

Fractional Brownian Motion and Blackbody Radiation

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Brownian motion plays a dominant role in the atomic theory of matter and in the atomic theory of electricity. In deriving the law of distribution of energy in the spectrum of blackbody radiation, Planck used a system of cavity oscillators. Hence, it is fitting to investigate if a similar formula can be derived via Brownian motion or a generalization of Brownian motion. In this paper, an alternative radiation formula based on fractional Brownian motion (FBM) is derived. It is comparable to Planck's radiation formula. The only properties that are required in our derivation are the self-similarity and stationary increments of FBM. Consequently, the basic theories of matter, electricity, and radiation are all unified under the same notion of FBM.

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I. INTRODUCTION

Thermal radiation has been a subject of immense interest since the time of Planck [1–3]. It has also appeared in the 3 K cosmic microwave background radiation [4]. Recently, the thermal radiation from thin film structures and photonic crystals have again attracted much attention [5–7]. According to Planck, experimental observation of cavity radiation has shown that the radiation originates from Hertzian resonators. In the radiating matter, standing electromagnetic waves are produced in the cavity. Because the number of resonators is so vast, the standing electromagnetic waves possess a very broad range of frequencies and energies. To agree with observation, Planck assumed that each oscillator would radiate discrete energy that was proportional to its frequency. He then obtained the famous Planck radiation formula. The theory of thermal radiation based on Planck's formula is usually called the quantum theory of radiation.

Brownian motion, on the other hand, was discovered by Robert Brown of Scotland when he noticed the irregular movement of pollen grains suspended in water around 1827 [8]. It was Einstein in 1905 who gave a quantitative description to this movement [9]. In the work of Robert Millikan, he stated that the theoretical work on Brownian motion had convinced Wilhelm Ostwald of the fundamental reality of the atomic hypothesis [10]. Later on, Brownian motion was used by him and Fletcher in the determination of the elementary electrical charge [11, 12]. Recently, it has been applied to city traffic in rush hour and solitons in proteins [13, 14]. However, a rigorous mathematical construction of Brownian movement was not proposed until the 1930s by Norbert Wiener [15]. In his derivation, the increments of Brownian motion were stationary and independent. As to practical applica-