New Localized Coherent Structures to the Dispersive Long-Wave Equation in (2+1)-Dimensional Space

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By means of the variable separation approach, a general variable separation solution of the (2+1)-dimensional dispersive long-wave equation is derived. Because of the existence of the arbitrary functions in the general solution, the multivalued functions can be used to construct a type of localized excitation, folded solitary waves (FSWs) and foldons. Two classes of novel localized coherent structures, like both the FSWs-antiFSWs and foldons-antifoldons solutions, are found by selecting appropriate functions. These new structures exhibit some novel interaction features.

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I. INTRODUCTION

In nonlinear science, soliton theory plays an essential role and has been applied in almost all the natural sciences, especially in all the physics branches such as condensed matter physics, field theory, fluid dynamics, plasma physics, optics, etc. [1]. Most of the previous studies on soliton theory, especially in high dimensions, are restricted to the single valued situations, such as the dromion, compacton, peakon and their interactions. However, in various cases, real natural phenomena are too intricate to be described only by virtue of single valued functions. For instance, in nature, there exist very complicated folded phenomena such as folded proteins [2], folded brain and skin surface, and many other kinds of folded biological systems [3]. The simplest multi-valued (folded) waves may be the bubbles on (or under) a fluid surface. Various ocean waves are really folded waves too. In Refs. [4–6], the authors discussed some simpler cases of folded natural phenomena. In this paper, we will be interested in the study of the explicit formulas of the FSWs-antiFSWs and foldons-antifoldons. In the meantime we also make a detailed analysis of the interaction between the FSWs and antiFSWs, and between the foldons and antifoldons, respectively, by taking the (2+1)-dimensional dispersive long-wave equation (2DDLWE) as an example:

$$u_{yt} + \eta_{xx} + u_x u_y + u u_{xy} = 0\,, \tag{1}$$

$$\eta_t + u_x + \eta u_x + u \eta_x + u_{xxy} = 0\,. \tag{2}$$

Eqs. (1) and (2) were first obtained by Boiti *et al.* [7] as compatibility conditions for a "weak" Lax pair. In Ref. [8] Paquin and Winternitz showed that the symmetry algebra of

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