

Observational Constraint on the Build-Up and Relaxation of Magnetic Fields in the Solar Atmosphere

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High-resolution data of the solar vector magnetogram can be used to examine the ways by which the magnetic fields in the solar atmosphere are built up and relaxed. In a relaxed state, the α parameter should have approached a constant value over an extended region, therefore yielding a sharply peaked distribution of $|\alpha|$. In this report, we present, for the first time, the discovery of power-law distributions for the $|\alpha|$ parameter. This finding suggests the existence of nontrivial build-up and relaxation mechanisms for the magnetic energy in the solar atmosphere.

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I. Introduction

It is well known in the laboratories that a turbulent MHD system tends to relax to the minimum-energy state allowed by various system constraints. One of the most important constraints was proposed by Taylor in 1974 [1]. Using dimensional analyses, Taylor was able to show that the turbulent MHD system tends to dissipate the magnetic energy while keeping the magnetic helicity intact, where the magnetic energy is defined to be

$$E_m \equiv \frac{1}{8\pi} \int B^2 d^3x \quad (1)$$

and the magnetic helicity

$$Ii, \equiv \int \mathbf{A} \cdot \mathbf{B} d^3x. \quad (2)$$

In the expression of K_m , the quantity \mathbf{A} is the vector potential.

A theoretical construction by minimizing the magnetic energy subjected to the constraint of a constant magnetic helicity yields a unique relaxed state. This relaxed state has a uniform distribution of the parameter α , which satisfies

$$\mathbf{J} = \alpha \mathbf{B} \quad (3)$$

over **the** available **space** [1,2]. (The quantity J is the electric current.) In regions where the magnetic fields reverse directions, one expects that the quantity α becomes singular. Therefore the constant α regions are expected to be confined within patches surrounded by the neutral (field reversal) lines. In the past, both theoretical and observational works have been reported, attempting to examining the Taylor states in solar atmosphere [3,4].

It can also be shown that dissipation is no longer possible in the presence of any slight deviation from the Taylor state since the energy content in the system is minimized. However, a typical solar flare can release a burst of energy of order 10^{27} - 10^{28} erg in 10 seconds [5], and thus the solar magnetic fields can definitely not be in a relaxed state. The issues of interest to us are to understand (1) how far the magnetic system in the solar atmosphere is from the relaxed state and (2) in what forms. In this report, we present, for the first time, the discovery of a power-law distribution of the α parameter, which may reveal insights into the above two issues.

Equation (3) means that the equality holds for all three components of both vectors separately. If we define the direction z to be normal to the photosphere, we may take $\alpha = J_z/B_z$. When looking into the center of the solar disk, B_z is the line-of-sight component of the magnetic field, and J_z is constructed from the spatial derivatives of the transverse magnetic fields. When looking into regions away from the center, proper correction due to the line of sight projection is required. Two-dimensional images of a high-resolution vector magnetogram, which measures both the line-of-sight and transverse components of the fields, are suitable for such determination of the α parameter over an extended area in the solar disk [6,7,8,9].

The vector magnetic field data presented in this report were obtained by the Beijing Observatory at the Huai-Zhou Solar Observing Station. Three "A" seeing images have been analyzed. We have adopted appropriate schemes in separating the z and transverse components of the fields resulting from projection along the line of sight and in determining the signs of the measured transverse fields. These schemes will not be detailed here, and interested readers should consult the original paper [10]. We shall in the next section briefly described how the magnetic field data is reduced.

II. Data reduction

The raw data is first smoothed using the smoothing scheme MEDIUM to get rid of the instrumental noises. The regions of interest are confined to the sunspot areas where the field strength exceeds few hundred gauss, and all data fields are less than one arcminute. Within such small data fields, the curvature of the solar surface can be ignored, and we may treat the photosphere as a plane. (Solar diameter -30 arc-minutes.)

When the data field is not located at the center of the solar disk, one needs to project the observed line-of-sight field component and transverse component into one normal to the photosphere and the other perpendicular to the photosphere, which we denote B_z and B_{\perp} , respectively.

Prior to the above step, a preparatory task has to be carried out; that is to determine the sign of the observed transverse field. This part of data reduction is the most controversial part of all, and we follow the scheme proposed by Gray and Hagyard [11]. This scheme involves identifying radiating centers of the transverse magnetic fields in the

field of interest, and works well when the field patterns are relatively simple. Simplicity of the field patterns becomes our selection criterion from all data available to us. This criterion can be rather subjective to the human eyes, and hence may not be an universal one. Nevertheless, our exercise is not for statistical purposes, and hence we are contented with the present method for the same reason as Gray and Hagyard advocated [11].

Once B_z and \mathbf{B}_\perp are determined, the z component of the current J_z is calculated from the spatial derivatives of \mathbf{B}_\perp through

$$\mathbf{J} = \nabla \times \mathbf{B}$$

or

$$J_z = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}, \quad (4)$$

and therefore α at each pixel can be determined using Eq. (3).

Because of the nature of the method used to determine the sign of \mathbf{B}_\perp , we can not rule out the possibility of obtaining a wrong sign for it, and we expect that if we should obtain the wrong sign, it should occur in patches. At the boundaries between patches of correct sign and those of wrong sign, the values of α will be unreasonably large. Within the patches of wrong sign, α will have wrong signs as well. To correct this problem in the large, we shall examine $|\alpha|$, instead of α itself. In this way, the errors are minimized and confined along the boundaries between patches of correct signs and wrong signs.

III. Results

III-1. HR91146

Figure 1 is an overlay of B_z (contours) and \mathbf{B}_\perp (arrows) of a monopole sunspot taken on 11 August, 1991 (HR91146). The solid and dashed contours represent positive and negative B_z respectively. The arrow lengths represent the relative strengths of \mathbf{B}_\perp . The strongest B_z of this sunspot exceeds 1000 gauss. The labels of the $x-y$ coordinates are the pixel numbers in the east-west and north-south directions of the Sun. Each pixel corresponds to 0.63 arcsec, or about 460 km.

It is interesting to note that the transverse field rotates clockwise, suggesting that there exists a sizable vertical current J_z which almost centers around the monopole sunspot. The strength of \mathbf{B}_\perp at the lower-left corner is rather weak, which may give rise to substantial errors in data reduction. In this region, one would expect to obtain unreasonably large $|\alpha|$ due to the need for taking spatial derivatives of \mathbf{B}_\perp in the determination of α . Figure 2 are, respectively, the plots of three-dimensional contours of α , cutting in arrays horizontally (a) and vertically (b) for visual comparison; the heights above or below each array of cutting line represent the relative values of α . Indeed, the lower-left corners contain large fluctuations in α . Large fluctuations also exist along the neutral line of B_z where the absolute value of α is expected to be large. (See Eq. (3) for the definition of α .) Surprisingly, large fluctuations of α are also found surrounding the central region with the coordinates $20 < x < 40$ and $20 < y < 40$. There is no obvious error source in this central region, and the large α should be real.

Plotted in Fig. 3 is the statistics of $|\alpha|$. Each pixel of the 60×60 -pixel field has its own value of $|\alpha|$. We plot the number distribution of $|\alpha|$ against the value of $|\alpha|$ in a log-

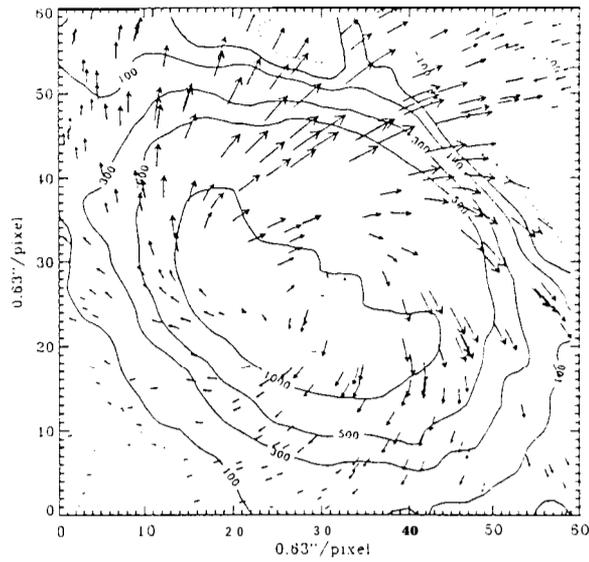


FIG. 1. Overlay of B_z (contours) and \mathbf{B}_\perp (arrows) in a field of 60 x 60 pixels for the sunspot HR91146. The labels of $x-y$ coordinates correspond to the pixel numbers in the east-west and north-south directions of the Sun, where each pixel has a physical size about 460 km.

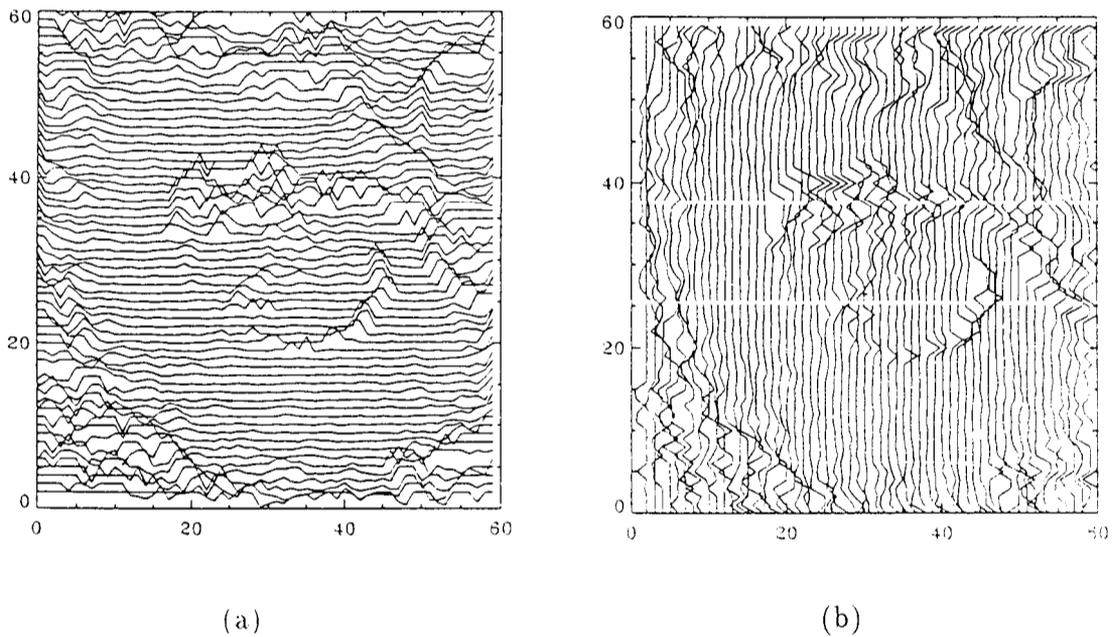


FIG. 2. Three dimensional plots of $|\alpha|$ cutting across the field in arrays horizontally (a) and vertically (b), where the heights above or below the straight lines represent the relative values of $|\alpha|$ for HR91146.

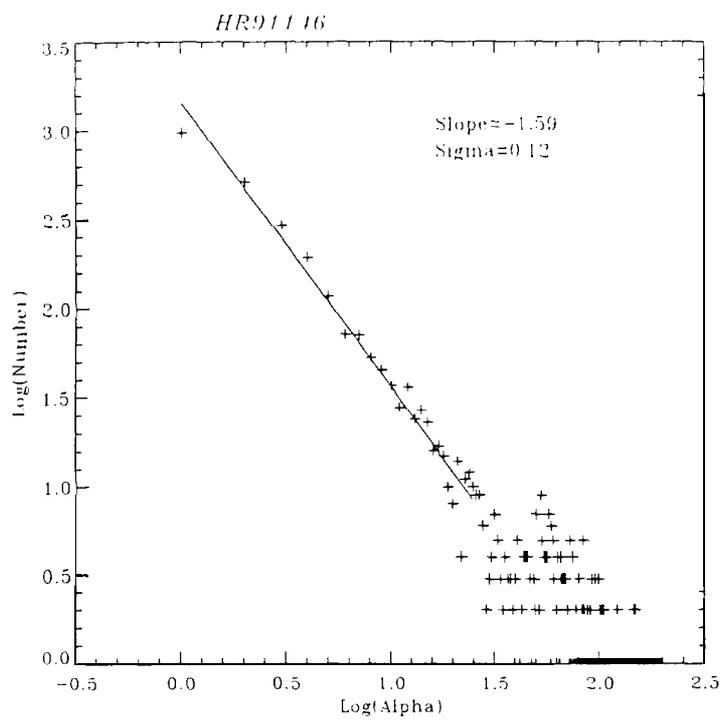


FIG. 3. Distribution of $|\alpha|$ for HR91146 within the GO x GO pixel field. For $1 < |\alpha| < 30$, the distribution can be fitted by a power law with the power index equal to -1.59 and the standard deviation σ equal to 0.12.

log diagram. Except for large-value α where systematic noises dominate, the distribution exhibits a power-law relation

$$f(|\alpha|) \propto |\alpha|^\beta, \quad (5)$$

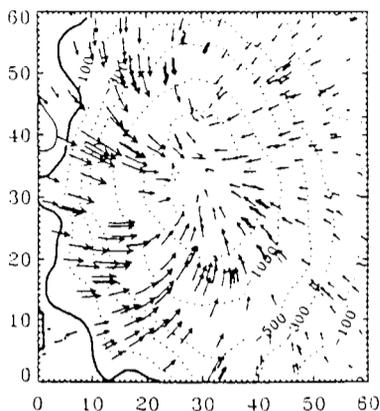
with the power index $\beta \sim -1.59$.

III-2. HR92124

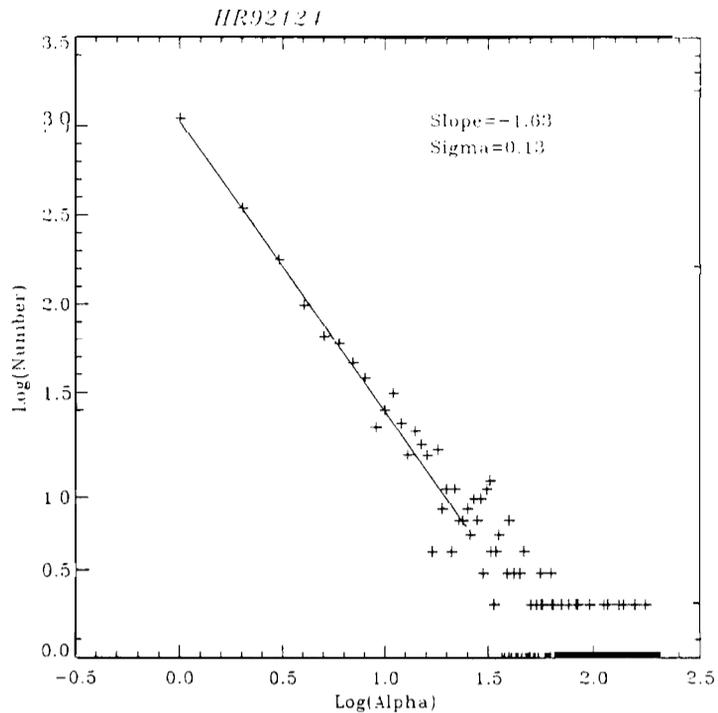
Plotted in Fig. 4 is an overlay of B_z and B_\perp for the sunspot taken on 19 June, 1992 (HR92124). Figure 5 shows the distribution of $|\alpha|$. It also exhibits a power-law distribution up to the similar value of α , with the power index $\beta \sim -1.63$.

III-3. HR92167

Figures 6 and 7 are the corresponding plots of Figs. 1 and 3 of HR91146, or Fig. 4 and 5 of HR92124. The power-law distribution for $|\alpha|$ persists in the same range, with the power index $\beta \sim -1.66$.



(Fig. 4)



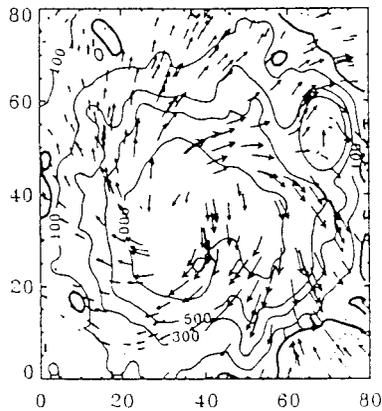
(Fig. 5)

FIG. 4. Overlay of B_z and B_{\perp} for the sunspot HR92124.FIG. 5. Distribution of $|\alpha|$ for HR92124. The power index equals -1.63 with the standard deviation $\sigma = 0.13$.

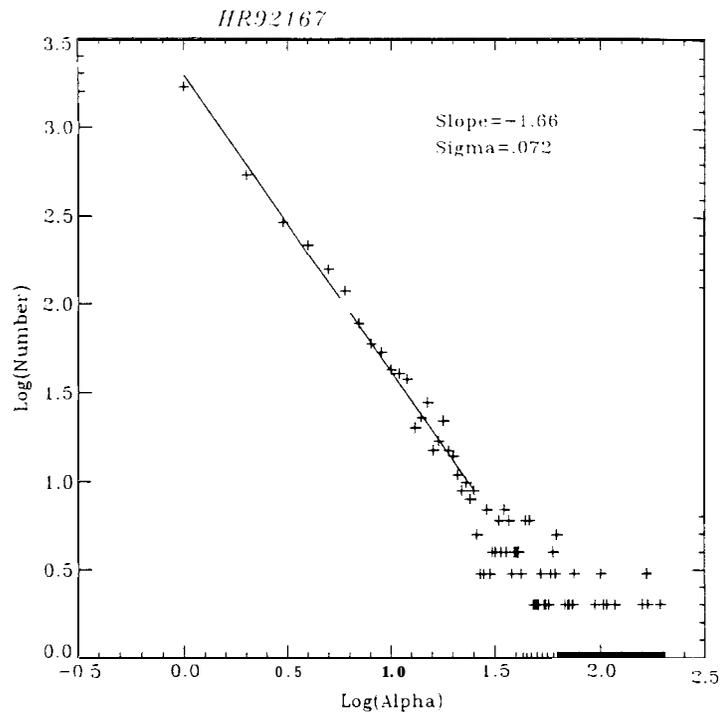
III-4. Discussions and conclusion:

Constant- α configurations have been widely believed to exist in magneto-plasmas, and observed in laboratories. This configuration is the minimum-energy state with a given magnetic helicity, representing a state of complete relaxation under a given constraint. However, in the solar atmosphere, the situation may be different from the laboratory, in that the field lines are closed within the system in the laboratory but the field lines in the solar atmosphere are connected to the much denser plasmas underneath the photosphere. The most important factor that may invalidate the constant- α conjecture is that the magnetic helicity in the solar atmosphere is not really kept constant, which can be either injected or absorbed by the plasmas underneath the photosphere.

Our discovery of the power-law distribution of $|\alpha|$ can be rather significant; it reflects some non-trivial mechanisms to be operative in the relaxation of the solar magnetic fields. From the three distinct active regions observed at different time, it is found that the power indices β are about the same, ranging from 1.6 to 1.7, a rather robust universal distribution. The power law region is confined within $1 < |\alpha| < 30$. In the larger $|\alpha|$ regime the distribution appears chaotic, and this chaotic behavior may very well be caused by the fact that our determination of the transverse B-field has wrong signs. A patch of transverse



(Fig. 6)



(Fig. 7)

FIG. 6. Overlay of B_z and B_{\perp} for the sunspot HR92167.FIG. 7. Distribution of $|\alpha|$ for HR92167. The power index equals -1.66 with the standard deviation $\sigma = 0.072$.

fields that have a wrong sign will result in sharp changes in the transverse fields along the boundary of the patch and therefore in wrong distribution at large $|\alpha|$. Hence, the actual power-law distribution is expected to extend to the regime of even larger $|\alpha|$. At the present time, we have no explanation for such a universal power-law behavior.

Universal power-law behaviors are typically observed in nonlinear dynamical systems, which are driven by external sources and balanced by some forms of dissipation. Examples include fluid (MHD) turbulence or self-organized critical processes. In the context of the solar magnetic field, it has been pointed out that the energy distribution of s-ray bursts also exhibits a power-law behavior [12]. Explanations have been put forth advocating the idea of self-organized criticality [13,14] or critical phenomenon [15]. It is not yet clear whether the mechanisms underlying the power-law distribution of $|\alpha|$ bear any relationship with those of the x-ray bursts of the Sun. In fact, the s-ray bursts occur all over the active regions in the Sun, but our analysis only applies to the sunspot, a much smaller region surrounded by the larger active region. Due to the contamination of systematic noises, we have not been able to accurately analyze the magnetic data in other areas of the active region that have weaker fields. Therefore a closer comparison with the x-ray bursts is not available at present. A study aiming at the accurate determination of α for the entire active regions will be a future direction much worth pursuing.

In summary, we have analyzed three sets of sunspot magnetic data in an attempt to determining the distribution of $|\alpha|$. In contrast to the conventional belief that the solar magnetic field should have relaxed to a constant- α configuration, we find that it is not. Furthermore, we find that the solar magnetic field is in a non-trivial configuration, where $|\alpha|$ exhibits a universal power-law distribution, typical of nonlinear relaxation of a driven system.

Our follow-up studies of more sunspot magnetic data also show similar behaviors as the ones reported in this work. Results of this more extensive study will be reported in the near future.

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References

- [1] J. B. Taylor, Phys. Rev. Letts. 33, 1139 (1974).
- [2] J. B. Taylor, Rev. Mod. Phys. 58, 741 (1986).
- [3] J. Heyvaerts and E. R. Priest, Astrophys. J. 390, 297 (1992).
- [4] A. P. Pevtsov, R. C. Canfield, and T. R. Metcalf, Astrophys. J. 425, L117 (1994).
- [5] P. Foukal, Solar Astrophysics (Wiley: New York 1990).
- [6] G. A. Gary, R. L. Moore, M. J. Hagyard, and B. M. Haisch, Astrophys. J. 314, 782 (1987).
- [7] J. Chen, H. Wang, H. Zirin, and G. Ai, Solar Phys. 154, 261 (1994).
- [8] G. Ai and Y. Hu, Acta Astrophysica Sinica 8, 1 (1986).
- [9] G. Ai, W. Li and H. Zhang, Acta Astrophysica Sinica 8, 11 (1986).
- [10] P. Venkatakrishnan, M. J. Hagyard, and D. H. Hathaway, Solar Phys. 122, 215 (1989).
- [11] G. A. Gray and M. J. Hagyard, Solar Phys. 126, 21 (1990).
- [12] B. R. Dennis, Solar Phys. 100, 465 (1985).
- [13] E. T. Lu and R. J. Hamilton, Astrophys. J. 380, L89 (1991).
- [14] E. T. Lu, R. J. Hamilton, J. M. McTiernan, and K. R. Bromund, Astrophys. J. 412, 841 (1993).
- [15] T. Chiueh, C. Lai, H. H. Li, and W.S. Tsay, submitted to Chin. J. Phys. (1995).