

Invariance of the Canonical Quantization Prescription Under Classical Canonical Transformations

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Dirac's postulate of canonical quantization, $[\hat{p}_i, \hat{q}_j] = -i\hbar\delta_{ij}$ for conjugate canonical variables, has been the most concise and general prescription on how to quantize a classical system. Since classical systems described by variables connected with canonical transformations are equivalent, $[\hat{p}_i, \hat{q}_j] = -i\hbar\delta_{ij}$ must remain invariant under classical canonical transformations. This invariance has not been proved except for the limited class of cascaded infinitesimal transformations. In this paper it is shown that if (\hat{P}_i, \hat{Q}_j) are related to (\hat{p}_i, \hat{q}_j) by a classical canonical transformation, then $[\hat{p}_i, \hat{q}_j] = -i\hbar\delta_{ij}$ implies $[\hat{P}_i, \hat{Q}_j] = -i\hbar\delta_{ij}$. In other words, the canonical quantization prescription is invariant for variables connected with classical canonical transformations.

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I. INTRODUCTION

In classical mechanics, canonical transformations are the transformations $(p_i, q_j) \rightarrow (P_i, Q_j)$ which preserve the Poisson brackets; $\{p_i, q_j\} = -\delta_{ij} \Rightarrow \{P_i, Q_j\} = -\delta_{ij}$. By noting the analogy between Poisson bracket and commutator, Dirac postulated that classical systems can be quantized by replacing the Poisson brackets $\{p_i, q_j\} = -\delta_{ij}$ with the commutation relations $[\hat{p}_i, \hat{q}_j] = -i\hbar\delta_{ij}$. This procedure, known as the canonical quantization, has been the most concise and general prescription for quantizing a classical system [1]. Since classical systems described by variables connected with canonical transformations are equivalent, it would be interesting to ask: Does the canonical quantization procedure preserves this equivalence?

It has been shown for classical canonical transformations that are infinitesimally close to the identity transformation, the corresponding transformations in quantum mechanics are unitary [2]. These transformations can be cascaded to form transformation groups. Because unitary transformations $\hat{P}_i = \hat{U}\hat{p}_i\hat{U}^{-1}$, $\hat{Q}_j = \hat{U}\hat{q}_j\hat{U}^{-1}$ preserve the commutation relations, for such transformation groups the invariance of the canonical quantization procedure is obvious. However, not all classical canonical transformations have corresponding unitary transformations. Many important transformations, such as transformation to the polar or spherical coordinates, or transformation to the action-angle variables, do not belong to this category [2]. Therefore, in the discussion of canonical quantization, one may not wish to be limited to canonical transformations that have corresponding unitary transformations