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Pionic Decay of Radially Excited Mesons*

T. Y. LEE (李曾適) and C. T. CHEN-TSAI (陳蔡鏡堂)

Physics Department, Tsing Hua University, Hsinchu, Taiwan

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The pionic decay widths of radially excited states of π, ρ, and A_1 are estimated. It is pointed out that mesons with mass larger than 2 GeV will be extremely difficult to identify in experiment, because of a sharp rise. in width as quantum excitations increase.

I. INTRODUCTION

RECENT successive discoveries of ψ mesons of mass greater than 3 GeV[1] make it of great interest to study the high quantum-excitation mesons composed of light quarks (i.e. the Gell-Mann-Zweig quarks, constract to the heavy quark, i.e., the charmed quark, in ψ mesons). From our previous study of meson spectroscopy,[2] it is found that the mass spectrum for highly excited mesons tends to be degenerated into harmonic oscillator spectrum, i. e. $m \simeq \Delta + N\omega$, with A=1023 MeV, ω=h. o. frequency=315 MeV, and N=number of quantum excitation. Table I shows the predicted mass values of the radially excited states of π, ρ, A, and B mesons. In this paper we shall estimate the pionic decay rates of these radially excited mesons so as to see whether the identification of these mesons from the ever intensified e-e+ scattering experiments, from which ψ mesons as well as ρ' (1600) have been discovered, can be facilitated.

Table I. Mass of the radially excited states of π, ρ, A_1, and B in MeV.

$n^\sigma L_J$	$n=0$	1	2	3	4	5	6
n^1S_0	140(π)	1340	2040	2710	3370	4030	4680
n^3S_1	770(ρ)	1520	2180	2820	3470	4100	4740
n^1P_1	1235(B)	1820	2440	3080	3730	4370	5020
n^3P_1	1100(A_1)	1800	2450	3100	3740	4390	5040

II. DECAY WIDTH FORMULAE

Consider the decay of a meson of mass M at the h. o. state $n^\sigma L_J$ (n=principal quantum number, $\sigma=2S+1$, S=quark spin, L=angular momentum, J=meson spin) decaying into a meson of mass M' at the h. o. state $n'^{\sigma'}L'_J$ and a π meson of mass μ, i.e.,

$$n^\sigma L_J \to n'^{\sigma'} L'_{J'} + \pi. \tag{1}$$

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(1) J. J. Aubert et. *al.*, Phys. Rev. Letters 33, 1404 (1974); J. E. Augustin et. *al.*, Phys. Rev. Letters 33, 1406 (1974); G.S. Abram *et. al.*, Phys. Rev. Letters 33, 1453 (1974).

(2) C.T. Chen-Tsai and T.Y. Lee, Phys. Rev. D 10, *2960 (1974);* ibid. Chinese J. Phys. 12, 98 (1974).

Following Dalitz[3], the decay amplitude is given by

$$A=\frac{f_q}{\mu}\frac{1}{2\sqrt{\omega}}\langle n'^{\sigma'}L'_{J'}|\sigma_q\cdot k e^{\frac{i}{2}k\cdot r_q}-\sigma_{\bar{q}}\cdot k e^{-\frac{i}{2}k\cdot r_{\bar{q}}}|n^{\sigma}L_J\rangle, \tag{2}$$

where f_q is the coupling constant of decay $q\to q+\pi$ ($-f_q$ for $\bar{q}\to\bar{q}+\pi$); k is the c. m. momentum; $\omega^2=k^2+\mu^2$.

The decay width can then be written

$$\Gamma=\frac{1}{6\pi}\frac{f_q}{\mu^2}k^3\frac{M-\omega}{M}\sum_{\ell=1}^{\infty}\sum_{j=|\ell-1|}^{\ell+1}|A_{\ell j}|^2, \tag{3}$$

where

$$A_{\ell j}=i^{\ell}\sqrt{(2\ell+1)(2J+1)(2j+1)}\begin{Bmatrix}S' & L' & J'\\ S & L & J\\ 1 & \ell & j\end{Bmatrix}\langle S'\|\sigma_q-(-)^{\ell}\sigma_{\bar{q}}\|S\rangle\cdot$$

$$\cdot\ \langle L'\|\vec{C}_{\ell}\|L\rangle\langle n'L'|j_{\ell}(kr/2)|nL\rangle, \tag{4}$$

where, with conventionally used symbols[4],

$$\langle S'\|\sigma_q\|S\rangle=(-)^{S+S'}\langle S'\|\sigma_{\bar{b}}\|S\rangle=(-)^{S}\sqrt{6(2S+1)(2S'+1)}\begin{Bmatrix}1/2 & S' & 1/2\\ S & 1/2 & 1\end{Bmatrix}, \tag{5}$$

$$\langle L'\|C_{\ell}\|L\rangle=(-)^{L'}\sqrt{(2L+1)(2L'+1)}\begin{pmatrix}L' & \ell & L\\ 0 & 0 & 0\end{pmatrix}, \tag{2}$$

$$\langle n'L'|j_{\ell}(kr/2)|nL\rangle=\int_0^{\infty}R_{n'L'}(r)R_{nL}(r)j_{\ell}(kr/2)r^2dr. \tag{7}$$

One notices that j is just the angular momentum carried out by the emitted π meson by (1), whereas ℓ is that of the π meson by $q\to q+\pi$.

For the decays of the radially excited states of π and ρ, i.e.,

$$n^{\sigma}S_J\to n'^{\sigma'}L'_{J'}+\pi, \tag{8}$$

Eq. (3) is reduced to

$$\Gamma(n^{\sigma}S_J\to n'^{\pi'}L'_{J'}+\pi)$$
$$=\frac{2}{3\pi}\frac{f_q^2}{\mu^2}\frac{M-\omega}{M}k^3(2J'+1)B(SS'L')|\langle n'L'|j_{L'}(kr/2)|nO\rangle|^2, \tag{9}$$

where we have following selection rules,

$$B(01L')=B(10L')=\begin{cases}1 \text{ for even } L',\\ 0 \text{ for odd } L',\end{cases} \tag{10}$$

$$B(00L')=0, \tag{11}$$

$$B(11L')=\begin{cases}2/3 \text{ for odd } L',\\ 0 \text{ for even } L'.\end{cases} \tag{12}$$

Thus the radially excited states of ρ can only decay into ^3L_J state mesons with $L=1, 3, 5, \cdots$, and ^1L_J state mesons with $L=0, 2, 4, \cdots$, whereas those of π can decay only into ^3L_J state mesons with $L=0, 2, 4, \cdots$.

(3) R.H. Dalitz, *Proceedings of* ***the second*** *Hawaii Topical Conference* ***in Particle Physics,*** Hawaii, U.S. A. (1967).

(4) .A. de-Shalit and I. Talmi, ***Nuclear Shell Theory,*** (Academic Press, New York, 1963).

Furthermore, approximating the radial integral in (9) by

$$\langle n'L'|j_{L'}(kr/2)|n0\rangle=(k\beta/2)^{L'}\frac{1}{(2L'+1)!!}\frac{L'!}{(n-n')!(n'+L'-n)}\sqrt{\frac{n!\Gamma(n'+L'+3/2)}{n!\Gamma(n+3/2)}}\,, \quad (13)$$

where $\beta=\sqrt{2/\omega M_q}$, M_q=quark mass, we then require

$$n'+L'\geq n, \quad (14)$$

which, together with $2n\geq 2n'+L'$, gives

$$2(n-n')\geq L'\geq n-n'. \quad (15)$$

Finally, we obtain from (9)

$$\Gamma(n^\sigma S_J\rightarrow n'^{\sigma'}L'_{J'}+\pi)=C(SS'L')\frac{2L'+1}{1.25}\left(1+\frac{M'^2}{M^2}\right)\left(\frac{M^2-M'^2}{2M}\right)^{2L'+3}\left(\frac{\omega}{2M_q}\right)^{L'}\Gamma(\rho\rightarrow 2\pi), \quad (16)$$

where

$$C(SS'L')=\begin{cases}1 \text{ for } S=1, S'=0, L'=\text{even},\\ 2 \text{ for } S=1, S'=1, L'=\text{odd},\\ 3 \text{ for } S=0, S'=1, L'=\text{even},\end{cases} \quad (17)$$

and

$$\Gamma(\rho\rightarrow 2\pi)=\frac{1}{3\pi}\frac{f_q}{\mu^2}k_\rho^3\times 0.83 = 145 \text{ MeV}. \quad (18)$$

The appearence of the numerical factor 0.83 comes from the due correction taken by the fact that the decayed π meson in ρ decay is not in the pure o^1S_0 state, as discussed in Ref. 2.

The total pionic decay width, Γ_T, which is of our primary interest, is then given by

$$\Gamma_T(n^\sigma S_J)=\sum_{S'L'}\Gamma(n^\sigma S_J\rightarrow n'^{\sigma'}L'_{J'}+\pi), \quad (19)$$

with the summation for L' confined in the region given by (15).

The Eq. (16) can be applied equally well to the decays,

$$n'^{\sigma'}L'_{J'}\rightarrow n^\sigma S_J+\pi, \quad (20)$$

In this case, however, the condition (15) should be replaced by

$$L'\geq 2(n-n'), \qquad n\geq n'. \quad (21)$$

For $L'=1$, i.e. the decays of the radially excited states of A_1, we have $n=n'$. The decay of B vanishes because of (17).

III. NUMERICAL RESULTS AND DISCUSSIONS

The evaluated total pionic decay widths of the radially excited states of π, ρ, and A_1 are shown in Table II. First, we observe that the decay width of $\rho'(1600)\rightarrow 2\pi$ is estimated to be about 29 MeV, i.e. the branching ratio with respect to the dominant decay mode $\rho'\rightarrow 4\pi$ is around 7%. Experimentally $\rho'\rightarrow K\bar{K}$ mode is found to be less than 8%, hence our estimation for $\rho'\rightarrow 2\pi$ is quite reasonable.

The decay widths of the radially excited states of mass around the ψ masses are somewhere one order of magnitude larger than that of $\rho\rightarrow 2\pi$. As in $\rho'\rightarrow 2\pi$, if we take the branching ratio, to be

Table It. Pionic decay widths l'_T of the radially excited states of π, ρ, and A_1 in unit of the decay width of $\rho \to 2\pi$.

$n^{\sigma} L_J$	$n=0$	1	2	3	4	5	6
n^1S_0	—	—	0.1	2	8	88	790
n^3S_1	1	0.2	0.4	3	26	160	1330
n^3P_1	1	2	4	8	13	21	29

about 10%, we would expect the total decay widths would be two order of magnitude larger than that of $\rho \to 2\pi$. In fact, from Table II, one would expect a strong overlapping in levels widths for states with quantum excitation $N>2$ (i. e. mass greater than 2 GeV). For example, the identification of the second radially excited ρ would be extremely difficult in the e^+e^- scattering experiment.