

Magnetic Field Effects on Superconductor-Normal Metal-Superconductor Junctions

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The temperature and magnetic field dependence of the Josephson supercurrent have been measured over the temperature range $0.4 < T < 7.2K$ in order to study the magnetic field effects. In the range where the Josephson penetration depth is large compared to the junction width, the junctions show well-defined diffraction patterns. For regions where the self-field effect is important, the junctions show Meissner effect at low magnetic field and reverse to diffraction pattern at higher fields. If the critical current density, J_c , is corrected for the self field using the linearized Vaglio's procedure, then J_c is proportional to $(1-T/T_c)^2$ as predicted by the de Gennes' theory.

INTRODUCTION

JOSEPHSON^(1,2) predicted in 1962 that a tunneling current through superconductor-insulator-superconductor (SIS) junctions can behave as a true supercurrent because there is a coherent tunneling of Cooper pairs. Many studies⁽³⁻⁵⁾ have verified the basic validity of these ideas for SIS junctions and extensive literatures have been accumulated. Indeed, many of these same ideas apply to SNS junctions as well and it is important to determine the similar and dissimilar features of these two different kinds of quantum interference devices.

One of the most striking features of the behavior of Josephson structures is the Fraunhofer diffraction pattern. This is a consequence of the wave-like nature of Cooper pairs and phase coherence through the junctions. The extremely high sensitivity of a Josephson supercurrent to magnetic fields is the key to many important applications⁽⁶⁾ of Josephson effect. Furthermore, a careful study of the Fraunhofer diffraction pattern represents a powerful method to investigate the junction quality, the junction behavior, and in particular the junction current distribution.

The magnetic field dependence of the Josephson supercurrent I_c has been extensively studied⁽⁷⁻⁹⁾ for small junctions with uniform current distribution and indeed one finds

$$I_c \propto \frac{\sin(\pi\phi/\phi_0)}{(\pi\phi/\phi_0)}, \quad (1)$$

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where $\phi = HW(d + 2\lambda_L)$ is the total flux in the junction, ϕ_0 is the flux quantum, W is the junction width, d_N is the junction thickness, and λ_L is the bulk London penetration depth in the superconductor. To achieve equation 1, it is necessary to have a junction for which W is small compared to the Josephson penetration depth, given by

$$\lambda_J = \{\hbar c^2 / [8\pi e J_c (d_N + 2\lambda_L)]\}^{1/2} \quad (2)$$

where \hbar is the Planck's constant divided by 2π , c is the speed of light, e is the electron charge, and J_c is the critical current density.

As the junction size or critical current increases, the influence of magnetic field induced by the Josephson current itself becomes important. The current density can be highly nonuniform even in zero external field because of the presence of the self field. In principle, the current density can be related to the phase difference across the junction, $\phi(y, z)$, by

$$J(y, z) = J_c \sin \phi(y, z), \quad (3)$$

where J_c is the maximum Josephson supercurrent density, and the y and z axes lie in the plane of the junction. The current flows in the x -direction. The behavior of the junction is governed by⁽²⁾

$$-\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = (1/\lambda_J)^2 \sin \phi. \quad (4)$$

Here, λ_J gives a measure of the distance in which d. c. Josephson currents are confined. It occurs as a consequence of a current screening due to the magnetic self field generated by the supercurrents. Equations 3 and 4 cannot be solved analytically in general; therefore one often has to resort to approximate procedures and numerical computations. In the case of one-dimensional junctions, Owen and Scalapino⁽¹⁾ have derived the current density and local magnetic field within the junctions. These predictions are in reasonable agreement with experimental results by Goldman and Kreisman⁽⁵⁾ and that of Schwidtal⁽¹⁰⁾. For a two-dimensional geometry, a complete solution with proper boundary conditions has not been solved yet due to mathematical difficulties. Vaglio^(11,12), however, has obtained the behavior of the junctions by solving equations 3 and 4 in linearized model by assuming $\sin \phi = \phi$ and gets reasonable agreement with the experimental results of various authors^(13,14).

The purpose of this work reported here was to study quantitatively the Fraunhofer effects, the Meissner effects, and the self-field effects on SNS junctions in a wide W/λ_J range. The I_c -versus- H curves for small and large junctions are of greatest interest. From the self-field effects, we wish to correct the J_c data when appropriate.

EXPERIMENTAL PROCEDURES

A high vacuum (1×10^{-8} torr) evaporator system was used to prepare the Pb-Ag_{1-x}, Mn, Al-Pb junctions on a 2.54×12.7 cm glass substrate. The Pb films had a thickness of $\sim 7000 \text{ \AA}$ and width of 7×10^{-3} cm. The alloy film was prepared by subsequently dropping the tiny alloy pellets to a hot boat. Lead and silver do not form intermetallic compounds⁽¹⁵⁾. They are almost immiscible in solid phase, so that the diffusion of one into the other is negligible. The glass substrate was cooled to a temperature of about -30°C by pumping alcohol out of the liquid alcohol reservoir during the evaporation. Pumped alcohol instead of the liquid nitrogen was used because the silver layer tends to crack during evaporation. The current to the junction was symmetric. A sensitive rf-SQUID (Superconducting Quantum Interference Device) voltmeter was used to measure the accurate critical current density with a voltage sensitivity of about 1×10^{-12}

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volt. A superconducting magnet was used to apply an external field to the junction. A temperature controller was used to keep the temperature constant while the I_c -versus- H curve was taken.

EXPERIMENTAL RESULTS AND DISCUSSION FRAUNHOFER EFFECTS

A sample which has both a uniform junction barrier and current distribution should show a good Fraunhofer diffraction pattern for the regime where the Josephson penetration depth, A_J , is greater than $\frac{1}{4}$ of the junction width W . Fig. 1 shows an I_c -versus- H curve for a sample at 4.52K for which $W/\lambda_J \approx 0.86$. The solid curve is the Fraunhofer diffraction pattern, using $I_c = |\sin(\pi\phi/\phi_0)/(\pi\phi/\phi_0)|$, which is normalized to the zero field I_c maximum and the second I_c minimum at 1.17G, where $\phi = HW(d_N + 2\lambda_L)$ is the total flux threading the junction, ϕ_0 is the flux quantum, W is the barrier width, and λ_L is the London penetration depth in the superconductor. The data show a nice fit to the theoretical prediction except that the first two maxima rise a little bit above the theoretical value. The calculated flux quantum, $\Delta\phi$, is $1.5 \times 10^{-7} G \text{ cm}^2$ if we use $\Delta\phi = HW(d_N + 2\lambda_L)$, where $\lambda_L(T) = 390[1 - (T/T_c)^4]^{-1/2}$ in units of \AA and AH is the magnetic field period. From the periodicity of the I_c -versus- H curve and $W = 7.0 \times 10^{-3} \text{ cm}$, it follows that $\lambda_L(0) = 1161 \text{\AA}$. This value is larger than the 390\AA that is usually assumed for the London penetration depth of clean Pb. Maybe Pb is dirty near the surface. The central maximum of the I_c -versus- H curve has been shifted from $H=0$ to a small negative value due to a partial asymmetric current feed caused by the induced current in the superconducting input Pb-wires when the magnetic field was applied. The I_c -versus- H curve is symmetric with respect to the central maximum which indicates the homogeneity of the sample.

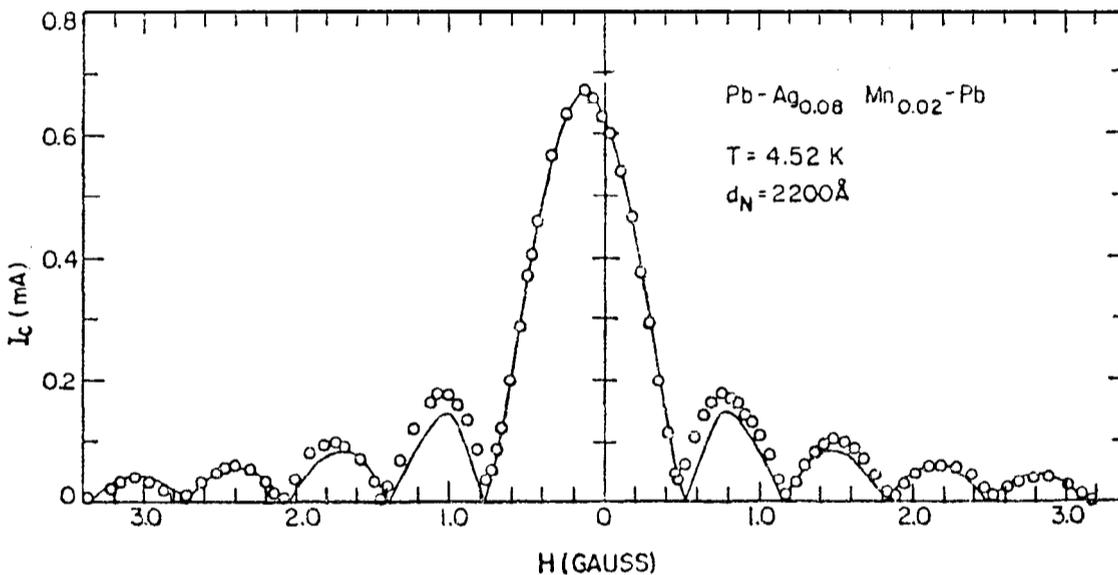


Fig. 1. I_c -versus- H curve for $\text{Pb-Ag}_{0.08}\text{Mn}_{0.02}\text{-Pb}$; $W/\lambda_J = 0.86$; measured flux quantum $= 1.5 \times 10^{-7} G \text{ cm}^2$.

MEISSNER EFFECTS

Fig. 2 shows an I_c -versus- H curve at $T = 6.74 \text{ K}$ for a sample which shows the Meissner effect. The maximum I_c value in zero field produced a corrected J_c for which $W/\lambda_L = 3.3$. When the applied field increases from zero, it is screened out from the junction and J_c decreases linearly. This is just the Meissner effect. The magnetic field at which it will penetrate the junction, H_c , is⁽²⁾

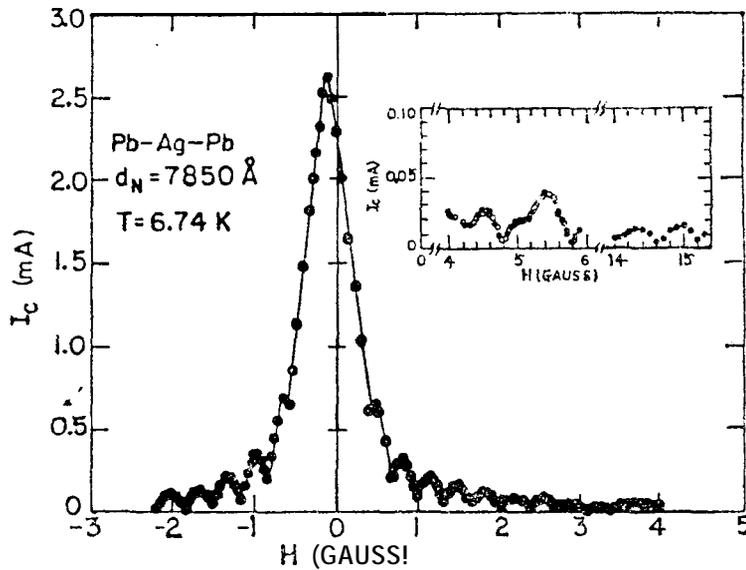


Fig. 2. Magnetic field dependence of the critical current I_c at $T=6.74$ K, $W/\lambda_L=3.3$; measured flux quantum is $2.34 \sim 10^{-7}$ G cm². The I_c behavior in high magnetic fields is shown in the inset.

$$H_c = (\pi/2)H_{c1} \quad (5)$$

where

$$H_{c1} = \{32\hbar J_c / [\pi e(d_N + 2\lambda_L)]\}^{1/2}. \quad (6)$$

The calculated value of H_c is 0.27 G when we assume a uniform current flow in the junction. Because $W/\lambda_L=3.3$, this is not a good approximation as confirmed by previous calculations⁽¹⁶⁾ based on Vaglio's theory^(11,12). In order to calculate the H_c value at lower temperatures, a corrected J_c should be used. Results of these calculations are shown in Table 1, along with the H_c 's assuming a uniform current distribution. To discuss the calculated H_c values with the observed values it is useful to define H_1 and H_i . H_1 is the magnetic field at which J_c has its first minimum, and H_i is the magnetic field extrapolated from the linear portion to $I_c=0$. It can be seen that the calculated H_c values agree well with the observed H_i values if we assume that the field will penetrate the junction at H_1 .

For fields greater than H_c , the mixed state begins to enter the junction and I_c behavior reverts

Table I. Comparison between the calculated H_c and H_i values, where H_1 is the first minimum in the diffraction pattern. The H_i values are also shown, where H_i is the magnetic field extrapolated from the linear portion of I_c -versus- H curve to $I_c=0$.

T (K)	H_c (assuming uniform current density) (Gauss)	H_c (with self-field correction) (Gauss)	H_1 (Gauss)	H_i (Gauss)
6.78	0.27	0.29	0.39	0.46
6.74	0.30	0.33	0.43	0.56
6.69	0.33	0.39	0.46	0.64
6.64	0.38	0.44	0.55	0.77
6.59	0.41	0.48	0.78	0.86

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to a Fraunhofer-like diffraction-pattern. with a field period, ΔH , of about $0.34G$. This will give us an observed flux quantum, $\Delta\phi$, of about $2.34 \times 10^{-7} G \text{ cm}^2$ if we evaluate $\Delta\phi = \Delta H W (d_N + 2\lambda_L)$. The observed $\Delta\phi$ must be considered to be in acceptable agreement within experimental errors.

For high fields, the situation becomes more complicated and the field periods begin to vary. Non-regular field periods were observed as, seen in the inset in Fig. 2, with the closed and open circles being data from two different runs. The variation in ΔH at high fields is probably due to the flux trapping in the barrier causing different amount of field to energy the junction when the field is applied.

SELF-FIELD EFFECTS

For the junctions reported here, the self-field effects become important for critical current densities larger than about 80 A/cm^2 where W/λ_L is greater than about 4. Fig. 3 shows the temperature dependence of J_c for a junction along with the corrected J_c 's. The circles are the experimental results for J_c if one assumes a uniform current distribution in the junction (i. e. no self-field correction). It can be seen that these data have a linear dependence near the transition temperature. Our previous calculations⁽¹⁶⁾ based on Vaglio's theory^(11,12), however, have shown that the self-field effects are important for a junction which has W/λ_L as large as those values reported here and a self-field correction is necessary if one wants to obtain the true J_c 's. Here Vaglio's theory is used to correct these data for the self-field effects. For a symmetrical current feed in zero external field the current distribution $J(y, z)$ is given by

$$J(y, z)/J_c = \{I/[8W\lambda_L J_c \sinh(W/2\lambda_L)]\} f(y, z), \quad (7)$$

where

$$f(y, z) = \cosh(z/2\lambda_L) + \cosh[(W-z)/2\lambda_L] + \cosh(y/2\lambda_L) + \cosh[(W-y)/2\lambda_L], \quad (8)$$

$(0 \leq y \leq W, 0 \leq z \leq W)$

and I is the current supplied by the battery. The experimental critical current J_c corresponds to the value of I for which the left-hand side of equation 7 is equal to unity. From these conditions, one can get

$$J_c = \{I_c/[4W\lambda_L \sinh(W/2\lambda_L)]\} [1 + \cosh(W/2\lambda_L)]. \quad (9)$$

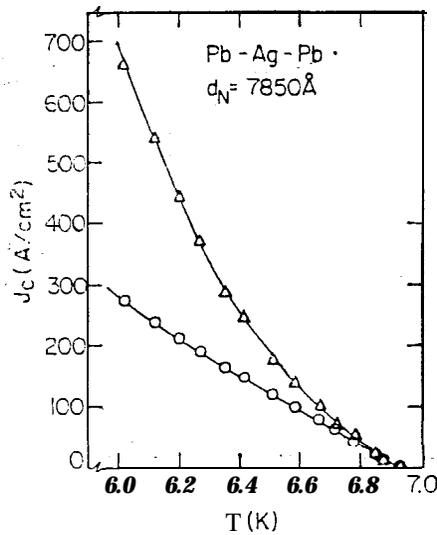


Fig. 3. Critical current density J_c as a function of temperature. The circles are the J_c 's assuming a uniform current distribution and the triangles the J_c 's after a self-field correction.

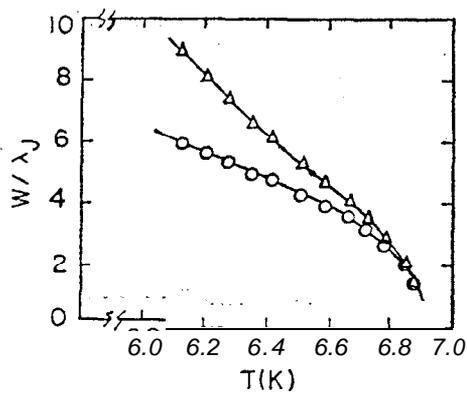


Fig. 4. The W/λ_J as function of temperature. The circles and triangles are as defined in Fig. 3.

It should be noticed that λ_J is also function of J_c . By solving equations 2 and 9 simultaneously, we can obtain the true J_c 's from the measured I_c 's. Results of the calculations after the self-field correction are shown in Fig. 3 (triangles). Here we have assumed $\lambda_L = 390 [1 - (T/T_c)^4]^{-1/2}$ in unit of \AA in our calculations. The apparent W/λ_L values obtained from the corrected J_c 's are shown in Fig. 4. The circles show the W/λ_J values which would have been obtained assuming a uniform current distribution. One can see that the self-field effects become more and more important as the temperature decreases. For this junction, the self-field effects on J_c are greater than 10% for all temperatures below 6.6K.

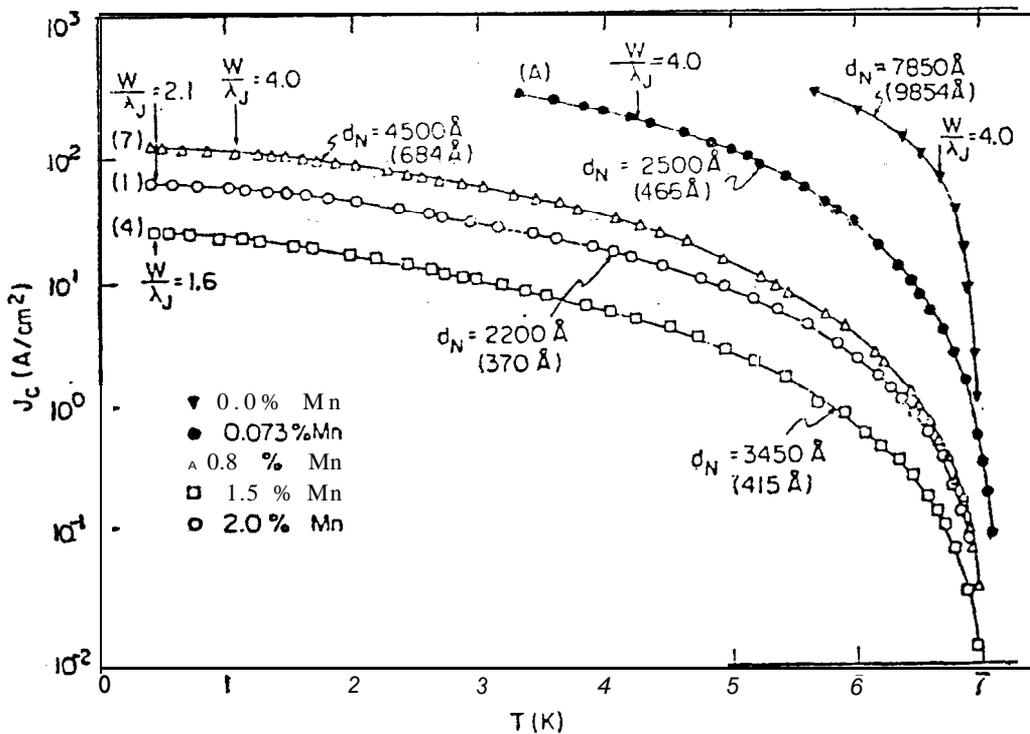


Fig. 5. Temperature dependence of J_c . $W/\lambda_J = 4.0$ indicates the onset of the region where self-field effects are important. I, is defined as the measured critical current divided by the junction area.

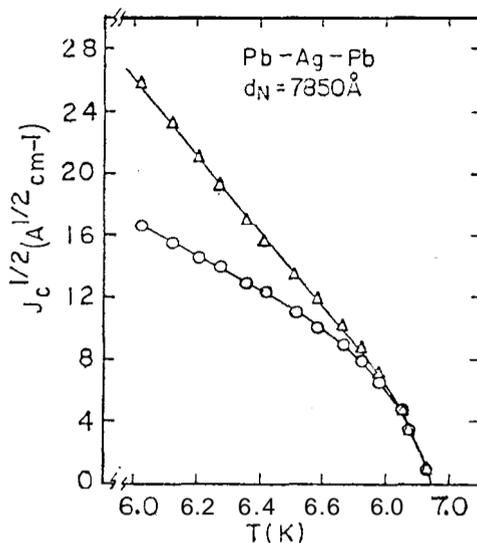


Fig. 6. $J_c^{1/2}$ as a function of temperature. The circles and triangles are as defined in Fig. 3.

For a two-dimensional junction with a symmetrical current input, the self-field effects become important when $\lambda_J \leq W/4$ ⁽¹⁷⁾. Fig. 5 shows the region of J_c which the self-field is important for several samples. In general terms, this is above 100 A/cm* for junction widths of about 6.5×10^{-3} cm and junction thicknesses of about 4500 \AA at 1.0K. For the data reported here, the true J_c 's can always be corrected when appropriate.

Fig. 6 shows the variation of $J_c^{1/2}$ as a function of temperature. The self-field correction data are linear up to $T/T_c \approx 0.97$ until the lead ceases to exhibit bulk behavior, after which J_c drops more rapidly. This linear behavior is consistent with the de Gennes theory which was derived originally for dirty limit but has been shown to apply as well for clean limit if proper modifications are made for energy dependent diffusivity⁽¹⁸⁾. The modified de Gennes theory shows that the behavior of J_c near T_c should be of the form

$$J_c \propto (1 - T/T_c)^2. \quad (10)$$

This prediction is actually observed in the experiments.

CONCLUSION

The general behavior of the Josephson supercurrent under magnetic fields been demonstrated in the work and the results are shown to be in reasonable agreement with the theories of the Josephson effect.

ACKNOWLEDGEMENT

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