

Schwinger Method and Path Integral with Generalized Canonical Transformation for a Harmonic Oscillator with Time-Dependent Mass and Frequency

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(Received November 12, 2008)

The exact propagator for a harmonic oscillator with time-dependent mass and frequency is found by the Schwinger method and a path integral with a generalized canonical transformation. In the Schwinger formalism, the propagator can be obtained by basic operator algebra and elementary integrations. In the path integral method, it can be shown that such a propagator can be derived from that for a unit mass and frequency oscillator in a new space-time coordinate system with the help of a generalized canonical transformation. The power of propagator methods for solving time-dependent Hamiltonian systems is also discussed.

PACS numbers: 03.65.Ge

I. INTRODUCTION

In recent years, the study of how to find quantum solutions for time-dependent Hamiltonian systems has become quite interesting [1-6]. Pepore *et al.* [7] applied the path integral method to calculate the propagator for a harmonic oscillator with a time-dependent mass and frequency, and then expanded the obtained propagator to derive the time-dependent wave function. The various applications in many areas of physics, such as quantum optics, [8] cosmology, [9] and nanotechnology [10] are the main reasons for these intensive studies in the physics literature. The aim of this paper is to derive the propagator for a harmonic oscillator with time-dependent mass and frequency by two methods. The first method is the Schwinger method [11], while the second method is the path integral with a generalized canonical transformation [12].

Recently, there has been a report on the calculating of non-relativistic propagators by the Schwinger method [13-17]. Aragao *et al.* [13] applied this method to derive the propagator for a charged particle in a uniform magnetic field in 2007. In 2003, Barone *et al.* [14] evaluated the propagator for a simple harmonic oscillator and compared it with the path integral method. In this paper we will generalize their results [13-17] to the case of time-dependent Hamiltonian systems.

Quantum canonical transformations [18-20] that include both point transformations and time evolution are adequate applications for transforming a system of interest into a