

## The Variances in a Single-Mode Superposed Squeezed State

A. S. Daoud

*Department of Mathematics, Faculty of Sciences, University of Zagazig, Zagazig, Egypt*  
(Received November 4, 2003)

We find explicit expressions for the variances in a single-mode superposed squeezed state as a function of a certain parameter. Formally, the parameter is given as a function of the mean free path (which can be measured experimentally) of the atoms of a material medium through which the natural light passes. Physically, this parameter is just the mean number of photons present in the coherent state of the mode under investigation. Some interesting new results follow as a consequence of using the P-representation of the density operator for the squeezed state of the mode.

PACS numbers: 42.50.Ar, 42.50.Ct

### I. INTRODUCTION

The calculation of the variances in a quantum state leads to the determination of the total noise of that state. The knowledge of the noise level of a state is essential to estimate the value of such a state in practice. According to Schumaker [1], the variances in a single-mode state are defined as the mean-square uncertainties in the real and imaginary parts of the annihilation operator of the mode. Hence the total noise of the state is given by the sum of the variances in that state.

In a beam of squeezed light, the quantum noise level could be reduced below the zero-point fluctuations that minimize the uncertainty product given by Heisenberg's uncertainty relation. The quantum fluctuations in a coherent state are equal to the zero-point fluctuations and are randomly distributed in phase. Even an ideal laser operating in a pure coherent state would still possess quantum noise, due to zero-point fluctuations [2]. In optical communication systems which use a coherent beam of laser light propagating in optical fibers, the ultimate limit to the noise is given by the quantum noise or zero-point fluctuations. If, instead, a beam of squeezed light were used to transmit information in the quadrature phase that had reduced fluctuations, the quantum noise level could be reduced below the zero-point fluctuations. Yuen and Shapiro [3, 4] propose optical communication systems based on light signals with phase sensitive quantum noise.

Indeed, when the density operator for a squeezed pure state represents the state of the mode, the variances are still functions of the squeeze factor, which varies from zero to infinity. In addition, the variances in such states are independent of the field amplitude of the coherent state. As well, the product of the uncertainties is still equal to the minimum allowed value, as in the coherent state. On the other hand, according to Schumaker [1], the sum of the variances in a state must be equal to the number of photons present in the