

Temperature-Dependent Anisotropy of Thermal Resistivity of Zinc and Cadmium

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The anisotropy parameter β ($\equiv W_{\parallel}/W_{\perp}$) in phononlimited thermal resistivity of hexagonal metals at low temperature is estimated numerically from a very simple formula. It is shown that the thermal resistivity and electrical resistivity for these metals have a very different temperature dependence. As a result of this, the anisotropy parameter β for both Zn and Cd are found to decrease as temperature decreases, in agreement with experiment.

I. INTRODUCTION

EXPERIMENTAL measurements of the thermal resistivity of hexagonal metal single crystals show peculiar phenomena for both zinc⁽¹⁾ and cadmium⁽²⁾ in the low temperature region. As shown in Fig. 1, the curves for the anisotropy parameter β appear to decrease as temperature decreases. These are exactly the opposite behavior for the electrical resistivity anisotropy of both metals. It has been shown^(3,4) that the distortion of the Fermi surface from a sphere is one important factor in the calculation of the anisotropic electrical resistivity of normal hexagonal metals. We attempt to show here that the same reason can also be applied to explain the anisotropic thermal resistivity of zinc and cadmium.

In Sec. II, we derive the formula we have used for the phonon-limited thermal resistivity anisotropy of hexagonal metals. Also, we give the results of our numerical computation for zinc, cadmium and magnesium. In Sec. III, we give some conclusions and discussions.

II. GENERAL CONSIDERATIONS AND CALCULATIONS

By the generalized relaxation time approximation⁽⁵⁾, the phonon-limited thermal resistivity of normal metals along a particular direction p can be written as

$$W_{pp} = \frac{6}{\pi^2 k_B^2 T X_{pp}} \sum_j \iint_{\text{F.S.}} |g_j(\mathbf{k}, \mathbf{k}')|^2 F(x) \left[\left(1 + \frac{x^2}{\pi^2}\right) \left(1 - \frac{v'_p}{v_p}\right) + \frac{3}{2} \left(\frac{x}{\pi}\right)^2 \left(\frac{v'_p}{v_p}\right) \right] \frac{dS_{\mathbf{k}'}}{v'} \frac{dS_{\mathbf{k}}}{v} \quad (1)$$

where $g_j(\mathbf{k}, \mathbf{k}')$ is the matrix element for the scattering of an electron from \mathbf{k} to \mathbf{k}' through the absorption or emission of a phonon with wave vector \mathbf{q} , polarization vector $\mathbf{e}_{\mathbf{q}j}$ and energy $\hbar\omega_{\mathbf{q}j}$. χ is the velocity factor defined by

$$\chi = \int_{\text{F.S.}} (\mathbf{v}\mathbf{v}/v) dS_{\mathbf{k}'}/S_{\mathbf{k}} \quad (2)$$

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The factor $F(x)$ with $x = \hbar\omega_{qj}/k_B T$ results from the integral over the energy variables⁽⁵⁾ and is given by

$$F(x) = x/(e^x - 1) \quad (1 - e^{-x}) \quad (3)$$

According to the analysis given by Lawrence and Wilkins⁽⁶⁾, the thermal resistivity W_i due to only one scattering process corresponding to the reciprocal lattice vector G_i can be written as

$$W_i = \begin{cases} a_i(T/\theta_{Di})^3 & \theta_{Di} > T > \theta_i \\ b_i(T/\theta_i)^2 & \theta_i > T \end{cases} \quad (4)$$

where θ_{Di} is the Debye temperature in the G_i direction,

$$\theta_i \equiv \theta_{Di} [V(G_i)/\epsilon_f] \quad (5)$$

is the characteristic temperature determined by the distortion parameter $V(G_i)$ of the Fermi surface,

$$a_i = G_i^3 [V'(G_i)]^2 / q_D \theta_{Di}^2 \quad (6)$$

and

$$b_i = a_i (\theta_i / \theta_{Di})^3 \quad (7)$$

The third power law in temperature is used here for W_i because Umklapp scatterings⁽⁷⁾ are dominant in the mediate to high temperature region and $G_i V'(G_i) \gg V(G_i)$ for all three metals Zn, Cd and Mg⁽⁸⁾.

For hexagonal divalent metals, there are three reciprocal lattice vectors intersecting with the Fermi sphere. After summing up contributions from all different scattering processes by tensor addition and taking approximately the derivative of the form factor $V'(G_1) = V'(G_2) = V'(G_3)$, one obtains the anisotropy parameter β which is defined as the ratio of thermal resistivity in the parallel and perpendicular direction ($W_{\parallel} / W_{\perp}$)

$$\beta = \begin{cases} \frac{(8/3)k_1 r_1^3 + 12k_3 r_3^3 \cos^2 \theta_c}{1 + 6k_3 r_3^3 \sin^2 \theta_c} & \theta_{D2} > T > \theta_3 \\ \frac{(8/3)k_1 r_1^3 (T/\theta_3) + 12k_3 r_3^3 \cos^2 \theta_c}{(T/\theta_3) + 6k_3 r_3^3 \sin^2 \theta_c} & \theta_3 > T > \theta_1 \\ \frac{(8/3)k_1 r_1^3 (\theta_1/\theta_3) + 12k_3 r_3^3 \cos^2 \theta_c}{(T/\theta_3) + 6k_3 r_3^3 \sin^2 \theta_c} & \theta_1 > T > \theta_2 \end{cases} \quad (8)$$

where $k_i = (G_i/G_2)^2$, $r_i = \theta_{D2}/\theta_{Di}$ and θ_c is the angle between slanting reciprocal lattice vector G_3 and the c-axis. The same data are used here in the calculation as given in table 1 in Ref. 4.

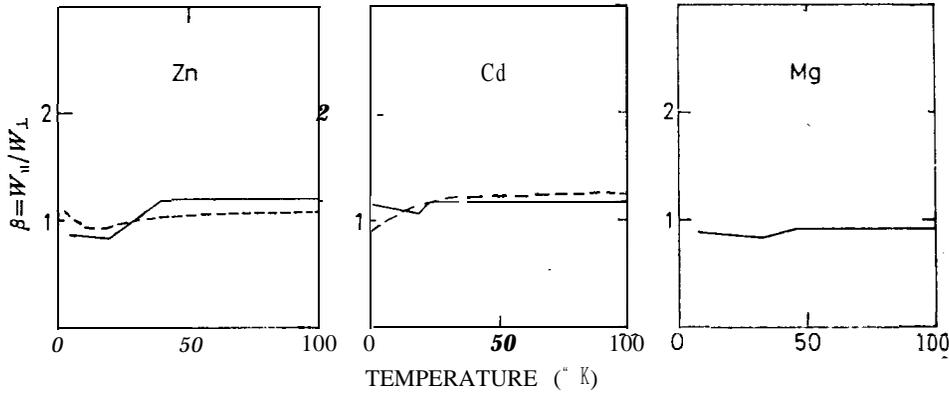


Fig. 1. Temperature dependence of the thermal resistivity anisotropy β of Zn, Cd and Mg: broken curve, experimental results from Ref. 1 and 2; full curve, theoretical results.

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Fig. 1 show our results for zinc, cadmium and magnesium together with experimental points for single crystals of Zn and Cd. Since we have no parameters adjusted on the experimental values for the thermal resistivity anisotropy β , the over all agreement can be regarded as reasonable good. The theoretical curve for Cd are found to be disagreed with experimental result at very low temperature. This may due to the non-Debye character of the lattice specific heat of Cd at very low temperature^(2,9).

III. DISCUSSION AND CONCLUSION

This calculation shows that the distortions of the Fermi surface are still one of the main cause for the temperature-dependent behavior of the thermal resistivity anisotropy of hexagonal metals. The decrease of the power law in temperature as shown in Eq. 4 is responsible for the decrease of the anisotropic parameter β as temperature decreases. In Fig. 1, we also show our result for Mg. As a check to our explanation, we suggest experimental measurement should be made on Mg too.

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