

The Jovian Plasmasphere in Tilted Dipole Model

A. TAN

*Department of Mathematics and Physics
Newberry College
Newberry, South Carolina 29108
U.S.A.*

and

S. T. WU

*Department of Mechanical Engineering
University of Alabama
Huntsville, Alabama 35807
U.S.A.*

(Received 12 March, 1981)

A model of the plasmasphere of Jupiter in the tilted dipole approximation is presented. The distributions of electrons under the conditions of isothermal diffusive equilibrium and adiabatic diffusive equilibrium are obtained. In either case, the maximum electron density is found near the geographic equatorial plane rather than the dipole equatorial plane. The surface on which gravitational and centrifugal forces balance is also oriented around the rotational axis rather than the dipole axis.

1. INTRODUCTION

THE planet Jupiter is known to have an extensive magnetosphere and hence a plasmasphere akin to that of the earth. But the nature of distribution of thermal plasma in the magnetosphere is still a matter of conjecture. Gledhill (1967)⁽¹⁾ calculated the distribution of ions in an isothermal Jovian magnetosphere under diffusive equilibrium. Mendis and Axford (1974)⁽²⁾ have done the same for the adiabatic case. Both models have taken into account the gravitational and centrifugal forces by assuming corotation of the magnetosphere with the planet. Both models further assume that the rotational and dipole axes coincide. In actuality, however, the dipole axis inclined to the axis of rotation by an angle of nearly 10° . In this article, the effects of this tilt in the models of Gledhill (1967) and Mendis and Axford (1974) are investigated. The effect of ambipolar diffusion is also included.

2. MODEL

In this paper, the various notations have their meanings as follows.

N -number density of ions or electrons,

P -pressure,

T -temperature,

K -Boltzmann's constant,

(1) Gledhill, J. A., Magnetosphere of Jupiter, *Nature*, **214**, 155-156, 1967.

(2) Mendis, D.A. and W. I. Axford, Satellites and magnetospheres of the outer planets, *Annual Review of Earth and Planetary Sciences*, **2**, 419-475, 1974.

m -mass of ions or electrons,
 μ =reduced mass of proton and electron,
 $\rho=\mu N$ =density,
 s -arc length along dipole field lines,
 g_s =component of acceleration due to gravity resolved along s ,
 f_s =component of centrifugal acceleration resolved along s ,
 E_s -electric field arising out of charge separation,
 θ -dipole colatitude,
 ϕ -dipole longitude,
 Q -System III colatitude,
 Φ =System III longitude,
 Θ_N =System III colatitude of the magnetic north pole,
 Φ_N =System III longitude of the magnetic north pole,
 G -universal gravitational constant,
 M -mass of Jupiter,
 r_e -equatorial value of the radial distance,
 Ω =angular velocity of Jupiter, and
 L =McIlwain's shell parameter.

The suffix i denotes ions whereas the suffix e denotes electrons. In this model, the following assumptions are made.

- (1) In the Jovian magnetosphere, the chief ion is H^+ (Cf. Tan and Capone, 1976)⁽³⁾.
- (2) Electrons and ions obey the perfect gas laws $P_{i,e} = N_{i,e} K T_{i,e}$.
- (3) The production, loss and diffusion of ions and electrons are supposed to be negligible so that diffusive equilibrium conditions hold.
- (4) The charge-neutrality condition holds so that $N_i = N_e = N$. Also, $T_i = T_e = T$ and $P_i = P_e = P$.
- (5) The plasmasphere is assumed to corotate with the planet.
- (6) The magnetic field of Jupiter is dipolar.
- (7) Ions and electrons are constrained to move along the dipole field lines; and
- (8) The electronic mass is negligible compared with the ionic mass, $m_e \ll m_i$.

The force equations for the protons and electrons resolved along s are given by

$$\frac{1}{N_i} \frac{dP_i}{ds} = m_i g_s + m_i f_s + e E_s, \quad (1)$$

$$\frac{1}{N_e} \frac{dP_e}{ds} = -e E_s. \quad (2)$$

Eliminating E_s between Eqs. (1) and (2), we get,

$$\frac{1}{\rho} \frac{dP}{ds} = g_s + f_s. \quad (3)$$

It is convenient to solve Eq. (3) in the spherical dipole coordinate system (r, θ, ϕ) . g_s and f_s can be expressed in terms of θ, ϕ and r_e only (Tan, 1979)⁽⁴⁾.

$$g_s = -\frac{2GM \cos \theta}{r_e^2 \sin^4 \theta (3 \cos^2 \theta + 1)^{1/2}}, \quad (4)$$

$$f_s = \frac{3\Omega^2 r_e \cos \theta \sin^2 \theta \{ \sin^2 \theta (1 - \sin^2 \Theta_N \cos^2 \phi) + \cos \theta \sin \theta \cos \Theta_N \sin \Theta_N \cos \phi \}}{(3 \cos^2 \theta + 1)^{1/2}}. \quad (5)$$

Eq. (3) can be integrated under isothermal and adiabatic conditions by employing the relation

- (3) Tan, A. and L. A. Capone, Diurnal variations of the Jovian ionosphere, *Publ. Astron. Soc. Japan*, 28, 155-161, 1916.
- (4) A. Tan, Model of mid- and low-latitude F-region ionosphere and the plasmasphere, Ph. D. Thesis, The University of Alabama in Huntsville, Huntsville, Alabama, 1979.

$$\frac{ds}{d\theta} = r_e \sin \theta (3 \cos^2 \theta + 1)^{1/2}. \quad (6)$$

Denoting quantities at the 'base' of the plasmasphere in the northern hemisphere by the subscript o , we finally arrive at, for an isothermal plasmasphere,

$$\begin{aligned} \text{N-N, } \exp [& A(\operatorname{cosec}^2 \theta - \operatorname{cosec}^2 \theta_o) + B(\sin^6 \theta - \sin^6 \theta_o) \\ & + C\{8(\sin^5 \theta \cos \theta - \sin^5 \theta_o \cos \theta_o) - 2(\sin^3 \theta \cos \theta - \sin^3 \theta_o \cos \theta_o) \\ & - 3(\sin \theta \cos \theta - \sin \theta_o \cos \theta_o) + 3(\theta - \theta_o)\}], \end{aligned} \quad (7)$$

where

$$A = \frac{\mu GM}{KT r_e}, \quad (8)$$

$$B = \frac{\mu \Omega^2 r_e^2}{2KT} (1 - \sin^2 \theta_N \cos^2 \phi), \quad (9)$$

and

$$C = \frac{\mu \Omega^2 r_e^2}{16KT} \cos \theta_N \sin \theta_N \cos \phi. \quad (10)$$

If the plasma behaves adiabatically, we can substitute $P = c\rho^\gamma$ (here $\gamma = 5/3$), differentiate Bq. (3) with the help of Eqs. (4), (5) and (6) and then eliminate c . After lengthy calculations, we obtain,

$$\begin{aligned} \text{N-N., } [& 1 + a\{b(\operatorname{cosec}^2 \theta - \operatorname{cosec}^2 \theta_o) + c(\sin^6 \theta - \sin^6 \theta_o) \\ & + d\{8(\sin^5 \theta \cos \theta - \sin^5 \theta_o \cos \theta_o) - 2(\sin^3 \theta \cos \theta - \sin^3 \theta_o \cos \theta_o) \\ & - 3(\sin \theta \cos \theta - \sin \theta_o \cos \theta_o) + 3(\theta - \theta_o)\}]^{3/2}, \end{aligned} \quad (11)$$

where

$$a = \frac{2\mu}{5KT_o}, \quad (12)$$

$$b = \frac{GM}{r_e}, \quad (13)$$

$$c = \frac{\Omega^2 r_e}{2} (1 - \sin^2 \theta_N \cos^2 \phi), \quad (14)$$

and

$$d = 3\Omega^2 r_e^2 \cos \theta_N \sin \theta_N \cos \phi. \quad (15)$$

Eqs. (7) and (11) give the distribution of electrons (and protons) in the Jovian plasmasphere under isothermal and adiabatic conditions.

If we equate the components of gravitational and centrifugal accelerations from Eqs. (4) and (5), we get,

$$\alpha \sin^3 \theta + \beta \sin^7 \theta \cos^2 \theta - r = 0, \quad (16)$$

where

$$\alpha = 1 - \sin^2 \theta_N \cos^2 \phi, \quad (17)$$

$$\beta = \cos \theta_N \sin \theta_N \cos \phi, \quad (18)$$

and

$$r = \frac{2GM}{3\Omega^2 r_e^3}. \quad (19)$$

Eq. (16) determines the surface on which the resultant force on the charged particles is zero. The same equation is also obtained by differentiating either of Eqs. (7) and (11) with respect to θ and setting the derivative equal to zero. Thus the points where the centrifugal and gravitational force components equal, also give the locations of a minimum of electron density along a field line.

If we neglect the inclination of the dipole axis to the rotational axis ($\theta_N = 0$), Eq. (7) reduces to

$$\text{N-N., } \exp \{A(\operatorname{cosec}^2 \theta - \operatorname{cosec}^2 \theta_o) + B'(\sin^6 \theta - \sin^6 \theta_o)\}, \quad (20)$$

where

$$B' = \frac{\mu \Omega^2 r_e^2}{2KT}. \quad (21)$$

Eq. (20) is the result of Gledhill (1967) provided μ is replaced by the mass of the ion and the reference level is set at the equator. Likewise, Eq. (11) reduces to

$$N = N_0 [1 + a \{b(\operatorname{cosec}^2 \theta - \operatorname{cosec}^2 \theta_0) + c'(\sin^6 \theta - \sin^6 \theta_0)\}]^{3/2}, \quad (22)$$

with

$$c' = \frac{\Omega^2 r_e}{2}. \quad (23)$$

Eq. (23) gives the result of Mendis and Axford (1974), again if μ is replaced by the mass of the ion and the reference level is at the equator.

Finally, Eq. (16) reduces to

$$\sin^8 \theta = \gamma, \quad (24)$$

the condition of Angerami and Thomas (1964)⁽⁵⁾ in terms of θ .

These results can be transformed into the System III spherical polar coordinates (r, θ, Φ) by the transformations

$$\cos \theta = \cos \theta \cos \theta_N - \sin \theta \sin \theta_N \cos \phi, \quad (25)$$

$$\sin(\Phi - \Phi_N) = \sin \theta \sin \phi \operatorname{cosec} \theta. \quad (26)$$

However, their forms become unmanageably complicated.

3. RESULTS AND CONCLUSIONS

The effect of the tilt of the dipole is greatest when the field line is in the System III meridional plane of the dipole axis. We shall illustrate the effects by considering two cases. In the first case, $\Phi = \Phi_N$. From Eqs. (25) and (26), we get, $\phi = 0^\circ$ and $\theta = \theta + \theta_N$. In the second case, $\Phi = \Phi_N - 180^\circ$. Then Eqs. (25) and (26) give $\phi = 180^\circ$, $\theta = \theta + \theta_N$.

Figs. 1 to 3 show the effects of the tilted dipole on the nature of distribution of electrons in the Jovian plasmasphere. The left halves of these figures correspond to our first case, while the right halves correspond to the latter case. The number densities are in cm^{-3} .

Fig. 1 shows the distribution of electrons in the Jovian plasmasphere under isothermal conditions. Note that the peak electron density for a field line with $L > 2$ is near the geographic equator rather than the dipole equator. This shows that the distribution of electrons depends strongly upon the rotation of the planet. In the figure the electron densities are normalized so that the maximum

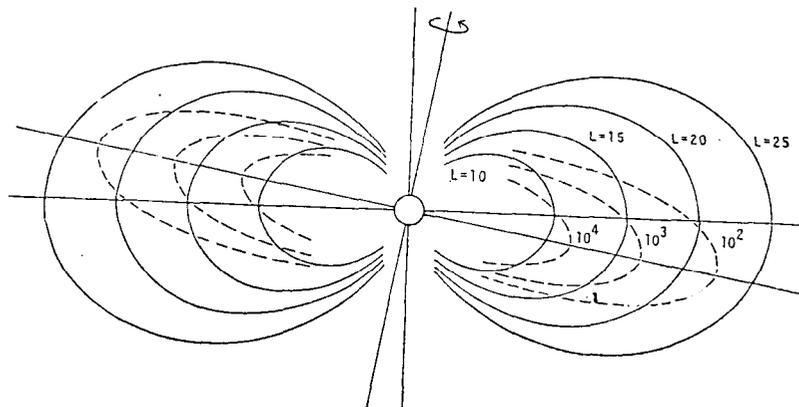


Fig. 1. Distribution of electrons in an isothermal Jovian plasmasphere under diffusive equilibrium.

(5) Angerami, J. J. and J. O. Thomas, Studies of planetary atmospheres, 1. The distribution of electrons and ions in the earth's exosphere, *J. Geophys. Res.*, 69, 4537-4560, 1963.

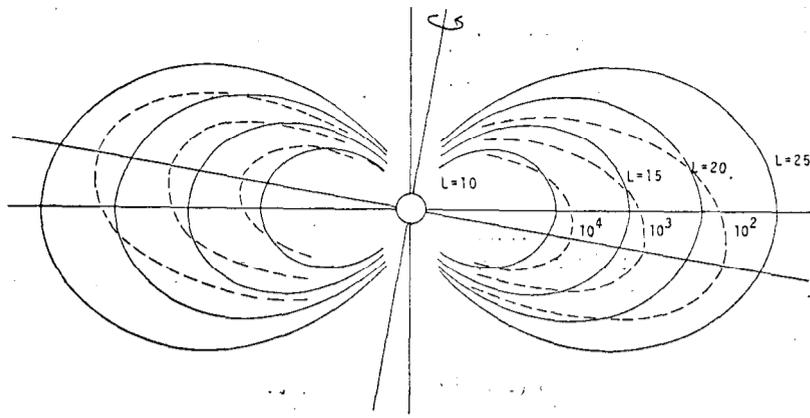


Fig. 2. Distribution of electrons in the Jovian plasmasphere under adiabatic and diffusive equilibrium conditions.

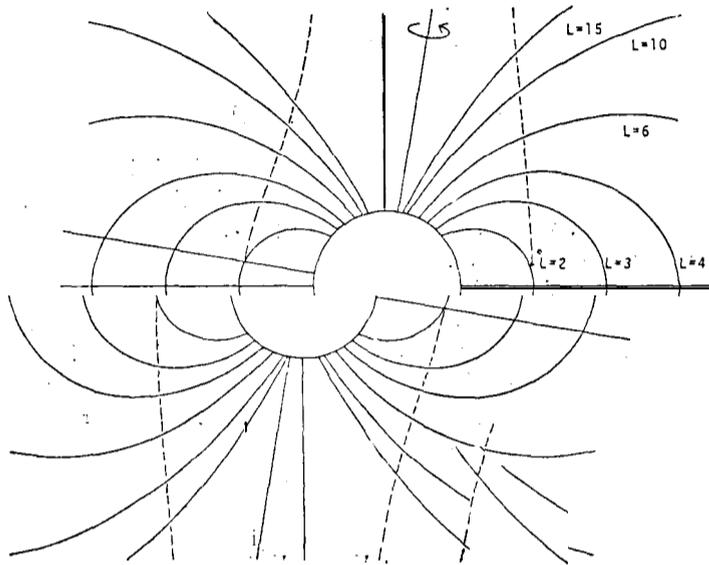


Fig. 3. Locus of points where the centrifugal and gravitational force components along the field lines equal.

electron density is determined by the condition of rotational instability (Cf. Gledhill, 1967; Mendis and Axford, 1974). The temperature of the isothermal plasma has been taken equal to 1000°K .

Fig. 2 shows the distribution of electrons under adiabatic conditions. Here again, the distribution shows a maximum electron density near the equatorial plane. The distribution is however, less 'disk'-shaped than in the isothermal case.

In Fig. 3, the dashed lines mark the positions where the centrifugal and gravitational force components balance each other. It is here that the minimum of density occurs for both- the isothermal and adiabatic conditions. The surface of zero force is oriented around the rotational axis rather than the dipole axis.

In conclusion, the distribution of thermal electrons in the Jovian plasmasphere is strongly influenced by the rotation of the planet.