipation made up by the power supply is shown by the solid curve [32].

above the strip, at which point they are brought straight together: then twisted, led out and connected to the voltmeter. The results of this calculation are shown by the dashed curves in Fig. 2. When $Z = 0$ and the leads are brought along the top surface of the conductor before they are twisted and brought out, $L_c = 0$, because both terms on the right-hand side of Eq. (1) are then zero. For $Z/W = 0.01$, the value of L_c is very small. However, as Z increases, the apparent loss quickly rises to the real, or true loss given in Eq. (2).

Note that the apparent loss in the parallel geometry is an overestimate of the true loss, but the apparent loss in the perpendicular geometry is an underestimate. We find that when X/W or Z/W is larger than 3, the apparent loss is within about 10% of the true loss. The predicted behavior has been confirmed, at least qualitatively by Fleshler et al. [51,52].

IV. Summary

There have been a number of recent theoretical advances in calculating the current-

density and magnetic-flux-density profiles, and in Sec. II we briefly discussed some of the results for strip and disk geometry. We note that although these results are directly applicable to thin films of the high-temperature superconductors, they also apply reasonably well to many single-crystalline samples of these materials, since single crystals often have a very high aspect ratio (thickness much less than the width).

In Sec. III we presented new results, which used the approach described in Sec. II to explain the measuring-circuit geometry dependence observed in recent measurements of hysteretic self-field transport losses in BSCCO high-temperature superconducting tapes.

Acknowledgments

Ames Laboratory is operated for the U.S. Department of Energy by Iowa State University under Contract No. W-7405-Eng-82. This work was supported by the Director for Energy Research, Office of Basic Energy Sciences. We thank E. Zeldov, V. G. Kogan, A. P. Malozemoff, and S. Fleshler for stimulating discussions and helpful advice.

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