

## Magnetic Levitation and Bean Model

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The levitation forces for both free sintered and melt-powder-melt-growth (MPMG) yttrium-barium-copper-oxide (YBCO) superconductors have been measured by a torsion balance. The results agree with data obtained by other experimental groups. These levitation forces also have been calculated theoretically by taking into account both the Meissner effect and the induced current due to the critical state of the superconductor. Acceptable agreements are obtained. It is found that the large area enclosed by the hysteretic levitation curve corresponds to a large critical current density of the superconductor.

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### I. INTRODUCTION

Ever since the discovery of high temperature superconductors [1], the hysteretic levitation curves of these materials have been measured by many authors [2-7]. It seems that a large critical current density of the superconducting sample corresponds to a large area enclosed by the hysteretic levitation curves. Theoretical investigations of these levitation forces [6-10] have given some qualitative agreement with experimental results. However, a complete calculation for the hysteretic levitation curves to compare with experimental results has not yet been seen. In this paper, we will first measure these levitation forces for both sintered and MPMG YBCO superconducting disks (SD). Then, we will use Bean model [11,12] to calculate the levitation forces quantitatively and to try to explain the difference in levitation forces between the sintered and MPMG samples.

## II. EXPERIMENTS

We use a torsion balance to measure the levitation forces as a function of height for both the sintered and MPMG YBCO sample. The whole setup is similar to the one used in Ref. [3]. A cylindrical, Nd-Fe-B permanent magnet (PM) of length 0.62 cm and diameter 0.62 cm is used for our measurements. The distribution of the radial component of the magnetic field of the PM are measured by a gaussmeter and the results are shown in Fig. 1. Our sintered sample is a disk of 2.5 cm diameter and 0.2 cm thickness. The MPMG sample is a disk of 2.5 cm diameter and 0.185 cm thickness. During our measurements, the axis of the PM is kept coaxial with the axis of the SD. The hysteretic levitation curves for both samples are shown in Figs. 2 and 3 respectively. One can see that for the field-rising part, the levitation forces of the MPMG sample is always larger than that of the sintered sample. But, for the field-falling part, the levitation forces of the MPMG sample have a large negative portion while the sintered sample shows nearly all repulsive forces. These results agree with data obtained by other experimental groups [2-7].

## III. THEORETICAL CALCULATIONS

Let us consider a small PM placed at a height  $h$  above the SD as shown in Fig. 4. The total vertical force  $dF_z$  acting on the ring at the position  $(r, z)$  due to the magnetic

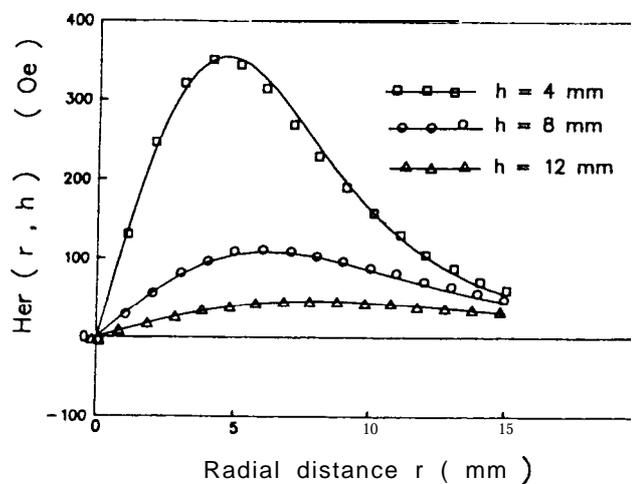


FIG. 1. Radial component of the magnetic field  $H_{er}$  of the PM as a function of  $r$  and  $h$ . Symbols are experimental points and lines are calculated values from our model.

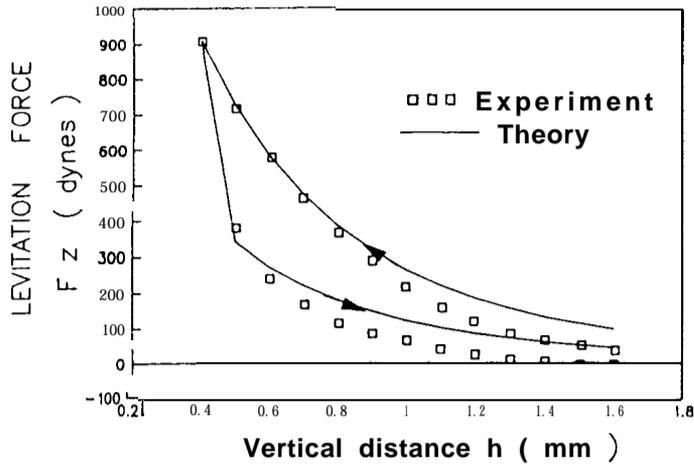


FIG. 2. Experimental and theoretical values of the magnetic levitation forces for the sintered sample.

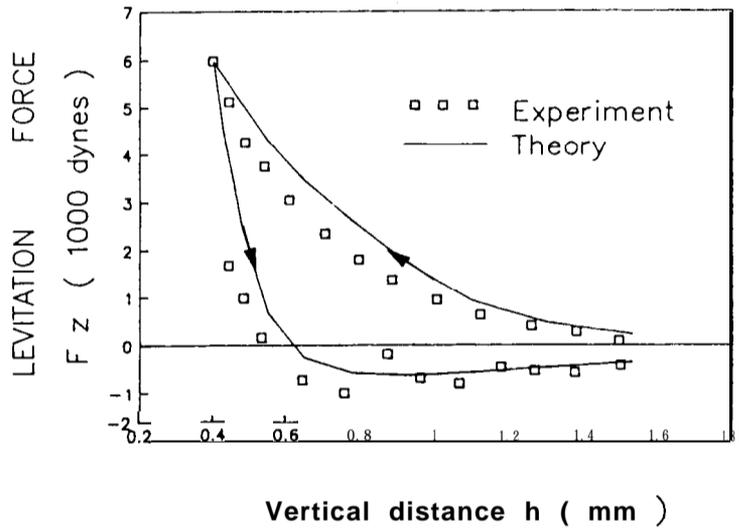


FIG. 3. Experimental and theoretical values of the magnetic levitation forces for the MPMG sample.

flux density  $B_e(r, z + \mathbf{h})$  generated by the PM and the current  $j_\phi drdz$  **flowing** inside the ring is:

$$dF_z = (2\pi r)(j_\phi drdz)B_{er}(r, z + h), \tag{1}$$

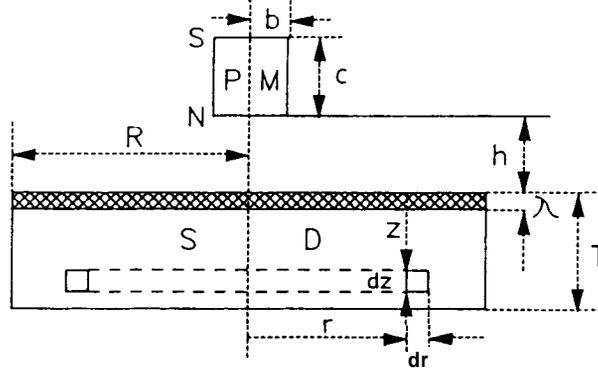


FIG. 4. Theoretical model for the magnetic levitation force calculations. Note that the figure is not drawn according to scale.

where  $B_{er}$  represents the radial component or  $B_e$ .

The total levitation forces acting on the magnet should equal the sum of all these small forces  $dF_z$  over the whole SD. By Maxwell's equation,

$$j_\phi = \partial H_{tr} / \partial z - \partial H_{tz} / \partial r, \quad (2)$$

$H_{tr}$  and  $H_{tz}$  represent the  $r$ -th and  $z$ -th component of the total magnetic field. For an SD with  $R \gg T$ , the second term in the above equation is always small in comparison with the first term [13,14]. So, in all our calculations, we will keep the first term only. That means that

$$j_\phi = \partial H_{tr} / \partial z. \quad (3)$$

As also shown in Fig. 4, the SD is divided by the penetration depth  $\lambda$  into two parts. When  $H_{tr}$  at the upper face of the SD does not exceed  $H_{c1}$ , the first critical field of the superconducting sample, the induced current is only due to the Meissner effect. After summing up over  $dz$ , the levitation force due to the Meissner effect only is:

$$dF_{zm} = \begin{cases} 2\pi B_{er} H_{tr} r dr & \text{for } H_{tr} < H_{c1}, \\ 2\pi B_{er} H_{c1} r dr & \text{for } H_{tr} > H_{c1}. \end{cases} \quad (4)$$

When  $H_{tr}$  is larger than  $H_{c1}$ , one also has to consider the induced current caused by the critical state of the sample [11,12]. By Bean model, one has

$$j_\phi = \pm j_c. \quad (5)$$

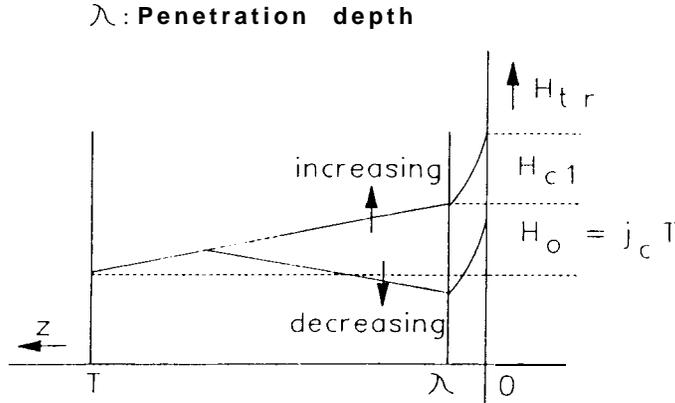


FIG. 5. Induced current distribution as a function of  $z$  inside the SD at a distance  $r$  from the symmetry axis. This figure is also not drawn according to scale.

Here  $j_c$  is the critical current density of the sample. The positive or negative sign depends upon whether  $H_{tr}$  is increasing or decreasing inside the SD. Fig. 5 shows the total magnetic field  $H_{tr}$  inside the SD as a function of  $z$  at a distance  $r$  from the symmetric axis for the case where  $H_{tr}(r, 0) \sim H_{c1} > j_c T$  with both increasing and decreasing  $H_{tr}$ .

In order to simplify our calculations, the magnetic field  $H_{er}(r, h)$ , of the PM, is approximately calculated with a  $z$ -direction distributed current loop ( $I$ ) of the same radius  $b$  and height  $c$  as the PM. The experimental points for  $H_{er}(r, h)$ , as shown in Fig. 1, are best fitted by:

$$dI/dz = k_1(1 - k_2 z^2), \quad |z| < c/2 \tag{6}$$

with the two parameters  $k_1 = 15$  and  $k_2 = 7/c^2$ . One can see that our fitting for the magnetic field  $H_{er}$  is quite good. Also, by self-consistent calculations, we have found that, for  $H_{c1} = 8 \text{ G}$  and a current density of the order  $10^8 \text{ A/m}^2$  or less, the  $r$ -th component of the magnetic field generated by the induced current inside the SD is negligibly small in comparison with the external magnetic field  $H_{er}$  (100 G). That means, the total magnetic field  $H_{tr}(r, 0)$  just outside the upper face of the SD can be approximated by:

$$H_{tr}(r, 0) = H_{er}(r, h). \tag{7}$$

A complete calculation on the levitation forces  $F_z$  as a function of the distance  $h$  between the PM and the SD was done on a personal computer. The calculated results are also shown in Figs. 2 and 3. The parameters we used in our calculations are:

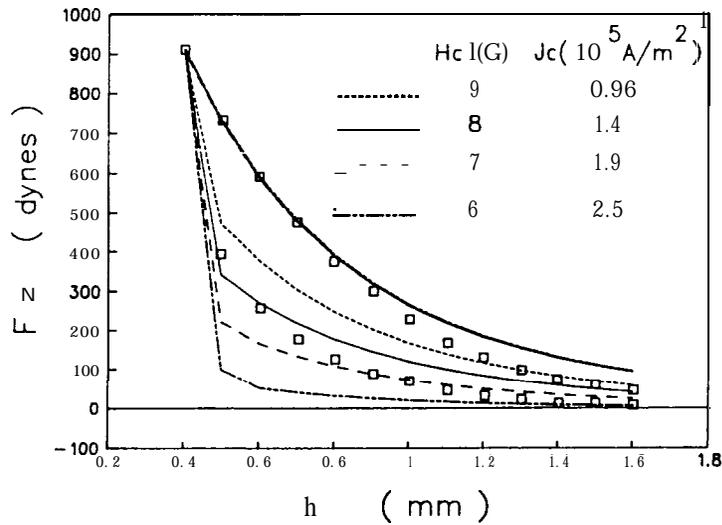


FIG. 6. The effect of  $H_{c1}$  on the area enclosed by the hysteretic levitation curve with the same maximum levitation force. This figure is used to find the best fitting parameters  $H_{c1}$  and  $j_c$  for the sintered sample.

$$H_{c1} = 8 \text{ G}, j_c = 1.4 \times 10^5 \text{ A/m}^2 \quad \text{for the sintered sample and}$$

$$H_{c1} = 20 \text{ G}, j_c = 3.3 \times 10^6 \text{ A/m}^2 \quad \text{for the MPMG sample.}$$

#### IV. DISCUSSION AND CONCLUSIONS

In our fitting procedure, we try to find the best fitting parameters  $H_{c1}$  and  $j_c$  according to the area enclosed by the hysteretic levitation curves for both samples. As expected, the critical current density for the sintered sample is much smaller than that for the MPMG sample. The effect of the critical current density  $j_c$  on the levitation forces is best shown in Fig. 6. For the same maximum levitation force, as  $j_c$  increases, the area enclosed by the hysteretic levitation curve also increases.

There are errors in our fittings. This is mainly because, in our calculations, we have overlooked some important problems in applying Bean model. For example, high temperature superconductors are highly anisotropic materials. Also, for polycrystal samples, one has to consider the problem of intragranular currents and intergranular currents. Therefore, the values of the fitting parameters we found in our calculations should not be taken

too seriously. They can only serve as a qualitative indication of the corresponding physical properties of the SD.

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